1a. Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$.

Write down the 10th term of the sequence. Do not simplify your answer.

**Markscheme**

\[ u_{10} = 3(0.9)^9 \quad A1 \quad N1 \]

[1 mark]

1b. Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$.

Find the sum of the infinite sequence.

**Markscheme**

recognizing $r = 0.9$ \((AI)\)

correct substitution \( A1 \)

e.g. \( S = \frac{3}{1-0.9} \)

\( S = \frac{3}{0.1} \quad (AI) \)

\( S = 30 \quad A1 \quad N3 \)

[4 marks]

Let \( f(x) = \log_3 \sqrt{x} \), for \( x > 0 \).

2a. Show that \( f^{-1}(x) = 3^{2x} \).

**Markscheme**

interchanging \( x \) and \( y \) (seen anywhere) \((MI)\)

e.g. \( x = \log_3 y \) (accept any base)

evidence of correct manipulation \( A1 \)

e.g. \( 3^x = \sqrt{y} \), \( 3^y = x^\frac{1}{2} \), \( x = \frac{1}{2} \log_3 y \), \( 2y = \log_3 x \)

\( f^{-1}(x) = 3^{2x} \quad AG \quad N0 \)

[2 marks]

Let \( f(x) = \log_3 \sqrt{x} \), for \( x > 0 \).

2b. Write down the range of \( f^{-1} \).

**Markscheme**

\( y > 0 \), \( f^{-1}(x) > 0 \) \( A1 \quad N1 \)

[1 mark]
3a. Find the value of $r$. 

**Markscheme**

- Evidence of substituting into formula for $n$th term of GP (M1)
- e.g. $u_4 = \frac{1}{81}$
- Setting up correct equation $\frac{1}{81}r^3 = \frac{1}{3}$ (A1)
- $r = 3$ (A1 N2) [3 marks]

3b. Find the smallest value of $n$ for which $S_n > 40$.

**Markscheme**

**METHOD 1**

- Setting up an inequality (accept an equation) (M1)
- e.g. $\frac{1}{2}(r^n - 1) > 40$, $\frac{1}{2}(1 - r^n) > 40$, $r^n > 6481$ (M1)
- Evidence of solving (M1)
- e.g. graph, taking logs
- $n > 7.9888\ldots$ (A1)
- $\therefore n = 8$ (A1 N2) [4 marks]

**METHOD 2**

- If $n = 7$, sum $= 13.49\ldots$; if $n = 8$, sum $= 40.49\ldots$ (A2)
- $n = 8$ (is the smallest value) (A2 N2) [4 marks]

4a. The first three terms of an arithmetic sequence are $5, 6.7, 8.4$.

Find the common difference.

**Markscheme**

- Valid method (M1)
- e.g. Subtracting terms, using sequence formula
- $d = 1.7$ (A1 N2) [2 marks]

4b. The first three terms of an arithmetic sequence are $5, 6.7, 8.4$.

Find the 28th term of the sequence.
4c. The first three terms of an arithmetic sequence are 5, 6.7, 8.4.
Find the sum of the first 28 terms.

Markscheme
correct substitution into term formula \((A1)\)
e.g. \(5 + 27(1.7)\)
28th term is 50.9 (exact) \(AI\) \(N2\)
[2 marks]

5a. Expand \((x - 2)^4\) and simplify your result.

Markscheme
evidence of expanding \(M1\)
e.g. \((x - 2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4\) \(A2\) \(N2\)
\((x - 2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16\)
[3 marks]

5b. Find the term in \(x^3\) in \((3x + 4)(x - 2)^4\).

Markscheme
finding coefficients, \(3 \times 24 = 72\), \(4 \times (-8) = -32\) \(AI\)(\(AI\))
term is 40\(x^3\) \(A1\) \(N3\)
[3 marks]

6a. The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.
Find the common ratio.

Markscheme
correct substitution into sum of a geometric sequence \((A1)\)
e.g. \(200 \left( \frac{1 - r^4}{1 - r} \right)\), \(200 + 200r + 200r^2 + 200r^3\)
attempt to set up an equation involving a sum and 324.8 \(MI\)
e.g. \(200 \left( \frac{1 - r^4}{1 - r} \right) = 324.8\), \(200 + 200r + 200r^2 + 200r^3 = 324.8\)
r = 0.4 (exact) \(A2\) \(N3\)
[4 marks]
6b. The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.

Find the tenth term.

**Markscheme**
- correct substitution into formula \( A1 \)
- e.g. \( u_{10} = 200 \times 0.4^0 \)
- \( u_{10} = 0.0524288 \) (exact), \( 0.0524 \) \( A1 \) \( N1 \) \( [2 \text{ marks}] \)

7. Let \( g(x) = \log_3 x \), for \( x > 0 \).

Find the value of \((f^{-1} \circ g)(2)\), giving your answer as an integer.

**Markscheme**
- METHOD 1
  - finding \( g(2) = \log_3 2 \) (seen anywhere) \( A1 \)
  - attempt to substitute \( (M1) \)
  - e.g. \((f^{-1} \circ g)(2) = 3^{2\log_3 2} \)
  - evidence of using log or index rule \( (A1) \)
  - e.g. \((f^{-1} \circ g)(2) = 3^{\log_3 4} \times 3^{\log_3 2} \)
  - \((f^{-1} \circ g)(2) = 4 \) \( A1 \) \( N1 \)

- METHOD 2
  - attempt to form composite (in any order) \( (M1) \)
  - e.g. \((f^{-1} \circ g)(x) = 3^{2\log_3 x} \)
  - evidence of using log or index rule \( (A1) \)
  - e.g. \((f^{-1} \circ g)(x) = 3^{\log_3 x^2} \times 3^{\log_3 x^2} \)
  - \((f^{-1} \circ g)(x) = x^2 \) \( A1 \)
  - \((f^{-1} \circ g)(2) = 4 \) \( A1 \) \( N1 \)

\( [4 \text{ marks}] \)

Let \( f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4 \), for \( x > 0 \).

8a. Show that \( f(x) = \log_3 2x \).

**Markscheme**
- combining 2 terms \( (A1) \)
- e.g. \( \log_3 8x - \log_3 4 \), \( \log_3 \frac{1}{2}x + \log_3 4 \)
- expression which clearly leads to answer given \( A1 \)
- e.g. \( \log_3 \frac{8x}{4} \), \( \log_3 \frac{4x}{2} \)
- \( f(x) = \log_3 2x \) \( AG \) \( N0 \)

\( [2 \text{ marks}] \)
Let \( f(x) = \log_{3} \frac{x}{2} + \log_{5} 16 - \log_{5} 4 \), for \( x > 0 \).

8b. The function \( f \) can also be written in the form \( f(x) = \frac{\ln x}{\ln 5} \).

(i) Write down the value of \( a \) and of \( b \).

(ii) Hence on graph paper, sketch the graph of \( f \), for \(-5 \leq x \leq 5 \), \(-5 \leq y \leq 5 \), using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

**Markscheme**

(i) \( a = 2 \), \( b = 3 \) AIAI NINI

(ii)

![Graph](image)

Note: Award A1 for sketch approximately through \((0.5 \pm 0.1, 0 \pm 0.1)\), A1 for approximately correct shape, A1 for sketch asymptotic to the y-axis.

(iii) \( x = 0 \) (must be an equation) A1 NI

[6 marks]

8c. Find the value of \( f(0.5) \) and of \( f(4.5) \).

**Markscheme**

attempt to substitute either value into \( f \) (MI)

e.g. \( \log_{3} 1 \), \( \log_{5} 9 \)

\( f(0.5) = 0 \), \( f(4.5) = 2 \) AIAI N3

[3 marks]

8d. Write down the value of \( f^{-1}(0) \).

**Markscheme**

\( f^{-1}(0) = 0.5 \) A1 NI

[1 mark]
8e. The point $A$ lies on the graph of $f$. At $A$, $x = 4.5$.

On your diagram, sketch the graph of $f^{-1}$, noting clearly the image of point $A$.

**Markscheme**

Note: Award $A1$ for sketch approximately through $(0 \pm 0.1, 0.5 \pm 0.1)$, $A1$ for approximately correct shape of the graph reflected over $y = x$, $A1$ for sketch asymptotic to $x$-axis, $A1$ for point $(2 \pm 0.1, 4.5 \pm 0.1)$ clearly marked and on curve.

9a. Consider the expansion of $\left(2x^3 + \frac{b}{x}\right)^8 = 256x^{24} + 3072x^{20} + \ldots + kx^0 + \ldots$.

**Markscheme**

valid attempt to find term in $x^{20}$  

(MI)

c.e.g. $\binom{8}{1} (2^7)(b) \cdot (2x^3)^7 \left(\frac{1}{x}\right) = 3072$

correct equation  

$A1$

c.e.g. $\binom{8}{1} (2^7)(b) = 3072$

$b = 3$  

$A1$  

$N2$

[3 marks]

9b. Find $k$.

[3 marks]
Consider the expansion of $(3x^2 + 2)^9$.

10a. Write down the number of terms in the expansion. [1 mark]

**Markscheme**

- 10 terms $\text{AI~NI}$

10b. Find the term in $x^4$. [5 marks]

**Markscheme**

- Evidence of binomial expansion $(\text{MI})$
  - e.g. $a^3b^6 + \left(\begin{array}{c}9 \\ 1 \end{array}\right) a^8b + \left(\begin{array}{c}9 \\ 2 \end{array}\right) a^7b^2 + \ldots + \left(\begin{array}{c}9 \\ r \end{array}\right) (a)^{9-r}(b)^r$, Pascal’s triangle
  
- Evidence of correct term $(\text{AI})$
  - e.g. 8th term, $r = 7$, $\left(\begin{array}{c}9 \\ 7 \end{array}\right)$, $(3x^2)^7 2^7$

- Correct expression of complete term $(\text{AI})$
  - e.g. $\left(\begin{array}{c}9 \\ 7 \end{array}\right) (3x^2)^7 2^7$, $\frac{9}{2}C(3x^2)^7 (2)^7$, $36 \times 9 \times 128$
  - $41472x^4$ (accept $41500x^4$) $\text{AI~N2}$

44 marks]

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

11a. Find $(f \circ g)(4)$. [3 marks]
**Markscheme**

**METHOD 1**

\[ g(4) = 5 \quad (A1) \]

evidence of composition of functions \( (M1) \)

\[ f(5) = 25 \quad AI \quad N3 \]

**METHOD 2**

\[ f \circ g(x) = (2x - 3)^2 \quad (M1) \]

\[ f \circ g(4) = (2 \times 4 - 3)^2 \quad (A1) \]

\[ = 25 \quad AI \quad N3 \]

[3 marks]

11b. Find \( g^{-1}(x) \). \quad [2 marks]

**Markscheme**

for interchanging \( x \) and \( y \) (may be done later) \( (M1) \)

e.g. \( x = 2y - 3 \)

\[ g^{-1}(x) = \frac{x + 3}{2} \quad (\text{accept } y = \frac{x + 3}{2}, \frac{x + 3}{2}) \quad AI \quad N2 \]

[2 marks]

Let \( f(x) = \frac{20x}{e^{0.3x}} \), for \( 0 \leq x \leq 20 \).

12a. Sketch the graph of \( f \). \quad [3 marks]

**Markscheme**

\[ AIAIAI \quad N3 \]

Note: Award \( AI \) for approximately correct shape with inflexion/change of curvature, \( AI \) for maximum skewed to the left, \( AI \) for asymptotic behaviour to the right.

[3 marks]

12b. (i) Write down the \( x \)-coordinate of the maximum point on the graph of \( f \). \quad [3 marks]

(ii) Write down the interval where \( f \) is increasing.
### Markscheme

**12c.** Show that \( f'(x) = \frac{20 - 6x}{e^{0.3x}} \). [5 marks]

**Markscheme**

valid approach \((M1)\)

e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) \((A1)(A1)\)

e.g. \(20, 0.3e^{0.3x}\) or \(-0.3e^{-0.3x}\)

correct substitution into product or quotient rule \(A1\)

e.g. \(e^{0.3x} - 6e^{0.3x}x\), \(20e^{-0.3x} + 20x(-0.3)e^{-0.3x}\)

correct working \(A1\)

e.g. \(\frac{20e^{0.3x} - 20x(0.3)e^{0.3x}}{(0.3e^{0.3x})^2}\), \(e^{-0.3x}(20 + 20x(-0.3))\)

\(f'(x) = \frac{20 - 6x}{e^{0.3x}}\) \(AG\) \(N0\) [5 marks]

**12d.** Find the interval where the rate of change of \(f\) is increasing. [4 marks]

**Markscheme**

consideration of \(f'\) or \(f''\) \((M1)\)

valid reasoning \(R1\)

e.g. sketch of \(f'\), \(f''\) is positive, \(f'' = 0\), reference to minimum of \(f'\)

correct value 6.6666666\ldots \((6\frac{2}{3})\) \((A1)\)

correct interval, with both endpoints \(A1\) \(N3\)

e.g. \(6.67 < x \leq 20, 6\frac{2}{3} < x < 20\) [4 marks]

Let \(h(x) = \frac{2^{x-1}}{x+1}\), \(x \neq -1\).

### 13a.

(i) Sketch the graph of \(h\) for \(-4 \leq x \leq 4\) and \(-5 \leq y \leq 8\), including any asymptotes. [7 marks]

(ii) Write down the equations of the asymptotes.

(iii) Write down the \(x\)-intercept of the graph of \(h\).
**Markscheme**

Note: Award A1 for approximately correct intercepts, A1 for correct shape, A1 for asymptotes, A1 for approximately correct domain and range.

(ii) $x = -1, y = 2$  A1A1 N2

(iii) $\frac{1}{2}$  A1 N1

[7 marks]

13b. Find $h^{-1}(x)$.

[4 marks]

**Markscheme**

$y = \frac{2x-1}{x+1}$

interchanging $x$ and $y$ (seen anywhere)  M1
e.g. $x = \frac{2y-1}{y+1}$
correct working  A1
e.g. $xy + x = 2y - 1$
collecting terms  A1
e.g. $x + 1 = 2y - xy, x + 1 = y(2 - x)$
$h^{-1}(x) = \frac{x+1}{2-x}$  A1 N2

[4 marks]

13c. Let $R$ be the region in the first quadrant enclosed by the graph of $h$, the $x$-axis and the line $x = 3$.

(i) Find the area of $R$.

(ii) Write down an expression for the volume obtained when $R$ is revolved through $360^\circ$ about the $x$-axis.

[5 marks]

**Markscheme**

(i) area = 2.06  A2 N2

(ii) attempt to substitute into volume formula (do not accept $\pi \int_a^b y^2 \, dx$)  M1

volume = $\pi \int_3^1 \left( \frac{2x-1}{x+1} \right)^2 \, dx$  A2 N3

[5 marks]
Let \( f(x) = e^x \sin 2x + 10 \), for \( 0 \leq x \leq 4 \). Part of the graph of \( f \) is given below.

There is an \( x \)-intercept at the point A, a local maximum point at M, where \( x = p \) and a local minimum point at N, where \( x = q \).

14a. Write down the \( x \)-coordinate of A.  

**Markscheme**  
2.31 \( A1 \ N1 \)  
[1 mark]

14b. Find the value of  
(i) \( p \);  
(ii) \( q \).

**Markscheme**  
(i) 1.02 \( A1 \ N1 \)  
(ii) 2.59 \( A1 \ N1 \)  
[2 marks]

14c. Find \( \int_p^q f(x) \, dx \). Explain why this is not the area of the shaded region.

**Markscheme**  
\( \int_p^q f(x) \, dx = 9.96 \) \( A1 \ N1 \)  
split into two regions, make the area below the \( x \)-axis positive \( RIRI \ N2 \)  
[3 marks]
15a. (i) Write down the equation of the axis of symmetry. [3 marks]

(ii) Find the $y$-coordinate of the vertex.

**Markscheme**

(i) $x = 2$ (must be equation) $\quad A1 \quad N1$

(ii) substituting $x = 2$ into $f(x)$ $\quad (M1)$

$y = 8 \quad A1 \quad N2$

[3 marks]

15b. Find the $x$-intercepts of the graph. [4 marks]

**Markscheme**

evidence of setting function to zero $\quad (M1)$

e.g. $f(x) = 0 \quad 8x = 2x^2$

evidence of correct working $\quad A1$

e.g. $0 = 2x(4-x) \quad \frac{-b+\sqrt{b^2-4ac}}{2a}$

$x$-intercepts are at 4 and 0 (accept $4, 0$ and $(0, 0)$, or $x = 4 \quad x = 0$) $\quad A1A1 \quad N1N1$

[4 marks]

Let $f(x) = 2x^2 + 4x - 6$.

16a. Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$. [3 marks]

**Markscheme**

evidence of obtaining the vertex $\quad (M1)$

e.g. a graph, $x = \frac{-b}{2a}$, completing the square

$f(x) = 2(x+1)^2 - 8 \quad A2 \quad N3$

[3 marks]
16b. Write down the equation of the axis of symmetry of the graph of $f$.  

**Markscheme**  
$x = -1$ (equation must be seen)  
$A1 \ N1$  
$[1 \ mark]$  

16c. Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$.  

**Markscheme**  
$f(x) = 2(x - 1)(x + 3)$  
$A1A1 \ N2$  
$[2 \ marks]$  

17. Consider the equation $x^2 + (k - 1)x + 1 = 0$, where $k$ is a real number.  
Find the values of $k$ for which the equation has two equal real solutions.  

**Markscheme**  
**METHOD 1**  
evidence of valid approach  
$M1$  
e.g. $b^2 - 4ac$, quadratic formula  
correct substitution into $b^2 - 4ac$ (may be seen in formula)  
$A1$  
e.g. $(k - 1)^2 - 4 \times 1 \times 1$, $(k - 1)^2 - 4$, $k^2 - 2k - 3$  
setting their discriminant equal to zero  
$M1$  
e.g. $\Delta = 0$, $(k - 1)^2 - 4 = 0$  
attempt to solve the quadratic  
$M1$  
e.g. $(k - 1)^2 = 4$, factorizing  
correct working  
$A1$  
e.g. $(k - 1) = \pm 2$, $(k - 3)(k + 1)$  
k = $-1$, $k = 3$ (do not accept inequalities)  
$A1A1 \ N2$  
$[7 \ marks]$  

**METHOD 2**  
recognizing perfect square  
$M1$  
e.g. $(x + 1)^2 = 0$, $(x - 1)^2$  
correct expansion  
$A1/A1$  
e.g. $x^2 + 2x + 1 = 0$, $x^2 - 2x + 1$  
equating coefficients of $x$  
$A1A1$  
e.g. $k - 1 = -2$, $k - 1 = 2$  
k = $-1$, $k = 3$  
$A1A1 \ N2$  
$[7 \ marks]$  

18. Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.  

$[7 \ marks]$
Markscheme

recognizing \( \log a + \log b = \log ab \) (seen anywhere) \((A1)\)

e.g. \( \log_2(x(x-2)) \), \( x^2 - 2x \)

recognizing \( \log_b x = x \leftrightarrow a^x = b \) \((A1)\)

e.g. \( 2^3 = 8 \)

correct simplification \( A1 \)

e.g. \( x(x-2) = 2^3 \), \( x^2 - 2x - 8 \)

evidence of correct approach to solve \((M1)\)

e.g. factorizing, quadratic formula

correct working \( A1 \)

e.g. \( (x - 4)(x + 2) = \frac{2x - \sqrt{36}}{2} \)

\( x = 4 \) \( A2 \) \( N3 \)

[7 marks]