Topics 1 and 2 Review *[134 marks]*

1a. Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$. *[1 mark]*

Write down the 10th term of the sequence. Do not simplify your answer.

Markscheme

$u_{10} = 3(0.9)^9$ ***A1 N1***

[1 mark]

1b. Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$. *[4 marks]*

Find the sum of the infinite sequence.

Markscheme

recognizing $r = 0.9$ ***(A1)***

correct substitution ***A1***

e.g. $S = \frac{3}{1-0.9}$

$S = \frac{3}{0.1}$ ***(A1)***

$S = 30$ ***A1 N3***

[4 marks]

Let $f(x) = log_3\sqrt{x}$, for $x > 0$.

2a. Show that $f^{-1}(x) = 3^{2x}$. *[2 marks]*

Markscheme

interchanging *x* and *y* (seen anywhere) ***(M1)***

e.g. $x = \log\sqrt{y}$ (accept any base)

evidence of correct manipulation ***A1***

e.g. $3^x = \sqrt{y}$, $3^y = x^{\frac{1}{2}}$, $x = \frac{1}{2}\log_3 y$, $2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ ***AG N0***

[2 marks]

Let $f(x) = log_3\sqrt{x}$, for $x > 0$.

2b. Write down the range of f^{-1} . *[1 mark]*

Markscheme

$y > 0$, $f^{-1}(x) > 0$ ***A1 N1***

[1 mark]

In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

3a. Find the value of r . *[3 marks]*

Markscheme

evidence of substituting into formula for nth term of GP ***(M1)***

e.g. $u_4 = \frac{1}{81}r^3$

setting up correct equation $\frac{1}{81}r^3 = \frac{1}{3}$ ***A1***

$r = 3$ ***A1 N2***

[3 marks]

In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

3b. Find the smallest value of n for which $S_n > 40$. *[4 marks]*

Markscheme

METHOD 1

setting up an inequality (accept an equation) ***M1***

e.g. $\frac{\frac{1}{81}(3^n - 1)}{2} > 40$, $\frac{\frac{1}{81}(1 - 3^n)}{-2} > 40$, $3^n > 6481$

evidence of solving ***M1***

e.g. graph, taking logs

$n > 7.9888\ldots$ ***(A1)***

$\therefore n = 8$ ***A1 N2***

METHOD 2

if $n = 7$, sum $= 13.49\ldots$; if $n = 8$, sum $= 40.49\ldots$ ***A2***

$n = 8$ (is the smallest value) ***A2 N2***

[4 marks]

4a. The first three terms of an arithmetic sequence are 5 , 6.7 , 8.4 . *[2 marks]*

Find the common difference.

Markscheme

valid method ***(M1)***

e.g. subtracting terms, using sequence formula

$d = 1.7$ ***A1 N2***

[2 marks]

4b. The first three terms of an arithmetic sequence are 5 , 6.7 , 8.4 . *[2 marks]*

Find the 28th term of the sequence.

Markscheme

correct substitution into term formula *(A1)*

e.g. $5 + 27(1.7)$

28^{th} term is 50.9 (exact) ***A1 N2***

[2 marks]

4c. The first three terms of an arithmetic sequence are 5 , 6.7 , 8.4 . *[2 marks]*

Find the sum of the first 28 terms.

Markscheme

correct substitution into sum formula *(A1)*

e.g. $S_{28} = \frac{28}{2}(2(5) + 27(1.7))$, $\frac{28}{2}(5 + 50.9)$

$S_{28} = 782.6$ (exact) [782, 783] ***A1 N2***

[2 marks]

5a. Expand $(x - 2)^4$ and simplify your result. *[3 marks]*

Markscheme

evidence of expanding ***M1***

e.g. $(x - 2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$ ***A2 N2***

$(x - 2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$

[3 marks]

5b. Find the term in x^3 in $(3x + 4)(x - 2)^4$. *[3 marks]*

Markscheme

finding coefficients, $3 \times 24 (= 72)$, $4 \times (-8)(= -32)$ ***(A1)(A1)***

term is $40x^3$ ***A1 N3***

[3 marks]

6a. The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8. *[4 marks]*

Find the common ratio.

Markscheme

correct substitution into sum of a geometric sequence *(A1)*

e.g. $200\left(\frac{1-r^4}{1-r}\right)$, $200 + 200r + 200r^2 + 200r^3$

attempt to set up an equation involving a sum and 324.8 ***M1***

e.g. $200\left(\frac{1-r^4}{1-r}\right) = 324.8$, $200 + 200r + 200r^2 + 200r^3 = 324.8$

$r = 0.4$ (exact) ***A2 N3***

[4 marks]

6b. The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8. *[2 marks]*

Find the tenth term.

Markscheme

correct substitution into formula *A1*

e.g. $u_{10} = 200 \times 0.4^9$

$u_{10} = 0.0524288$ (exact), 0.0524 *A1 N1*

[2 marks]

7. Let $g(x) = \log_3 x$, for $x > 0$. *[4 marks]*

Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

Markscheme

METHOD 1

finding $g(2) = log_3 2$ (seen anywhere) *A1*

attempt to substitute *(M1)*

e.g. $(f^{-1} \circ g)(2) = 3^{2\log_3 2}$

evidence of using log or index rule *(A1)*

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 4}$, $3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ *A1 N1*

METHOD 2

attempt to form composite (in any order) *(M1)*

e.g. $(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule *(A1)*

e.g. $(f^{-1} \circ g)(x) = 3^{\log_3 x^2}$, $3^{\log_3 x}{}^2$

$(f^{-1} \circ g)(x) = x^2$ *A1*

$(f^{-1} \circ g)(2) = 4$ *A1 N1*

[4 marks]

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

8a. Show that $f(x) = \log_3 2x$. *[2 marks]*

Markscheme

combining 2 terms *(A1)*

e.g. $\log_3 8x - \log_3 4$, $\log_3 \frac{1}{2}x + \log_3 4$

expression which clearly leads to answer given *A1*

e.g. $\log_3 \frac{8x}{4}$, $\log_3 \frac{4x}{2}$

$f(x) = \log_3 2x$ *AG N0*

[2 marks]

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

8b. The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$. *[6 marks]*

(i) Write down the value of a and of b .

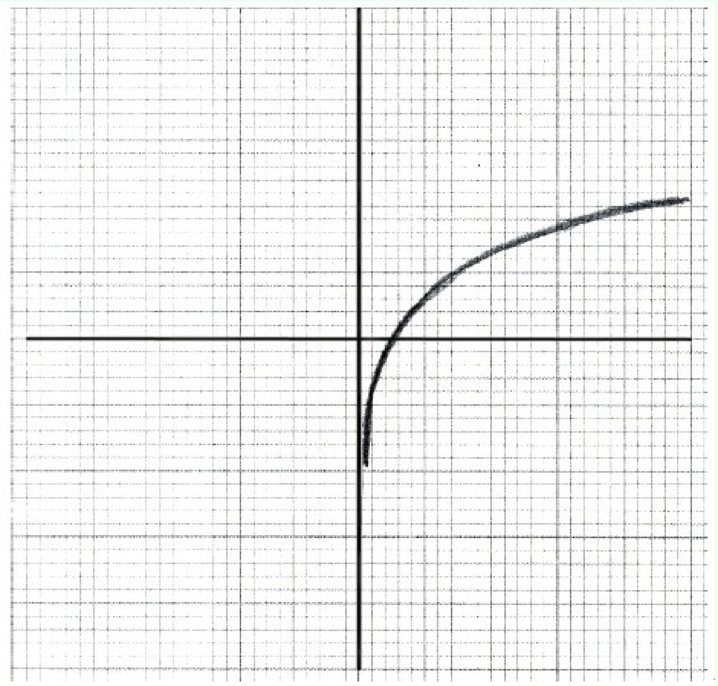
(ii) Hence on graph paper, **sketch** the graph of f , for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

Markscheme

(i) $a = 2$, $b = 3$ ***A1A1 N1N1***

(ii)



A1A1A1 N3

Note: Award ***A1*** for sketch approximately through $(0.5 \pm 0.1, 0 \pm 0.1)$, ***A1*** for approximately correct shape, ***A1*** for sketch asymptotic to the y-axis.

(iii) $x = 0$ (must be an equation) ***A1 N1***

[6 marks]

8c. Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

Markscheme

attempt to substitute either value into f ***(M1)***

e.g. $\log_3 1$, $\log_3 9$

$f(0.5) = 0$, $f(4.5) = 2$ ***A1A1 N3***

[3 marks]

8d. Write down the value of $f^{-1}(0)$. *[1 mark]*

Markscheme

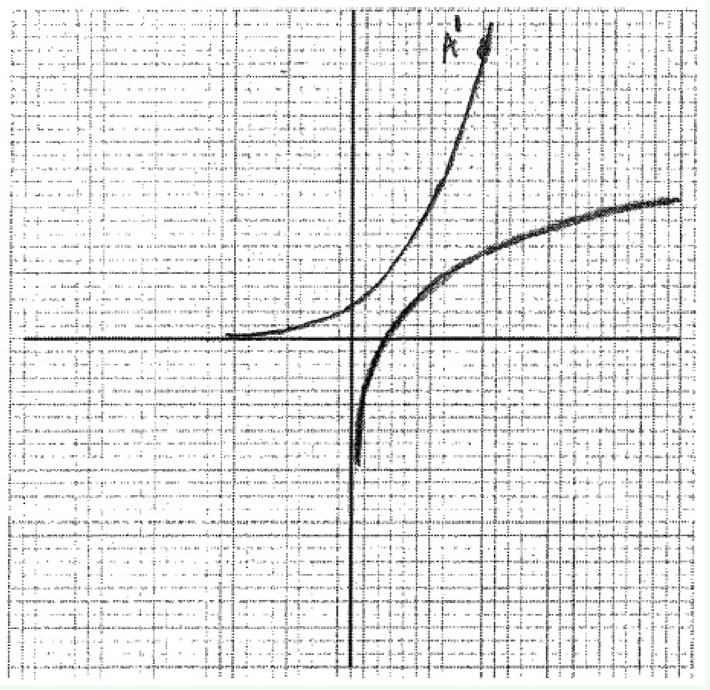
$f^{-1}(0) = 0.5$ ***A1 N1***

[1 mark]

8e. The point A lies on the graph of f. At A, $x = 4.5$. *[4 marks]*

On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A.

Markscheme



A1A1A1A1 ***N4***

Note: Award ***A1*** for sketch approximately through $(0 \pm 0.1, 0.5 \pm 0.1)$, ***A1*** for approximately correct shape of the graph reflected over $y = x$, ***A1*** for sketch asymptotic to *x*-axis, ***A1*** for point $(2 \pm 0.1, 4.5 \pm 0.1)$ clearly marked and on curve.

[4 marks]

Consider the expansion of $\left(2x^3 + \frac{b}{x}\right)^8 = 256x^{24} + 3072x^{20} + \ldots + kx^0 + \ldots$.

9a. Find b. *[3 marks]*

Markscheme

valid attempt to find term in x^{20} ***(M1)***

e.g. $\binom{8}{1}(2^7)(b)$, $(2x^3)^7\left(\frac{b}{x}\right) = 3072$

correct equation ***A1***

e.g. $\binom{8}{1}(2^7)(b) = 3072$

$b = 3$ ***A1*** ***N2***

[3 marks]

9b. Find k. *[3 marks]*

Markscheme

evidence of choosing correct term *(M1)*

e.g. 7th term, $r = 6$

correct expression *A1*

e.g. $\binom{8}{6}(2x^3)^2\left(\frac{3}{x}\right)^6$

$k = 81648$ (accept 81600) *A1* *N2*

[3 marks]

Consider the expansion of $(3x^2 + 2)^9$.

10a. Write down the number of terms in the expansion. *[1 mark]*

Markscheme

10 terms *A1* *N1*

[1 mark]

10b. Find the term in x^4 . *[5 marks]*

Markscheme

evidence of binomial expansion *(M1)*

e.g. $a^9b^0 + \binom{9}{1}a^8b + \binom{9}{2}a^7b^2 + \ldots$, $\binom{9}{r}(a)^{n-r}(b)^r$, Pascal's triangle

evidence of correct term *(A1)*

e.g. 8th term, $r = 7$, $\binom{9}{7}$, $(3x^2)^2 2^7$

correct expression of complete term *(A1)*

e.g. $\binom{9}{7}(3x^2)^2(2)^7$, ${}^9_2C(3x^2)^2(2)^7$, $36 \times 9 \times 128$

$41472x^4$ (accept $41500x^4$) *A1* *N2*

[4 marks]

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

11a. Find $(f \circ g)(4)$. *[3 marks]*

Markscheme

METHOD 1

$g(4) = 5$ ***(A1)***

evidence of composition of functions ***(M1)***

$f(5) = 25$ ***A1 N3***

METHOD 2

$f \circ g(x) = (2x - 3)^2$ ***(M1)***

$f \circ g(4) = (2 \times 4 - 3)^2$ ***(A1)***

$= 25$ ***A1*** ***N3***

[3 marks]

11b. Find $g^{-1}(x)$. *[2 marks]*

Markscheme

for interchanging *x* and *y* (may be done later) ***(M1)***

e.g. $x = 2y - 3$

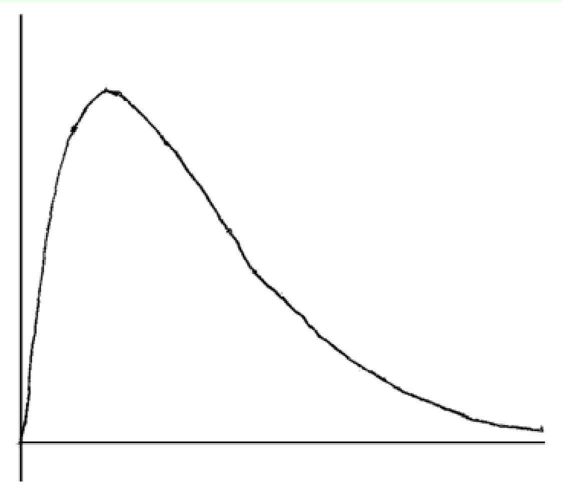
$g^{-1}(x) = \frac{x+3}{2}$ (accept $y = \frac{x+3}{2}, \frac{x+3}{2}$) ***A1*** ***N2***

[2 marks]

Let $f(x) = \frac{20x}{e^{0.3x}}$, for $0 \le x \le 20$.

12a. Sketch the graph of *f* . *[3 marks]*

Markscheme



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape with inflexion/change of curvature, ***A1*** for maximum skewed to the left, ***A1*** for asymptotic behaviour to the right.

[3 marks]

12b. (i) Write down the *x*-coordinate of the maximum point on the graph of *f* . *[3 marks]*

(ii) Write down the interval where *f* is increasing.

Markscheme

(i) $x = 3.33$ ***A1*** ***N1***

(ii) correct interval, with right end point $3\frac{1}{3}$ ***A1A1*** ***N2***

e.g. $0 < x \le 3.33$, $0 \le x < 3\frac{1}{3}$

Note: Accept any inequalities in the right direction.

[3 marks]

12c. Show that $f'(x) = \frac{20-6x}{e^{0.3x}}$. *[5 marks]*

Markscheme

valid approach ***(M1)***

e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) ***(A1)(A1)***

e.g. 20 , $0.3e^{0.3x}$ or $-0.3e^{-0.3x}$

correct substitution into product or quotient rule ***A1***

e.g. $\frac{20e^{0.3x}-20x(0.3)e^{0.3x}}{(e^{0.3x})^2}$, $20e^{-0.3x} + 20x(-0.3)e^{-0.3x}$

correct working ***A1***

e.g. $\frac{20e^{0.3x}-6xe^{0.3x}}{e^{0.6x}}$, $\frac{e^{0.3x}(20-20x(0.3))}{(e^{0.3x})^2}$, $e^{-0.3x}(20+20x(-0.3))$

$f'(x) = \frac{20-6x}{e^{0.3x}}$ ***AG*** ***N0***

[5 marks]

12d. Find the interval where the rate of change of *f* is increasing. *[4 marks]*

Markscheme

consideration of f' or f'' ***(M1)***

valid reasoning ***R1***

e.g. sketch of f' , f'' is positive, $f'' = 0$, reference to minimum of f'

correct value $6.6666666\ldots$ $\left(6\frac{2}{3}\right)$ ***(A1)***

correct interval, with **both** endpoints ***A1*** ***N3***

e.g. $6.67 < x \le 20$, $6\frac{2}{3} \le x < 20$

[4 marks]

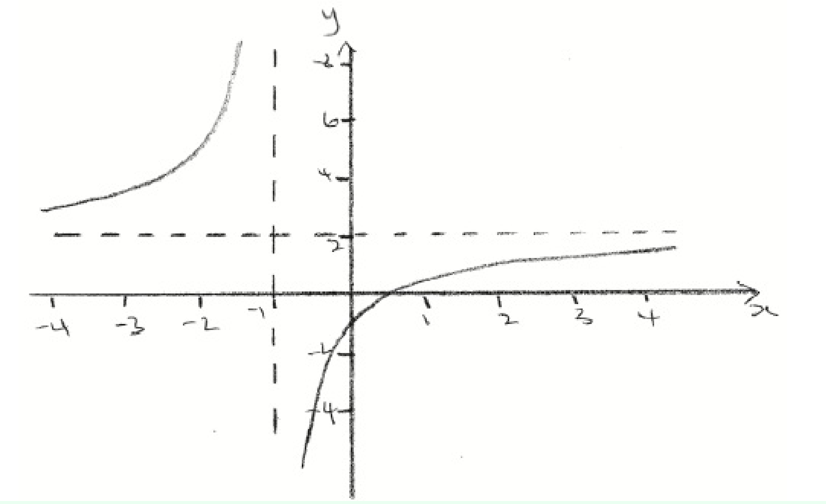
Let $h(x) = \frac{2x-1}{x+1}$, $x \ne -1$.

13a. (i) Sketch the graph of h for $-4 \le x \le 4$ and $-5 \le y \le 8$, including any asymptotes. *[7 marks]*

(ii) Write down the equations of the asymptotes.

(iii) Write down the x-intercept of the graph of h .

Markscheme



A1A1A1A1 *N4*

Note: Award ***A1*** for approximately correct intercepts, ***A1*** for correct shape, ***A1*** for asymptotes, ***A1*** for approximately correct domain and range.

(ii) $x = -1$, $y = 2$ ***A1A1*** ***N2***

(iii) $\frac{1}{2}$ ***A1*** ***N1***

[7 marks]

13b. Find $h^{-1}(x)$. *[4 marks]*

Markscheme

$y = \frac{2x-1}{x+1}$

interchanging *x* and *y* (seen anywhere) ***M1***

e.g. $x = \frac{2y-1}{y+1}$

correct working ***A1***

e.g. $xy + x = 2y - 1$

collecting terms ***A1***

e.g. $x + 1 = 2y - xy$, $x + 1 = y(2 - x)$

$h^{-1}(x) = \frac{x+1}{2-x}$ ***A1*** ***N2***

[4 marks]

13c. Let *R* be the region in the first quadrant enclosed by the graph of h , the *x*-axis and the line $x = 3$. *[5 marks]*

(i) Find the area of *R*.

(ii) Write down an expression for the volume obtained when *R* is revolved through 360° about the *x*-axis.

Markscheme

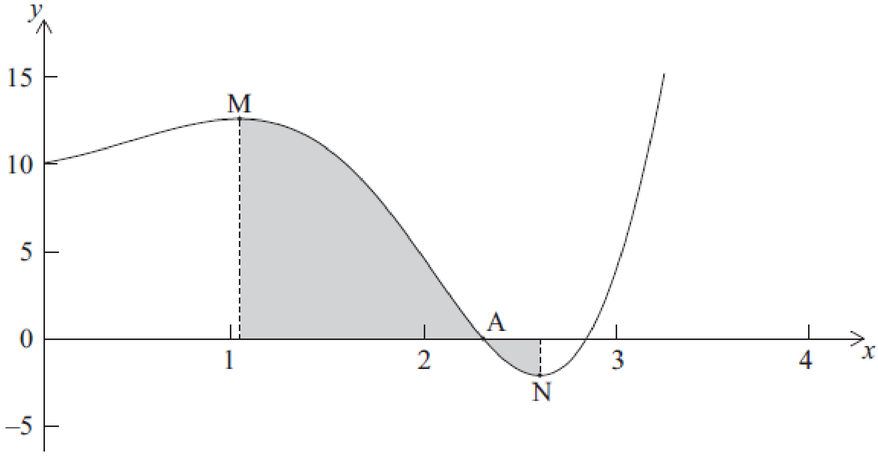
(i) area $= 2.06$ ***A2*** ***N2***

(ii) attempt to substitute into volume formula (do not accept $\pi \int_a^b y^2 \mathrm{d}x$) ***M1***

volume $= \pi \int_{\frac{1}{2}}^{3} \left(\frac{2x-1}{x+1}\right)^2 \mathrm{d}x$ ***A2*** ***N3***

[5 marks]

Let $f(x) = e^x \sin 2x + 10$, for $0 \le x \le 4$. Part of the graph of f is given below.



There is an x-intercept at the point A, a local maximum point at M, where $x = p$ and a local minimum point at N, where $x = q$.

14a. Write down the x-coordinate of A. *[1 mark]*

Markscheme

2.31 ***A1*** ***N1***

[1 mark]

14b. Find the value of *[2 marks]*

(i) p ;

(ii) q .

Markscheme

(i) 1.02 ***A1*** ***N1***

(ii) 2.59 ***A1*** ***N1***

[2 marks]

14c. Find $\int_p^q f(x)\mathrm{d}x$. Explain why this is not the area of the shaded region. *[3 marks]*

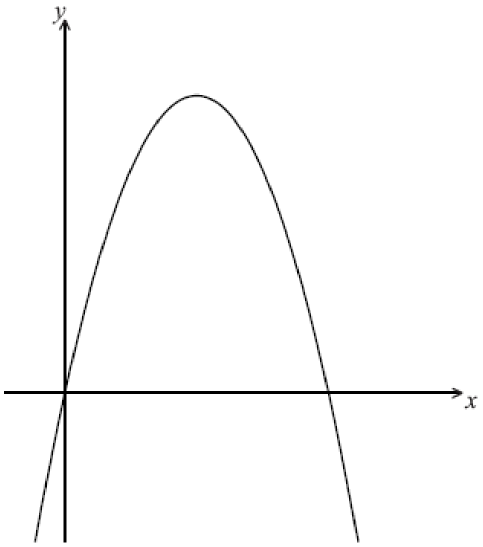
Markscheme

$\int_p^q f(x)\mathrm{d}x = 9.96$ ***A1*** ***N1***

split into two regions, make the area below the x-axis positive ***R1R1*** ***N2***

[3 marks]

Let $f(x) = 8x - 2x^2$. Part of the graph of *f* is shown below.



15a. (i) Write down the equation of the axis of symmetry. *[3 marks]*

(ii) Find the *y*-coordinate of the vertex.

Markscheme

(i) $x = 2$ (must be equation) ***A1*** ***N1***

(ii) substituting $x = 2$ into $f(x)$ ***(M1)***

$y = 8$ ***A1*** ***N2***

[3 marks]

15b. Find the *x*-intercepts of the graph. *[4 marks]*

Markscheme

evidence of setting function to zero ***(M1)***

e.g. $f(x) = 0$, $8x = 2x^2$

evidence of correct working ***A1***

e.g. $0 = 2x(4 - x)$, $\frac{-8 \pm \sqrt{64}}{-4}$

x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0) , or $x = 4$, $x = 0$) ***A1A1*** ***N1N1***

[4 marks]

Let $f(x) = 2x^2 + 4x - 6$.

16a. Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$. *[3 marks]*

Markscheme

evidence of obtaining the vertex ***(M1)***

e.g. a graph, $x = -\frac{b}{2a}$, completing the square

$f(x) = 2(x + 1)^2 - 8$ ***A2*** ***N3***

[3 marks]

16b. Write down the equation of the axis of symmetry of the graph of f . *[1 mark]*

Markscheme

$x = -1$ (equation must be seen) ***A1*** ***N1***

[1 mark]

16c. Express $f(x)$ in the form $f(x) = 2(x-p)(x-q)$. *[2 marks]*

Markscheme

$f(x) = 2(x-1)(x+3)$ ***A1A1*** ***N2***

[2 marks]

17. Consider the equation $x^2 + (k-1)x + 1 = 0$, where k is a real number. *[7 marks]*

Find the values of k for which the equation has two **equal** real solutions.

Markscheme

METHOD 1

evidence of valid approach ***(M1)***

e.g. $b^2 - 4ac$, quadratic formula

correct substitution into $b^2 - 4ac$ (may be seen in formula) ***(A1)***

e.g. $(k-1)^2 - 4 \times 1 \times 1$, $(k-1)^2 - 4$, $k^2 - 2k - 3$

setting **their** discriminant equal to zero ***M1***

e.g. $\Delta = 0, (k-1)^2 - 4 = 0$

attempt to solve the quadratic ***(M1)***

e.g. $(k-1)^2 = 4$, factorizing

correct working ***A1***

e.g. $(k-1) = \pm 2$, $(k-3)(k+1)$

$k = -1$, $k = 3$ (do not accept inequalities) ***A1A1*** ***N2***

[7 marks]

METHOD 2

recognizing perfect square ***(M1)***

e.g. $(x+1)^2 = 0$, $(x-1)^2$

correct expansion ***(A1)(A1)***

e.g. $x^2 + 2x + 1 = 0$, $x^2 - 2x + 1$

equating coefficients of x ***A1A1***

e.g. $k - 1 = -2$, $k - 1 = 2$

$k = -1$, $k = 3$ ***A1A1*** ***N2***

[7 marks]

18. Solve $\log_2 x + \log_2 (x-2) = 3$, for $x > 2$. *[7 marks]*

Markscheme

recognizing $\log a + \log b = \log ab$ (seen anywhere) ***(A1)***

e.g. $\log_2(x(x-2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ ***(A1)***

e.g. $2^3 = 8$

correct simplification ***A1***

e.g. $x(x-2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working ***A1***

e.g. $(x-4)(x+2)$, $\frac{2 \pm \sqrt{36}}{2}$

$x = 4$ ***A2*** ***N3***

[7 marks]

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