Exam Review **Calculus**

1. Attempting to integrate. (M1)

$y = x^3 - 5x + c$ (A1)(A1)(A1)

substitute (2, 6) to find c $\left(6 = 2^3 - 5(2) + c\right)$ (M1)

$c = 8$ (A1)

$y = x^3 - 5x + 8$ (Accept $x^3 - 5x + 8$) (C6)

[6]

2. (a) $x = \frac{1}{5}$ or $5x - 1 = 0$ (A1) (N1) 1

(b) $f'(x) = \frac{(5x-1)(6x) - (3x^2)(5)}{(5x-1)^2}$ (M1)(A1)

$= \frac{30x^2 - 6x - 15x^2}{(5x-1)^2}$ (may be implied) (A1)

$= \frac{15x^2 - 6x}{(5x-1)^2}$ (accept a = 15, b = –6) (A1) (N2) 4

[5]

3. (a) (i) intersection points x = 3.77, x = 8.30 (may be seen as the limits) (A1)(A1)

approach involving subtraction and integrals (M1)

fully correct expression A2

eg $\int_{3.77}^{8.30}\left(\left(-4\cos(0.5x)+2\right)-\left(\ln(3x-2)+1\right)\right)dx$,

$\int_{3.77}^{8.30} g(x)dx - \int_{3.77}^{8.30} f(x)dx$ N5

(ii) A = 6.46 A1 N1

(b) (i) $f'(x) = \frac{3}{3x-2}$ A1A1 N2

Note: *Award A1 for numerator (3), A1 for denominator (3x – 2), but penalize 1 mark for additional terms.*

(ii) $g'(x) = 2\sin(0.5x)$ A1A1 N2

Note: *Award A1 for 2, A1 for sin (0.5x), but penalize 1 mark for additional terms.*

(c) evidence of using derivatives for gradients (M1)

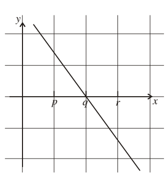
correct approach (A1)

eg $f'(x) = g'(x)$, points of intersection

x = 1.43, x = 6.10 A1A1 N2N2

[14]

4. (a)



A1A1 N2

Note: *Award A1 for negative gradient throughout, A1 for x- intercept of q. It need not be linear.*

(b)

	x - coordinate
(i) Maximum point on f	r
(i) Inflexion point on f	q

A1 N1

A1 N1

(c) **METHOD 1**

Second derivative is zero, second derivative changes sign. R1R1 N2

METHOD 2

There is a maximum on the graph of the first derivative. R2 N2

[6]

5. (a) evidence of factorizing 3/division by 3 A1

eg $\int_1^5 3f(x)dx = 3\int_1^5 f(x)dx$, $\frac{12}{3}$, $\int_1^5 \frac{3f(x)dx}{3}$

(do not accept 4 as this is show that)

evidence of stating that reversing the limits changes the sign A1

eg $\int_5^1 f(x)dx = -\int_1^5 f(x)dx$

$\int_5^1 f(x)dx = -4$ AG N0

(b) evidence of correctly combining the integrals (seen anywhere) (A1)

eg $I = \int_1^2 (x + f(x))dx + \int_2^5 (x + f(x))dx = \int_1^5 (x + f(x))dx$

evidence of correctly splitting the integrals (seen anywhere) (A1)

eg $I = \int_1^5 x dx + \int_1^5 f(x)dx$

$\int x dx = \frac{x^2}{2}$ (seen anywhere) A1

$\int_1^5 x dx = \left[\frac{x^2}{2}\right]_1^5 = \frac{25}{2} - \frac{1}{2}$ $\left(= \frac{24}{2}, 12\right)$ A1

I = 16 A1 N3

[7]

6. (a) $f'(x) = x^2 + 4x - 5$ A1A1A1 N3

(b) evidence of attempting to solve $f'(x) = 0$ (M1)

evidence of correct working A1

eg $(x + 5)(x - 1)$, $\frac{-4 \pm \sqrt{16+20}}{2}$, sketch

$x = -5$, $x = 1$ (A1)

so $x = -5$ A1 N2

(c) **METHOD 1**

$f''(x) = 2x + 4$ (may be seen later) A1

evidence of setting second derivative = 0 (M1)

eg $2x + 4 = 0$

$x = -2$ A1 N2

METHOD 2

evidence of use of symmetry (M1)
eg midpoint of max/min, reference to shape of cubic
correct calculation A1

eg $\frac{-5+1}{2}$,

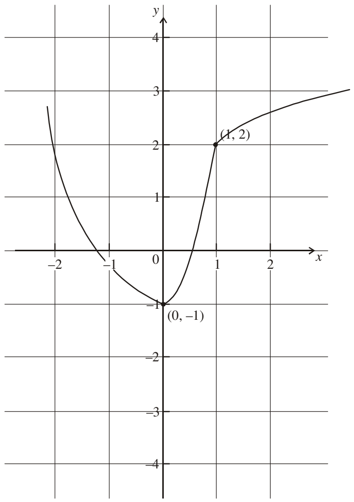
$x = -2$ A1 N2

(d) attempting to find the value of the derivative when $x = 3$ (M1)
$f'(3) = 16$ A1

valid approach to finding the equation of a line M1
eg $y - 12 = 16(x - 3)$, $12 = 16 \times 3 + b$
$y = 16x - 36$ A1 N2

[14]

7.



A1A1A1A1A1A1 N6

Notes: *On interval [−2,0], award A1 for decreasing, A1 for concave up.*
On interval [0,1], award A1 for increasing, A1 for concave up.
On interval [1,2], award A1 for change of concavity, A1 for concave down.

[6]

8. (a) $\frac{dy}{dx} = 3\cos 3x$ A1 N1

(b) $\frac{dy}{dx} = \frac{x}{\cos^2 x} + \tan x$ accept $x\sec^2 x + \tan x$ A1A1 N2

(c) **METHOD 1**

Evidence of using the quotient rule (M1)

$$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - \ln x}{x^2}$$ A1A1

$$\frac{dy}{dx} = \frac{1-\ln x}{x^2}$$ N3

METHOD 2

$y = x^{-1}\ln x$

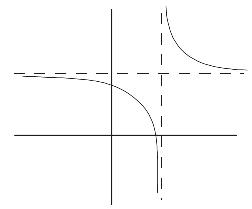
Evidence of using the product rule (M1)

$$\frac{dy}{dx} = x^{-1} \times \frac{1}{x} + \ln x(-1)(x^{-2})$$ A1A1

$$\frac{dy}{dx} = \frac{1}{x^2} - \frac{\ln x}{x^2}$$ N3

[6]

9. (a)



A1A1A1 N3

Notes: *Award A1 for **both** asymptotes shown. The asymptotes need not be labelled.*
*Award A1 for the left branch in **approximately** correct position,*
*A1 for the right branch in **approximately** correct position.*

(b) (i) $y = 3$, $x = \frac{5}{2}$ (must be equations) A1A1 N2

(ii) $x = \frac{14}{6}$ $\left(\frac{7}{3} \text{ or } 2.33,\ \text{also accept} \left(\frac{14}{6}, 0\right)\right)$ A1 N1

(iii) $y = \frac{14}{6}$ $(y = 2.8)$ $\left(\text{accept} \left(0, \frac{14}{5}\right) \text{or } (0, 2.8)\right)$ A1 N1

(c) (i) $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right)dx = 9x +$

$3\ln(2x-5) - \frac{1}{2(2x-5)} + C$ A1A1A1

A1A1 N5

(ii) Evidence of using $V = \int_a^b \pi y^2 dx$ (M1)

Correct expression A1

eg $\int_3^a \pi \left(3 + \frac{1}{2x-5}\right)^2 dx,\ \pi\int_3^a \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right)dx,$

$$\left[9x + 3\ln(2x-5) - \frac{1}{2(2x-5)}\right]_3^a$$

Substituting $\left(9a+3\ln(2a-5)-\dfrac{1}{2(2a-5)}\right)-\left(27+3\ln 1-\dfrac{1}{2}\right)$ A1

Setting up an equation (M1)

$$9a-\frac{1}{2(2a-5)}-27+\frac{1}{2}+3\ln(2a-5)-3\ln 1=\left(\frac{28}{3}+3\ln 3\right)$$

Solving gives $a = 4$ A1 N2

[17]

10. $f'(x) = 12x^2 + 2$ A1A1

When $x = 1$, $f(1) = 6$ (seen anywhere) (A1)

When $x = 1$, $f'(1) = 14$ (A1)

Evidence of taking the negative reciprocal (M1)

eg $\dfrac{-1}{14}x$, $\dfrac{1}{-14}$, -0.0714

Equation is $y - 6 = -\dfrac{1}{14}(x-1)$ $\left(y=-\dfrac{1}{14}x+\dfrac{85}{14}, y=-0.0714x+6.07\right)$ A1 N4

[6]

11. (a) (i) $p = 1$, $q = 5$ (or $p = 5$, $q = 1$) A1A1 N2

(ii) $x = 3$ (must be an equation) A1 N1

(b) $y = (x - 1)(x - 5)$

$= x^2 - 6x + 5$ (A1)

$= (x - 3)^2 - 4$ (accept $h = 3$, $k = -4$) A1A1 N3

(c) $\dfrac{dy}{dx}=2(x-3)$ $(=2x-6)$ A1A1 N2

(d) When $x = 0$, $\dfrac{dy}{dx}=-6$ (A1)

$y - 5 = -6(x - 0)$ ($y = -6x + 5$ or equivalent) A1 N2

[10]

12. (a) Evidence of using $a=\dfrac{dv}{dt}$ (M1)

eg $3e^{3t-2}$

$a(1) = 3e$ $(= 8.15)$ A1 N2

(b) Attempt to solve $e^{3t-2} = 22.3$ (M1)

eg $(3t - 2)(\ln e) = \ln 22.3$, sketch

$t = 1.70$ A1 N2

(c) Evidence of using $s = \int v\,dt$ (limits not required) M1

e.g. $\int e^{3t-2}\,dt, \dfrac{1}{3}\left[e^{3t-2}\right]_0^1$

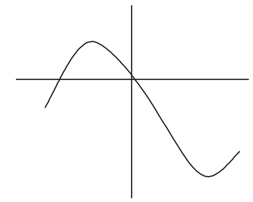
$\dfrac{1}{3}\left(e^1-e^{-2}\right)$ $\left[=\dfrac{1}{3}\left(e-e^{-2}\right)=0.861\right]$ A1 N1

[6]

13. (a) **EITHER**

Recognizing that tangents parallel to the x-axis mean maximum and minimum (may be seen on sketch) R1

Sketch of graph of f M1



OR

Evidence of using $f'(x) = 0$ M1

Finding $f'(x) = 3x^2 - 6x - 24$ A1

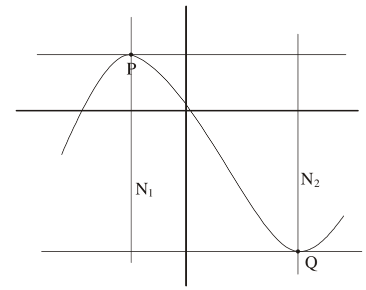
$3x^2 - 6x - 24 = 0$

Solutions $x = -2$ or $x = 4$

THEN

Coordinates are P(−2, 29) and Q(4, −79) A1A1 N1N1

(b)



(i) (4, 29) A1 N1

(ii) (−2, −79) A1 N1

[6]

14. Using $V = \int \pi y^2\,dx$ (M1)

Correctly integrating $\int\left(x^{\frac{1}{2}}\right)^2 dx=\dfrac{x^2}{2}$ A1

$V = \pi\left[\dfrac{x^2}{2}\right]_0^a$ A1

$= \dfrac{\pi a^2}{2}$ (A1)

Setting up **their** equation $\left(\dfrac{1}{2}\pi a^2=0.845\pi\right)$ M1

$a^2 = 1.69$

$a = 1.3$ A1 N2

[6]