Topic 5 Review *[81 marks]*

A **four-sided** die has three blue faces and one red face. The die is rolled.

Let B be the event a blue face lands down, and R be the event a red face lands down.

1a. Write down *[2 marks]*

(i) $\mathrm{P}(B)$;
(ii) $\mathrm{P}(R)$.

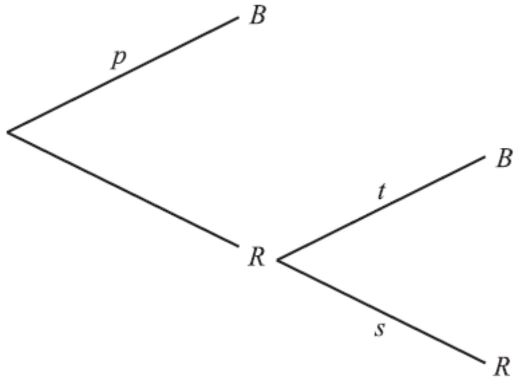
Markscheme

(i) $\mathrm{P}(B) = \frac{3}{4}$ ***A1 N1***

(ii) $\mathrm{P}(R) = \frac{1}{4}$ ***A1 N1***

[2 marks]

1b. If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities. *[2 marks]*



Find the value of p, of s and of t.

Markscheme

$p = \frac{3}{4}$ ***A1 N1***

$s = \frac{1}{4}, t = \frac{3}{4}$ ***A1 N1***

[2 marks]

1c. Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let X be the total score obtained. *[3 marks]*

(i) Show that $\mathrm{P}(X = 3) = \frac{3}{16}$.
(ii) Find $\mathrm{P}(X = 2)$.

Markscheme

(i) $\mathrm{P}(X=3)$

$=\mathrm{P}$ (getting 1 and 2) $=\frac{1}{4}\times\frac{3}{4}$ ***A1***

$=\frac{3}{16}$ ***AG*** ***N0***

(ii) $\mathrm{P}(X=2)=\frac{1}{4}\times\frac{1}{4}+\frac{3}{4}\left(\text{or } 1-\frac{3}{16}\right)$ ***(A1)***

$=\frac{13}{16}$ ***A1*** ***N2***

[3 marks]

1d. (i) Construct a probability distribution table for X. *[5 marks]*

(ii) Calculate the expected value of X.

Markscheme

(i)

X	2	3
$\mathrm{P}(X=x)$	$\frac{13}{16}$	$\frac{3}{16}$

A2 ***N2***

(ii) evidence of using $\mathrm{E}(X)=\sum x\mathrm{P}(X=x)$ ***(M1)***

$\mathrm{E}(X)=2\left(\frac{13}{16}\right)+3\left(\frac{3}{16}\right)$ ***(A1)***

$=\frac{35}{16}\left(=2\frac{3}{16}\right)$ ***A1*** ***N2***

[5 marks]

1e. If the total score is 3, Guiseppi wins \$10. If the total score is 2, Guiseppi gets nothing. *[4 marks]*

Guiseppi plays the game twice. Find the probability that he wins exactly \$10.

Markscheme

win \$10 $\Rightarrow$ scores 3 one time, 2 other time ***(M1)***

$\mathrm{P}(3)\times\mathrm{P}(2)=\frac{13}{16}\times\frac{3}{16}$ (seen anywhere) ***A1***

evidence of recognising there are different ways of winning \$10 ***(M1)***

e.g. $\mathrm{P}(3)\times\mathrm{P}(2)+\mathrm{P}(2)\times\mathrm{P}(3)$, $2\left(\frac{13}{16}\times\frac{3}{16}\right)$, $\frac{36}{256}+\frac{3}{256}+\frac{36}{256}+\frac{3}{256}$

$\mathrm{P}(\text{win }\$10)=\frac{78}{256}\left(=\frac{39}{128}\right)$ ***A1*** ***N3***

[4 marks]

The ages of people attending a music concert are given in the table below.

Age	$15\le x<19$	$19\le x<23$	$23\le x<27$	$27\le x<31$	$31\le x<35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

2a. Find p . *[2 marks]*

Markscheme

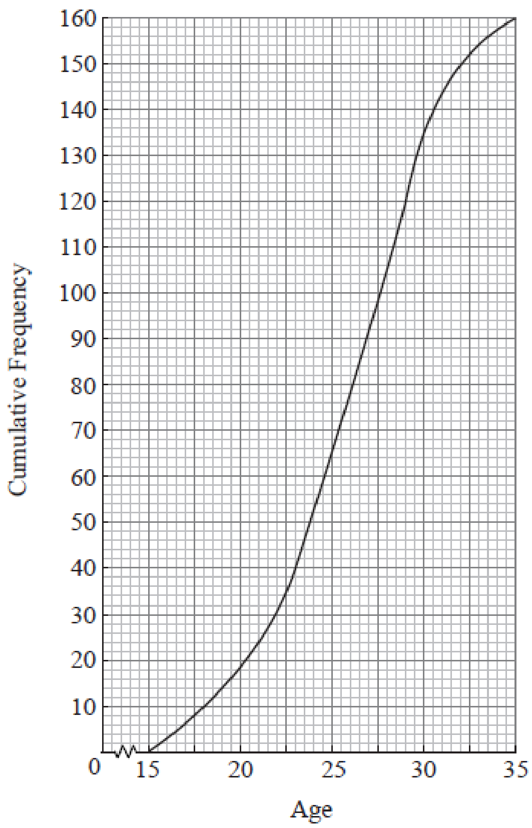
evidence of valid approach ***(M1)***

e.g. $92 + 52$, line on graph at $x = 31$

$p = 144$ ***A1*** ***N2***

[2 marks]

2b. The cumulative frequency diagram is given below. *[5 marks]*



Use the diagram to estimate

(i) the 80th percentile;

(ii) the interquartile range.

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. line on graph, 0.8×160 , using complement

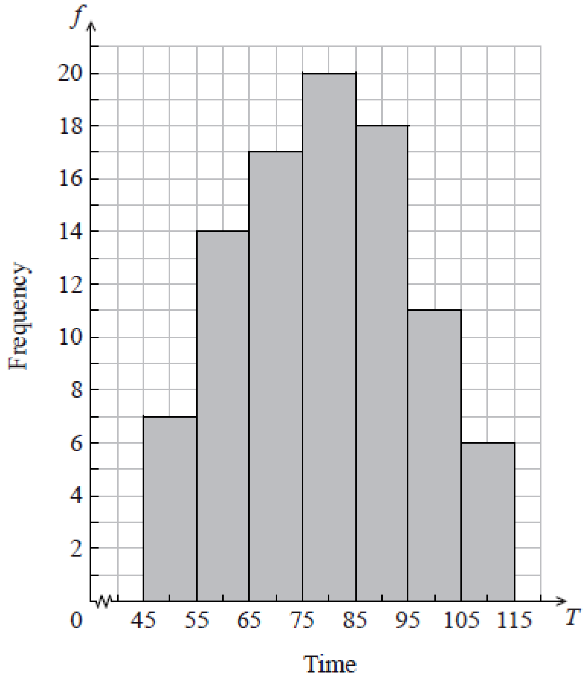
$= 29.5$ ***A1*** ***N2***

(ii) $Q_1 = 23$; $Q_3 = 29$ ***(A1)(A1)***

IQR $=$ 6 (accept any notation that suggests an interval) ***A1*** ***N3***

[5 marks]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T .

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

3a. (i) Write down the value of p and of q . *[3 marks]*

(ii) Write down the median class.

Markscheme

(i) $p = 17$, $q = 11$ ***A1A1 N2***

(ii) $75 \le T < 85$ ***A1 N1***

[3 marks]

3b. A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. adding frequencies

$\frac{76}{93} = 0.8172043\ldots$

$\mathrm{P}(T < 95) = \frac{76}{93} = 0.817$ ***A1 N2***

[2 marks]

3c. Consider the class interval $45 \leq T < 55$. *[2 marks]*

(i) Write down the interval width.

(ii) Write down the mid-interval value.

Markscheme

(i) 10 ***A1 N1***

(ii) 50 ***A1 N1***

[2 marks]

3d. Hence find an estimate for the *[4 marks]*

(i) mean;

(ii) standard deviation.

Markscheme

(i) evidence of approach using mid-interval values (may be seen in part (ii)) ***(M1)***

79.1397849

$\bar{x} = 79.1$ ***A2 N3***

(ii) 16.4386061

$\sigma = 16.4$ ***A1 N1***

[4 marks]

3e. John assumes that T is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle. *[2 marks]*

Find John's estimate.

Markscheme

e.g. standardizing, $z = 0.9648\ldots$

0.8326812

$\mathrm{P}(T < 95) = 0.833$ ***A1 N2***

[2 marks]

There are nine books on a shelf. For each book, x is the number of pages, and y is the selling price in pounds (£). Let r be the correlation coefficient.

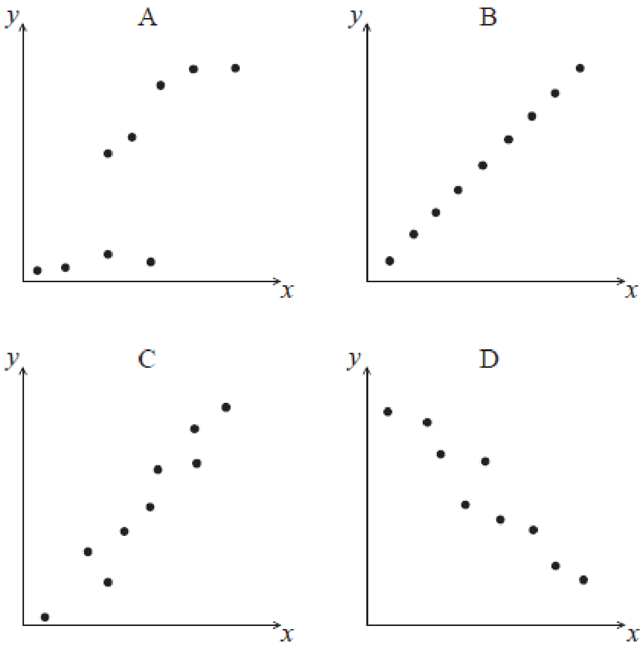
4a. Write down the possible minimum and maximum values of r . *[2 marks]*

Markscheme

min value of r is -1, max value of r is 1 ***A1A1 N2***

[2 marks]

4b. Given that $r = 0.95$, which of the following diagrams best represents the data. *[1 mark]*

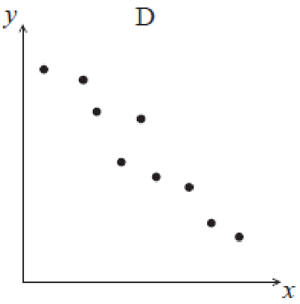


Markscheme

C *A1* *N1*

[1 mark]

4c. *[2 marks]*



For the data in diagram D , which **two** of the following expressions describe the correlation between x and y ?

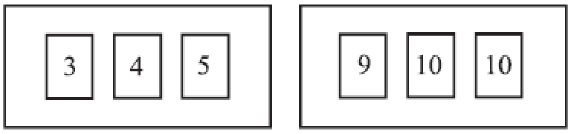
perfect, zero, linear, strong positive, strong negative, weak positive, weak negative

Markscheme

linear, strong negative *A1A1* *N2*

[2 marks]

Two boxes contain numbered cards as shown below.



Two cards are drawn at random, one from each box.

5a. Copy and complete the table below to show all nine equally likely outcomes. *[2 marks]*

3 , 9		
3 , 10		
3 , 10		

Markscheme

3, 9	**4, 9**	**5, 9**
3, 10	**4, 10**	**5, 10**
3, 10	**4, 10**	**5, 10**

A2 N2

[2 marks]

5b. Let S be the sum of the numbers on the two cards. *[2 marks]*

Find the probability of each value of S.

Markscheme

$\mathrm{P}(12) = \frac{1}{9}$, $\mathrm{P}(13) = \frac{3}{9}$, $\mathrm{P}(14) = \frac{3}{9}$, $\mathrm{P}(15) = \frac{2}{9}$ ***A2 N2***

[2 marks]

5c. Find the expected value of S. *[3 marks]*

Markscheme

correct substitution into formula for $\mathrm{E}(X)$ ***A1***

e.g. $\mathrm{E}(S) = 12 \times \frac{1}{9} + 13 \times \frac{3}{9} + 14 \times \frac{3}{9} + 15 \times \frac{2}{9}$

$\mathrm{E}(S) = \frac{123}{9}$ ***A2 N2***

[3 marks]

5d. Anna plays a game where she wins \$50 if S is even and loses \$30 if S is odd. *[3 marks]*

Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games.

Markscheme

METHOD 1

correct expression for expected gain $\mathrm{E}(A)$ for 1 game ***(A1)***

e.g. $\frac{4}{9} \times 50 - \frac{5}{9} \times 30$

$\mathrm{E}(A) = \frac{50}{9}$

amount at end = expected gain for 1 game $\times 36$ ***(M1)***

= 200 (dollars) ***A1 N2***

METHOD 2

attempt to find expected number of wins and losses ***(M1)***

e.g. $\frac{4}{9} \times 36$, $\frac{5}{9} \times 36$

attempt to find expected gain $\mathrm{E}(G)$ ***(M1)***

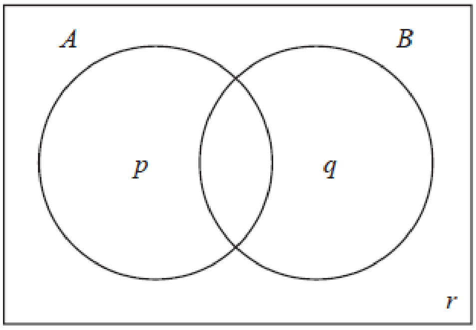
e.g. $16 \times 50 - 30 \times 20$

$\mathrm{E}(G) = 200$ (dollars) ***A1 N2***

[3 marks]

Consider the events A and B, where $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.



6a. Write down the value of *[3 marks]*

(i) p ;

(ii) q ;

(iii) r.

Markscheme

(i) $p = 0.2$ ***A1 N1***

(ii) $q = 0.4$ ***A1 N1***

(iii) $r = 0.1$ ***A1 N1***

[3 marks]

6b. Find the value of $\mathrm{P}(A|B')$. *[2 marks]*

Markscheme

$\mathrm{P}(A|B') = \frac{2}{3}$ ***A2*** ***N2***

Note: Award ***A1*** for an unfinished answer such as $\frac{0.2}{0.3}$.

[2 marks]

6c. Hence, or otherwise, show that the events A and B are **not** independent. *[1 mark]*

Markscheme

valid reason ***R1***

e.g. $\frac{2}{3} \neq 0.5$, $0.35 \neq 0.3$

thus, A and B are not independent ***AG*** ***N0***

[1 mark]

7a. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested. *[4 marks]*

Find the probability that there is at least one defective lamp in the sample.

Markscheme

evidence of recognizing binomial (seen anywhere) ***(M1)***

e.g. $\mathrm{B}(n, p)$, 0.95^{30}

finding $\mathrm{P}(X = 0) = 0.21463876$ ***(A1)***

appropriate approach ***(M1)***

e.g. complement, summing probabilities

0.785361

probability is 0.785 ***A1*** ***N3***

[4 marks]

7b. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested. *[4 marks]*

Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps.

Markscheme

identifying correct outcomes (seen anywhere) ***(A1)***

e.g. $\mathrm{P}(X = 1) + \mathrm{P}(X = 2)$, 1 or 2 defective, $0.3389\ldots + 0.2586\ldots$

recognizing conditional probability (seen anywhere) ***R1***

e.g. $\mathrm{P}(A|B)$, $\mathrm{P}(X \leq 2|X \geq 1)$, P(at most 2|at least 1)

appropriate approach involving conditional probability ***(M1)***

e.g. $\frac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X\geq 1)}$, $\frac{0.3389\ldots+0.2586\ldots}{0.785\ldots}$, $\frac{1 \text{ or } 2}{0.785}$

0.760847

probability is 0.761 ***A1*** ***N2***

[4 marks]

A company produces a large number of water containers. Each container has two parts, a bottle and a cap. The bottles and caps are tested to check that they are not defective.

A cap has a probability of 0.012 of being defective. A random sample of 10 caps is selected for inspection.

8a. Find the probability that exactly one cap in the sample will be defective. *[2 marks]*

Markscheme

Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

evidence of recognizing binomial (seen anywhere in the question) ***(M1)***

e.g. ${}_nC_r p^r q^{n-r}$, $\mathrm{B}(n, p)$, ${}^{10}C_1(0.012)^1(0.988)^9$

$p = 0.108$ ***A1*** ***N2***

[2 marks]

8b. The sample of caps passes inspection if at most one cap is defective. Find the probability that the sample passes inspection. *[2 marks]*

Markscheme

valid approach ***(M1)***

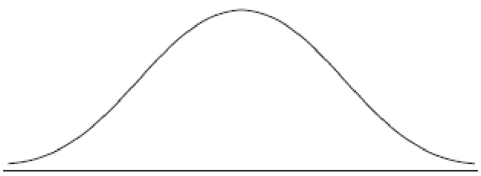
e.g. $\mathrm{P}(X \le 1)$, $0.88627\ldots + 0.10764\ldots$

$p = 0.994$ ***A1*** ***N2***

[2 marks]

8c. The heights of the bottles are normally distributed with a mean of 22 cm and a standard deviation of 0.3 cm. *[5 marks]*

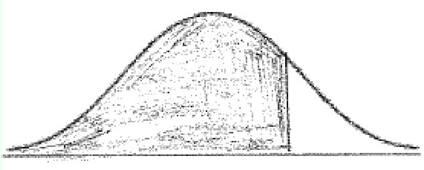
(i) Copy and complete the following diagram, shading the region representing where the heights are less than 22.63 cm.



(ii) Find the probability that the height of a bottle is less than 22.63 cm.

Markscheme

(i)



A1A1 *N2*

Note: Award ***A1*** for vertical line to right of mean, ***A1*** for shading to left of **their** vertical line.

(ii) valid approach ***(M1)***

e.g. $\mathrm{P}(X < 22.63)$

working to find standardized value ***(A1)***

e.g. $\frac{22.63-22}{0.3}$, 2.1

$p = 0.982$ ***A1*** ***N3***

[5 marks]

8d. (i) A bottle is accepted if its height lies between 21.37 cm and 22.63 cm. Find the probability that a bottle selected at random is accepted. *[5 marks]*

(ii) A sample of 10 bottles passes inspection if all of the bottles in the sample are accepted. Find the probability that the sample passes inspection.

Markscheme

valid approach ***(M1)***

e.g. $\mathrm{P}(21.37 < X < 22.63)$, $\mathrm{P}(-2.1 < z < 2.1)$

correct working ***(A1)***

e.g. $0.982 - (1 - 0.982)$

$p = 0.964$ ***A1*** ***N3***

(ii) correct working ***(A1)***

e.g. $X \sim \mathrm{B}(10, 0.964)$, $(0.964)^{10}$

$p = 0.695$ (accept 0.694 from tables) ***A1*** ***N2***

[5 marks]

8e. The bottles and caps are manufactured separately. A sample of 10 bottles and a sample of 10 caps are randomly selected for testing. Find the probability that both samples pass inspection. *[2 marks]*

Markscheme

valid approach ***(M1)***

e.g. $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$, $(0.994) \times (0.964)^{10}$

$p = 0.691$ (accept 0.690 from tables) ***A1*** ***N2***

[2 marks]