**Vectors**

1. Consider the points A (1, 5, 4), B (3, 1, 2) and D (3, *k*, 2), with (AD) perpendicular to (AB).

(a) Find

(i)

(ii) , giving your answer in terms of *k*.

(b) Show that *k* = 7

The point C is such that  = 

(c) Find the position vector of C

(d) Find cos 

2. Let ***v*** = 3***i*** + 4 ***j*** + ***k*** and ***w*** = ***i*** + 2 ***j*** – 3***k***. The vector ***v*** + *p****w*** is perpendicular to ***w***.

Find the value of *p*.

3. The point O has coordinates (0, 0, 0), point A has coordinates (1, –2, 3) and point B has coordinates (–3, 4, 2).

(a) (i) Show that  = 

(ii) Find 

(b) The line *L*1 has equation 

Write down the coordinates of two points on *L*1.

(c) The line *L*2 passes through A and is parallel to.

(i) Find a vector equation for *L*2, giving your answer in the form ***r*** = ***a*** + *t****b***.

 (ii) Point C (*k*, –*k*, 5)is on *L*2. Find the coordinates of C.

(d) The line *L*3 has equation  and passes through the point C.

Find the value of *p* at C.

4. The line *L*1 is represented by ***r***1 =  and the line *L*2 by ***r***2 = 

The lines *L*1 and *L*2 intersect at point T. Find the coordinates of T.

5. The diagram below shows a cuboid (rectangular solid) OJKLMNPQ. The vertex O is (0, 0, 0), J is (6, 0, 0), K is (6, 0, 10), M is (0, 7, 0) and Q is (0, 7, 10).



(a) (i) Show that  = .

(ii) Find .

(b) An equation for the line (MK) is ***r*** = $\left[\begin{matrix}0\\7\\0\end{matrix}\right]+s\left[\begin{matrix}6\\-7\\10\end{matrix}\right]$.

(i) Write down an equation for the line (JQ) in the form ***r*** = ***a*** + *t****b***.

 (ii) Find the acute angle between (JQ) and (MK).

(c) The lines (JQ) and (MK) intersect at D. Find the position vector of D.

**Solution :** (a) (i) evidence of combining vectors

*eg*  =    (or  =  +  in part (ii))

 = 

(ii)  = 

(b) evidence of using perpendicularity  scalar product = 0



4  4(*k*  5) + 4 = 0

4*k* + 28 = 0 (accept any correct equation clearly leading to *k* = 7)

 k = 7

(c)  = 

 = 

evidence of correct approach

*eg* 

 = 

(d) **METHOD 1**

choosing appropriate vectors, 

finding the scalar product

*eg* 2(1) + 4(1) + 2(1), 2(1) + (4)(1) + (2)(1)

cos  = 0 **METHOD 2**

 parallel to  (may show this on a diagram with points labelled)

   (may show this on a diagram with points labelled)

 = 90

cos  = 0

2. **Solution :** *p****w*** = *p****i*** + 2*p****j***  3*pk* (seen anywhere) (A1)

attempt to find ***v*** + *p****w***

*eg* 3***i*** + 4***j*** + ***k*** + p(***i*** + 2***j***  3***k***)

collecting terms (3 + *p*)***i*** + (4 + 2*p*)***j*** + (1  3*p*) ***k***

attempt to find the dot product

*eg* 1(3 + *p*) + 2(4 + 2*p*)  3(1  3*p*)

setting **their** dot product equal to 0

*eg* 1(3 + *p*) + 2(4 + 2*p*)  3(1  3*p*) = 0

simplifying

*eg* 3 + *p* + 8 + 4*p*  3 + 9*p* = 0, 14*p* + 8 = 0

*P* =  0.571 

3. **Solution :** (a) (i)evidence of approach

*eg*  +  =  B  A

 = 

(ii) for choosing **correct** vectors, (with , or  with
)

***Note:*** *Using  with  will lead to*
 *  0.799. If they then say BO*
 *= 0.799, this is a correct solution.*

calculating •,  *eg* ***d***1•***d***2 = (1)(4) + (2)(6) + (3)(1) (= 19)

 


evidence of using the formula to find the angle

 *eg* cos ** = 


 = 0.799 radians (accept 45.8)

(b) two correct answers

*eg* (1,  2, 3), (3, 4, 2), (7, 10, 1), (11, 16, 0)

(c) (i) ***r*** = 

(ii) C on *L*2, so 

evidence of equating components

*eg* 1  3*t* = *k*,  2 + 4*t* = *k*, 5 = 3 + 2*t*

one correct value *t* = 1, *k* = 2 (seen anywhere)

coordinates of C are (2, 2, 5)

(d) for setting up one (or more) correct equation using


*eg* 3 + *p* = 2, 8 2*p* = 2,  *p* = 5

*p* =  5

4. **Solution :** evidence of equating vectors (M1)

*eg* *L*1 = *L*2

for any **two** correct equations

*eg* 2 + *s* = 3  *t,* 5 + 2*s* = 3 + 3*t*, 3 + 3*s* = 8  4*t*

attempting to solve the equations

finding **one** correct parameter (*s* = 1, *t* = 2)

the coordinates of T are (1, 3, 0)

5. **Solution :** (a) (i)Evidence of approach

*eg*  = 



(ii) 

(b) (i) ***r*** =  + *t*  or ***r*** =  + *t* 

***Note:*** *Award A1 if “****r*** *= ” is missing.*

(ii) Evidence of choosing correct vectors , 

Evidence of calculating magnitudes

 *eg* 


 •  =  36  49 + 100 (= 15) (accept  15)

For evidence of substitution into the correct formula

 *eg* cos ** = 


** = 1.49 (radians), 85.3

(c) **METHOD 1**

Geometric approach

Valid reasoning

*eg* diagonals bisect each other, 

Calculation of mid point

*eg* 

 (accept (3,3.5,5))

**METHOD 2**

Correct approach

*eg*  + ***t***  =  + ***s*** 

Two correct equations

*eg* 6  6*t* = 6*s*,7*t* = 7  7*s*, 10*t* = 10*s*

Attempt to solve

One correct parameter

*s* = 0.5 *t* = 0.5

 (accept (3, 3.5, 5))

**METHOD 3**

Correct approach

*eg*  + *t*  =  + *s* 

Two correct equations

*eg*  6*t* = 6*s*, 7 + 7*t* = 7  7*s*, 10 + 10*t* = 10*s*

Attempt to solve

One correct parameter

*s* = 0.5 *t* =  0.5

 (accept (3, 3.5, 5))