The following diagram shows a circular play area for children.

![Diagram of a circular play area](image)

The circle has centre O and a radius of 20 m, and the points A, B, C and D lie on the circle. Angle AOB is 1.5 radians.

1a. Find the length of arc ADC.

**Markscheme**

appropriate method to find angle AOC \( (M1) \)

correct substitution into arc length formula \( (A1) \)

e.g. \( 2\pi - 1.5 - 2.4 \)

e.g. \( (2\pi - 3.9) \times 20 = 2.3831853 \times 20 \)

arc length = 47.6637...

arc length = 47.7 (47.6, 47.7) (i.e. do not accept 47.6) \( (A1) \ N2 \)

**Notes:** Candidates may misread the question and use \( \overline{AOC} = 2.4 \). If working shown, award \( M0 \) then \( A0MRA1 \) for the answer 48. Do not then penalize \( \overline{AOC} \) in part (d) which, if used, leads to the answer 679.498...

**However,** if they use the prematurely rounded value of 2.4 for \( \overline{AOC} \), penalise 1 mark for premature rounding for the answer 48 in (c). Do not then penalize for this in (d).

[3 marks]

1b. Find the length of the chord \([AB]\).
METHOD 1
choosing cosine rule (must have cos in it) \( (M1) \)
e.g. \( c^2 = a^2 + b^2 - 2ab \cos C \)
correct substitution into rhs \( A1 \)
e.g. \( 20^2 + 20^2 - 2(20)(20) \cos 1.5 \), \( AB = \sqrt{800 - 800 \cos 1.5} \)

\( AB = 27.3 \) \( [27.2, 27.3] \) \( A1 \) \( N2 \)

[3 marks]

METHOD 2
choosing sine rule \( (M1) \)
e.g. \( \frac{\sin A}{a} = \frac{\sin B}{b} \), \( \frac{AB}{\sin O} = \frac{AO}{\sin B} \)
correct substitution \( A1 \)
e.g. \( \frac{AB}{\sin 1.5} = \frac{20}{\sin(0.5(\pi - 1.5))} \)

\( AB = 27.3 \) \( [27.2, 27.3] \) \( A1 \) \( N2 \)

[3 marks]

1c. Find the area of triangle AOB.

[2 marks]

METHOD 1

correct substitution into area formula \( A1 \)
e.g. \( \frac{1}{2}(20)(20) \sin 1.5 \), \( \frac{1}{2}(20)(27.26555) \sin(0.5(\pi - 1.5)) \)
area = 199.498997... (accept 199.75106 = 200, from using 27.3)
area = 199 \( [199, 200] \) \( A1 \) \( N1 \)

[2 marks]

1d. Angle BOC is 2.4 radians.

Find the area of the shaded region.

[3 marks]

METHOD 1
calculating sector area using their angle AOC \( (A1) \)
e.g. \( \frac{1}{2}(2.38...)(20^2) \), 200(2.38...), 476.6370614...
shaded area = their area of triangle AOB + their area of sector \( (M1) \)
e.g. 199.498997... + 476.6370614..., 199 + 476.637
shaded area = 676.136... (accept 675.637... = 676 from using 199)
shaded area = 676 \( [676, 677] \) \( A1 \) \( N2 \)

[3 marks]
Angle BOC is 2.4 radians.

The shaded region is to be painted red. Red paint is sold in cans which cost $32 each. One can covers 140 m². How much does it cost to buy the paint?

**Markscheme**

dividing to find number of cans \((MI)\)

e.g. \(\frac{376}{140} \approx 2.76 \ldots\)

5 cans must be purchased \((AI)\)

multiplying to find cost of cans \((MI)\)

e.g. \(5(32) \times \frac{676}{140} = 160\) (dollars) \((AI)\)

[4 marks]

Let \(f(x) = \cos 2x\) and \(g(x) = 2x^2 - 1\).

2a. Find \(f\left(\frac{\pi}{2}\right)\).

**Markscheme**

\(f\left(\frac{\pi}{2}\right) = \cos\pi\) \((AI)\)

\(= -1\) \((AI)\)

[2 marks]

Let \(f(x) = \cos 2x\) and \(g(x) = 2x^2 - 1\).

2b. Find \((g \circ f)\left(\frac{\pi}{2}\right)\).

**Markscheme**

\((g \circ f)\left(\frac{\pi}{2}\right) = g(-1) = 2(-1)^2 - 1\) \((AI)\)

\(= 1\) \((AI)\)

[2 marks]

2c. Given that \((g \circ f)(x)\) can be written as \(\cos(kx)\), find the value of \(k, k \in \mathbb{Z}\).

**Markscheme**

\((g \circ f)(x) = 2(\cos(2x))^2 - 1\) \((= 2\cos^2(2x) - 1)\) \((AI)\)

evidence of \(2\cos^2\theta - 1 = \cos 2\theta\) (seen anywhere) \((MI)\)

\((g \circ f)(x) = \cos 4x\)

\(k = 4\) \((AI)\)

[3 marks]
Let \( f(x) = (\sin x + \cos x)^2 \).

Show that \( f(x) \) can be expressed as \( 1 + \sin 2x \). \([2 \text{ marks}]\)

**Markscheme**

attempt to expand \((M1)\)
e.g. \((\sin x + \cos x)(\sin x + \cos x)\); at least 3 terms
correct expansion \(A1\)
e.g. \(\sin^2 x + 2 \sin x \cos x + \cos^2 x\)
\(f(x) = 1 + \sin 2x\) \(AG\) \(N0\)
\([2 \text{ marks}]\)

The diagram below shows part of the graph of \( f(x) = a \cos(b(x - c)) - 1 \), where \( a > 0 \).

The point \( P \left( \frac{\pi}{4}, 2 \right) \) is a maximum point and the point \( Q \left( \frac{3\pi}{4}, -4 \right) \) is a minimum point.

4a. Find the value of \( a \). \([2 \text{ marks}]\)

**Markscheme**
evidence of valid approach \((M1)\)
e.g. \(\frac{\text{max } y \text{ value} - \text{min } y \text{ value}}{2}\), distance from \( y = -1 \)
\(a = 3\) \(A1\) \(N2\)
\([2 \text{ marks}]\)
The diagram below shows part of the graph of \( f(x) = a \cos (b(x - c)) - 1 \), where \( a > 0 \).

\[
\begin{align*}
P \left( \frac{\pi}{4}, 2 \right) & \\
Q \left( \frac{3\pi}{4}, -4 \right) & \\
\end{align*}
\]

The point \( P \left( \frac{\pi}{4}, 2 \right) \) is a maximum point and the point \( Q \left( \frac{3\pi}{4}, -4 \right) \) is a minimum point.

4b. (i) Show that the period of \( f \) is \( \pi \).

(ii) Hence, find the value of \( b \).

**Markscheme**

(i) evidence of valid approach \( (M1) \)

- e.g. finding difference in \( x \)-coordinates, \( \frac{\pi}{2} \)
  - evidence of doubling \( A1 \)
  - e.g. \( 2 \times \left( \frac{\pi}{2} \right) \)
  - period = \( \pi \) \( AG \) \( N0 \)

(ii) evidence of valid approach \( (M1) \)

- e.g. \( b = \frac{2\pi}{\pi} \)
  - \( b = 2 \) \( A1 \) \( N2 \)

[4 marks]

4c. Given that \( 0 < c < \pi \), write down the value of \( c \).

**Markscheme**

\( c = \frac{\pi}{4} \) \( A1 \) \( N1 \)

[1 mark]

Let \( f(x) = \frac{3x}{2} + 1 \), \( g(x) = 4 \cos \left( \frac{x}{2} \right) - 1 \). Let \( h(x) = (g \circ f)(x) \).

5a. Find an expression for \( h(x) \).

[3 marks]
Markscheme

5a. 

Let \( f(x) = \frac{3x}{2} + 1 \) , \( g(x) = 4 \cos \left( \frac{x}{3} \right) - 1 \). Let \( h(x) = (g \circ f)(x) \).

5b. Write down the period of \( h \). 

Markscheme

period is \( 4\pi(12.6) \) \( AI \quad N1 \) 

[1 mark]

5c. Write down the range of \( h \). 

Markscheme

range is \( -5 \leq h(x) \leq 3 \) \( [-5, 3] \) \( A1A1 \quad N2 \) 

[2 marks]

6a. Show that \( 4 - \cos 2\theta + 5 \sin \theta = 2\sin^2 \theta + 5\sin \theta + 3 \).

Markscheme

attempt to substitute \( 1 - 2\sin^2 \theta \) for \( \cos 2\theta \) \( (MI) \) 
correct substitution \( A1 \) 
e.g. \( 4 - (1 - 2\sin^2 \theta) + 5\sin \theta \) 
\( 4 - \cos 2\theta + 5 \sin \theta = 2\sin^2 \theta + 5 \sin \theta + 3 \) \( AG \quad N0 \) 

[2 marks]

6b. Hence, solve the equation \( 4 - \cos 2\theta + 5 \sin \theta = 0 \) for \( 0 \leq \theta \leq 2\pi \).

Markscheme

evidence of appropriate approach to solve \( (MI) \) 
e.g. factorizing, quadratic formula 
correct working \( A1 \) 
e.g. \( (2\sin \theta + 3)(\sin \theta + 1) \) , \( (2x + 3)(x + 1) = 0 \) , \( \sin x = \frac{-5 + \sqrt{49}}{4} \) 
correct solution \( \sin \theta = -1 \) (do not penalise for including \( \sin \theta = -\frac{3}{2} \)) \( (AI) \) 
(\( \theta = \frac{3\pi}{2} \) \( A2 \quad N3 \) 

[5 marks]
The following diagram represents a large Ferris wheel at an amusement park.

The points P, Q and R represent different positions of a seat on the wheel.

The wheel has a radius of 50 metres and rotates clockwise at a rate of one revolution every 30 minutes.

A seat starts at the lowest point P, when its height is one metre above the ground.

7a. Find the height of a seat above the ground after 15 minutes. [2 marks]

**Markscheme**
valid approach  \((M1)\)
e.g. 15 mins is half way, top of the wheel, \(d + 1\)
height = 101 (metres) \(A1\) \(N2\)

[2 marks]

7b. After six minutes, the seat is at point Q. Find its height above the ground at Q. [5 marks]

**Markscheme**
evidence of identifying rotation angle after 6 minutes \(A1\)
e.g. \(\frac{2\pi}{5} \times \frac{1}{5}\) of a rotation, 72°
evidence of appropriate approach \((M1)\)
e.g. drawing a right triangle and using cosine ratio
correct working (seen anywhere) \(A1\)
e.g. \(\cos \frac{2\pi}{5} = \frac{50}{x} \), 15.4(508…)
evidence of appropriate method \(M1\)
e.g. height = radius + 1 − 15.45…
height = 35.5 (metres) (accept 35.6) \(A1\) \(N2\)

[5 marks]

7c. The height of the seat above ground after \(t\) minutes can be modelled by the function \(h(t) = 50\sin(b(t - c)) + 51\). [6 marks]

Find the value of \(b\) and of \(c\).
**Markscheme**

**METHOD 1**

evidence of substituting into \( b = \frac{2\pi}{\text{period}} \) (MI)
correct substitution
e.g. period = 30 minutes, \( b = \frac{2\pi}{30} \) \( A1 \)

\( b = 0.209 \ (\frac{\pi}{15}) \) \( A1 \ N2 \)

substituting into \( h(t) \) (MI)
e.g. \( h(0) = 1, h(15) = 101 \)
correct substitution \( A1 \)

\( 1 = 50 \sin \left( \frac{\pi}{15}c \right) + 51 \)

\( c = 7.5 \) \( A1 \ N2 \)

**METHOD 2**
evidence of setting up a system of equations (MI)
two correct equations
e.g. \( 1 = 50 \sin b(0 - c) + 51, 101 = 50 \sin b(15 - c) + 51 \) \( A1A1 \)

attempt to solve simultaneously (MI)
e.g. evidence of combining two equations

\( b = 0.209 \ (\frac{\pi}{15}) \) \( A1A1 \)

\( c = 7.5 \) \( N2N2 \)

[6 marks]

**7d.** The height of the seat above ground after \( t \) minutes can be modelled by the function \( h(t) = 50 \sin(b(t - c)) + 51 \) .

Hence find the value of \( t \) the first time the seat is 96 m above the ground.

**Markscheme**
evidence of solving \( h(t) = 96 \) (MI)
e.g. equation, graph

\( t = 12.8 \) (minutes) \( A2 \ N3 \)

[3 marks]

Consider the following circle with centre \( O \) and radius \( r \).

![](image)

The points \( P, R \) and \( Q \) are on the circumference, \( P\hat{O}Q = 2\theta \), for \( 0 < \theta < \frac{\pi}{2} \).

**8a.** Use the cosine rule to show that \( PQ = 2r \sin \theta \). [4 marks]
**Markscheme**

correct substitution into cosine rule \( A1 \)
e.g. \( PQ^2 = r^2 + r^2 - 2(r)(r) \cos(2\theta) \), \( PQ^2 = 2r^2 - 2r^2(\cos(2\theta)) \)
substituting \( 1 - 2\sin^2\theta \) for \( \cos(2\theta) \) (seen anywhere) \( A1 \)
e.g. \( PQ^2 = 2r^2 - 2r^2(1 - 2\sin^2\theta) \)
working towards answer \( A1 \)
e.g. \( PQ^2 = 4r^2\sin^2\theta \), \( PQ = \sqrt{4r^2\sin^2\theta} \)
PQ = 2r\sin\theta \( AG \) \( N0 \)

\( [4 \text{ marks}] \)

8b. Let \( l \) be the length of the arc \( PRQ \).

Given that \( 1.3PQ - l = 0 \), find the value of \( \theta \).

**Markscheme**

\( PRQ = r \times 2\theta \) (seen anywhere) \( A1 \)
correct set up \( A1 \)
e.g. \( 1.3 \times 2r\sin\theta - r \times (2\theta) = 0 \)
attempt to eliminate \( r \) \( M1 \)
correct equation in terms of the one variable \( \theta \) \( A1 \)
e.g. \( 1.3 \times 2\sin\theta - 2\theta = 0 \)
1.221496215
\( \theta = 1.22 \) (accept 70.0° (69.9)) \( A1 \) \( N3 \)

\( [5 \text{ marks}] \)

8c. Consider the function \( f(\theta) = 2.6\sin\theta - 2\theta \), for \( 0 < \theta < \frac{\pi}{2} \).

(i) Sketch the graph of \( f \).

(ii) Write down the root of \( f(\theta) = 0 \).
**Markscheme**

(i)

![Diagram]

**Note:** Award A1 for approximately correct shape, A1 for x-intercept in approximately correct position, A1 for domain. Do not penalise if sketch starts at origin.

(ii) 1.221496215

$\theta = 1.22$ \hspace{1cm} A1 \hspace{1cm} N1

[4 marks]

8d. Use the graph of $f$ to find the values of $\theta$ for which $l < 1.3PQ$.

[3 marks]

**Markscheme**

Evidence of appropriate approach (may be seen earlier) \hspace{1cm} M2

e.g. $2\theta < 2.6\sin \theta$, $0 < f(\theta)$, showing positive part of sketch

$0 < \theta < 1.221496215$

$0 < \theta < 1.22$ (accept $\theta < 1.22$) \hspace{1cm} A1 \hspace{1cm} N1

[3 marks]

Consider the triangle ABC, where $AB = 10$, $BC = 7$ and $\angle BAC = 30^\circ$.

9a. Find the two possible values of $\angle ABC$.

[4 marks]

**Markscheme**

**Note:** accept answers given in degrees, and minutes.

Evidence of choosing sine rule \hspace{1cm} M1

e.g. $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$

correct substitution \hspace{1cm} A1

e.g. $\frac{\sin \theta}{10} = \frac{\sin 30^\circ}{7}$, $\sin \theta = \frac{5}{7}$

$\angle ABC = 45.6^\circ$, $\angle ACB = 134^\circ$ \hspace{1cm} A1A1 \hspace{1cm} N1N1

**Note:** If candidates only find the acute angle in part (a), award no marks for (b).

[4 marks]
9b. Hence, find $\hat{A}\hat{B}\hat{C}$, given that it is acute.

**Markscheme**

attempt to substitute their larger value into angle sum of triangle \((M1)\)

e.g. $180^\circ - (134.415^\circ + 30^\circ)$

$A\hat{B}\hat{C} = 15.6^\circ \quad AI \quad N2$

[2 marks]