Key

Practice Test on Chapter 12: Vectors [Calculator prohibited]

1. The vectors u, v are given by $\mathbf{u} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Find real numbers a and b such that $a(\mathbf{u} + \mathbf{v}) = \begin{bmatrix} 8 \\ b-2 \end{bmatrix}$

$4a = 8$ $a = 2$

$3a = b - 2$ $b = 8$

2. A line passes through the point (4 , -1) and its direction is perpendicular to the vector 2i + 3j. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $m = \frac{3}{2}$

Find the equation of the line in the form ax + by = c, where a, b and c are integers to be determined.

$m = \frac{-2}{3}$ $(4, -1)$

$y + 1 = -\frac{2}{3}(x - 4)$

$y + 1 = -\frac{2}{3}x + \frac{8}{3}$

$3y + 3 = -2x + 8$

$2x + 3y = 5$

3. A particle is moving with a constant velocity along line L. Its initial position is A(6, −2, 10) . After one second the particle has moved to B(9, −6, 15).

ai) Find the velocity vector, $\vec{AB}$.

$$\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$$

aii) Find the speed of the particle.

$$|\vec{AB}|$$

$$\sqrt{+3^2+(-4)^2+5^2} = \sqrt{50}$$

b) Write down an equation of the line L.

$$r = \begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix} + t\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$$

4. The vector equations of two lines are given below:

$$r_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + s\begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad r_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The lines intersect at the point P. Find the position vector of P.

$$5 + 3s = -2 + 4t$$

$$1 - 2s = 2 + t$$

$$3s - 4t = -7$$

$$+8s + 4t = -4$$

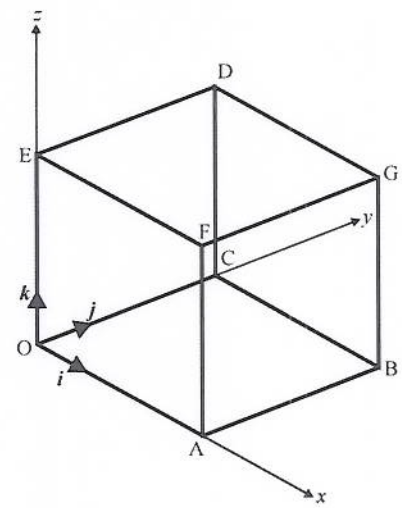
$$11s = -11$$

$$s = -1$$

$$P: \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

5. The diagram shows a cube, OABCDEFG where the length of each edge is 5cm. Express the following vectors in terms of unit vectors i, j and k.

(a) $\overrightarrow{OG}$ $\begin{pmatrix}5\\5\\5\end{pmatrix}$ or $5i+5j+5k$

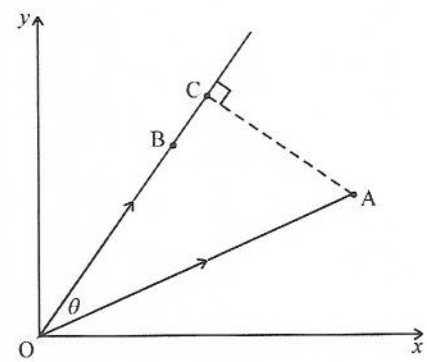


(b) $\overrightarrow{BD}$ $-5i+5k$

(c) $\overrightarrow{EB}$ $5i+5j-5k$

6. The following diagram shows the point O with coordinates (0, 0), the point A with position vector a = 12i + 5j, and the point B with position vector b = 6i + 8j. The angle between (OA) and (OB) is θ.

Find



(i) | ***a*** |; $\sqrt{12^2+5^2} = 13$

(ii) a unit vector in the direction of ***b***;

$|b| = \sqrt{6^2+8^2} = 10$

$\frac{1}{10}\begin{pmatrix}6\\8\end{pmatrix}$

(iii) the **exact** value of cos θ in the form $\frac{p}{q}$, where *p* and *q* are integers.

$a \cdot b = 112$

$\cos\theta = \frac{a\cdot b}{|a||b|} = \frac{112}{(13)(10)} = \frac{112}{130}$

$= \frac{56}{65}$

7. The vectors $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$ and $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$ are perpendicular for two values of x.

(a) Write down the quadratic equation which the two values of x must satisfy.

$$2x(x+1) + (x-3)(5) = 0$$

$$2x^2 + 2x + 5x - 15 = 0$$

$$2x^2 + 7x - 15 = 0$$

(b) Find the two values of x which satisfy the equation.

$$(2x-3)(x+5) = 0$$

$2x - 3 = 0$ $\quad\quad$ $x + 5 = 0$

$x = \frac{3}{2}$ $\quad\quad$ $x = -5$