8. This question refers to quadrilateral ABCD as described below. *[8 marks]*

a) The points A and B on the quadrilateral ABCD have the position vectors $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ respectively.

(i) Find the vector $\overrightarrow{AB}$. [2]

$$\begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

(ii) Find $|\overrightarrow{AB}|$. [2]

$$\sqrt{(-5)^2 + 1^2} = \sqrt{26}$$

b) The point D on the quadrilateral ABCD has position vector $\begin{pmatrix} 7 \\ 23 \end{pmatrix}$.

i) The vector $\overrightarrow{AB}$ is equal to what other vector on the rectangle?

$$\overrightarrow{DC}$$

ii) Hence, find the position vector of the point C. [4]

$$D + \overrightarrow{CD} = C$$

$$\begin{pmatrix} 7 \\ 23 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 24 \end{pmatrix}$$

9. [13 marks]

In this question, distance is in kilometers, time is in hours. A balloon is moving at a constant height with a speed of 18 km h^{-1}, in the direction of the vector $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$. At time $t = 0$, the balloon is at point B, coordinates (0, 0, 5).

(a) Show that the position vector $\boldsymbol{b}$ of the balloon at time t is given by

$$\boldsymbol{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}.$$

[4]

$\left\| \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \right\| = 5$

$5k = 18$

$k = 3.6$

Velocity $= 3.6 \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}$

@ $t=0$, $\begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$

SO

$$b = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}$$

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9. *Continued from the previous page.*

At time $t = 0$, a helicopter goes to deliver a message to the balloon. The position vector $\boldsymbol{h}$ of the helicopter at time t is given by

$$\boldsymbol{h} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 49 \\ 32 \\ 0 \end{pmatrix} + t \begin{pmatrix} -48 \\ -24 \\ 6 \end{pmatrix}$$

bi) Write down the coordinates of the starting position of the helicopter. [1]

$\begin{pmatrix} 49 \\ 32 \\ 0 \end{pmatrix}$

bii) Find the speed of the helicopter. [3]

$\left| \begin{pmatrix} -48 \\ -24 \\ 6 \end{pmatrix} \right| = 54$ km/hr.

The helicopter reaches the balloon at point R.

(i) Find the time the helicopter takes to reach the balloon in minutes. [3]

$\begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix} = \begin{pmatrix} 49 \\ 32 \\ 0 \end{pmatrix} + t \begin{pmatrix} -48 \\ -24 \\ 6 \end{pmatrix}$

$5 = 6t$

$\frac{5}{6} = t$

$t = 50$ min.

(ii) Find the coordinates of R. [2]

$R = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + \frac{5}{6} \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}$

$R = \begin{pmatrix} 9 \\ 12 \\ 5 \end{pmatrix}$

10. [18 marks]

The point O has coordinates (0, 0, 0), point A has coordinates (1, -2, 3) and point B has coordinates (-3, 4, 2).

ai) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$ [1]

$\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$

aii) Find $B\hat{A}O$. [7]

$\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$ $\overrightarrow{AO} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

$|\overrightarrow{AB}| = 7.28 \quad (\sqrt{53})$

$|\overrightarrow{AO}| = \sqrt{14}$

$\overrightarrow{AB} \cdot \overrightarrow{AO} = -13$

$\cos\theta = \frac{-13}{\sqrt{53}\sqrt{14}}$

$\angle BAO = 119°$

b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of any two points on L_1. [2]

$\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -7 \\ 10 \\ 1 \end{pmatrix}$ (when $s=1$)

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10. *This question continues from the previous page.*

c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

i) Find a vector equation for L_2, giving your answer in the form $r = a + tb$. [2]

$$r = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$$

ii) Point C(k, -k, 5) is on L_2. Find the coordinates of C. [4]

$$\begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$$

$k = 1 - 3t$ $\quad -k = -2 + 4t$ $\quad 5 = 3 + 2t$

$k = -2$ $\quad k = -2$ $\quad 1 = t$

$$C = \begin{pmatrix} -2 \\ +2 \\ 5 \end{pmatrix}$$

d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C. Find the value of p at C. [2]

$$\begin{pmatrix} -2 \\ +2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

$5 = -p$

$p = -5$