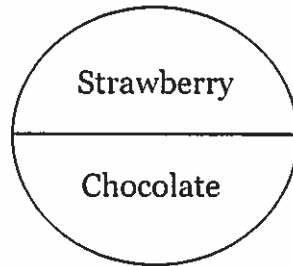


Chapter 3 Test Review: Fair Division

1. Jody buys a chocolate-strawberry mousse cake for \$15.00. Jody values strawberry four times as much as she values chocolate.



$$\begin{array}{r} S \quad C \\ 4x + x = 13.50 \\ 5x = 13.50 \\ x = 2.70 \end{array}$$

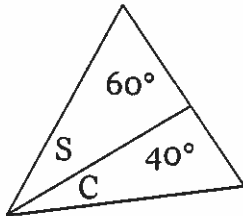
- a. What is the value of the chocolate half of the cake to Jody?

\$ 2.70

- b. What is the value of the strawberry half of the cake to Jody?

\$ 10.80

- c. A piece of cake is cut and shown below. What is the value of the piece to Jody? Round to the nearest hundredth.



$$\frac{60}{180} (10.80) + \frac{40}{180} (2.70) = \mathbf{\$ 4.20}$$

3.60 .60

2. Malia buys a chocolate-strawberry-vanilla cake for \$11.20 (please reference the picture on page 112 of your textbook). Malia values strawberry twice as much as chocolate and values chocolate twice as much as vanilla.

- a. What is the value of the chocolate part of the cake to Malia?

\$ 3.20

- b. What is the value of the strawberry part of the cake to Malia?

\$ 6.40

- c. What is the value of the vanilla part of the cake to Malia?

\$ 1.60

$$\begin{array}{r} S \quad C \quad V \\ 4x + 2x + x = 11.20 \\ 7x = 11.20 \\ x = 1.60 \end{array}$$

- d. If the cake is cut into six equal size pieces, find the value to Malia of each of the six pieces. You need to go to use the picture on the last page of this packet.

$$\text{Piece 1} = \frac{60}{120} (1.60) = \$0.80$$

$$\text{Piece 2} = \frac{30}{120} (1.60) + \frac{30}{120} (6.40) = \$2$$

$$\text{Piece 3} = \frac{60}{120} (6.40) = \$3.20$$

$$\text{Piece 4} = \frac{30}{120} (6.40) + \frac{30}{120} (3.20) = \$2.40$$

$$\text{Piece 5} = \frac{60}{120} (3.20) = \$1.60$$

$$\text{Piece 6} = \frac{30}{120} (3.20) + \frac{30}{120} (1.60) = \$1.20$$

3. Three players (Alexis, Beatrice, and Cynthia) must divide a cake among themselves. Suppose the cake is divided into three slices (s_1 , s_2 , and s_3). The following table shows the percentage of the value of the entire cake that each slice represents to each player.

$$\frac{100}{3} = 33.3\%$$

	s_1	s_2	s_3
Alexis	30%	40%	30%
Beatrice	35%	25%	40%
Cynthia	33.3%	50%	16.7%

- a. Indicate which of the three slices are fair shares to Alexis.

s_2

- b. Indicate which of the three slices are fair shares to Beatrice.

$s_1 + s_3$

- c. Indicate which of the three slices are fair shares to Cynthia.

$s_1 + s_2$

- d. Describe the fair division of the cake.

Alexis = s_2

Beatrice = s_3

Cynthia = s_1

4. Abe, Betty, Cory, and Dana are dividing a cake among themselves. The cake is divided into four slices $s_1, s_2, s_3,$ and s_4 . The values of the cake and of each of the four slices in the eyes of each of the players are shown in the following table.

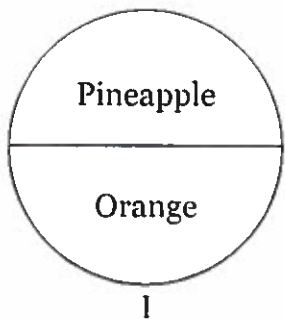
	Whole cake	s_1	s_2	s_3	s_4
Abe	\$15.00	\$3.00	\$5.00	\$5.00	\$2.00
Betty	\$18.00	\$4.50	\$4.50	\$4.50	\$4.50
Cory	\$13.00	\$4.00	\$3.50	\$1.50	\$3.00
Dana	\$10.00	\$2.75	\$2.40	\$2.45	\$2.40

- Who was the divider?
- Indicate which of the four slices are fair shares to Abe.
 $15/4 = 3.75$
- Indicate which of the four slices are fair shares to Betty.
 $18/4 = 4.50$
- Indicate which of the four slices are fair shares to Cory.
 $13/4 = 3.25$
- Indicate which of the four slices are fair shares to Dana.
 $10/4 = 2.50$
- Using $s_1, s_2, s_3,$ and s_4 as the four shares, describe a fair division of the cake.

Betty
 $s_2 + s_3$
 s_1, s_2, s_3, s_4
 $s_1 + s_2$
 s_1

Abe = s_3 Betty = s_4 Cory = s_2 Dana = s_1

5. Ziva and Abby want to divide an orange-pineapple cake shown to the left (I). They are using the divider-chooser method. Ziva values orange four times as much as she values pineapple. Abby is the divider and cuts the cake as shown to the left (II).



- What percent of the value of the cake is the pineapple half in Abby's eyes?

50%

- What percent of the value of the cake is each piece in (II) in Ziva's eyes?

$$\begin{aligned} 4x + x &= 100 \\ x &= 20 \end{aligned}$$

$$\text{left piece} = \frac{60}{180} (.8) + \frac{120}{180} (.2) = .4$$

orange = 80%
pineapple = 20%

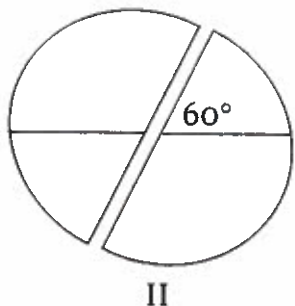
Left piece = 40%

Right piece = 60%

- Describe the final fair division of the cake.

Ziva = right piece

Abby = left piece



6. Three partners (Divine, Chase, and Courtney) are dividing a plot of land among themselves using the lone-divider method. Using a map, the divider (Divine) divides the property into three parcels, s_1 , s_2 , and s_3 . When the choosers' bid lists are opened, Chase's bid list is $\{s_1, s_2, s_3\}$ and Courtney's bid list is $\{s_1\}$.

- a. Describe the fair division where Divine's fair share is s_2 .

Divine = s_2 Chase = s_3 Courtney = s_1

- b. Describe the fair division where Divine's fair share is s_3 .

Divine = s_3 Chase = s_2 Courtney = s_1

7. Four partners (Ross, Rachel, Monica, and Chandler) are dividing a plot of land among themselves using the lone-divider method. Using a map, Ross divides the land into four parcels s_1 , s_2 , s_3 , and s_4 . When the chooser's bid lists are opened, Rachel bid list is $\{s_2, s_3\}$, Monica's bid list is $\{s_3, s_4\}$, and Chandler's bid list is $\{s_4\}$. Describe a fair division of the land.

Ross = s_1 Rachel = s_2 Monica = s_3 Chandler = s_4

8. Desi, Cher, Cheech, and Chong are dividing a cake among themselves using the lone-divider method. Desi cuts the cake into four slices s_1 , s_2 , s_3 , and s_4 . When the chooser's bid lists are opened, Cher's bid list is $\{s_2, s_3\}$, Cheech's bid list is $\{s_1, s_3\}$, and Chong's bid list is $\{s_1, s_2\}$.

- a. Describe the fair division where Cher's fair share is s_2 .

Desi = s_4 Cher = s_2 Cheech = s_3 Chong = s_1

- b. Describe the fair division where Chong's fair share is s_2 .

Desi = s_4 Cher = s_3 Cheech = s_1 Chong = s_2

9. Five players (Justin, JC, Lance, Joey, and Chris) are dividing a cake among themselves using the lone-divider method. Justin cuts the cake into five slices s_1 , s_2 , s_3 , s_4 , s_5 . When the choosers' bid lists are opened, JC's bid list is $\{s_2, s_5\}$, Lance's bid list $\{s_1, s_2\}$, Joey's bid list is $\{s_1, s_4, s_5\}$, and Chris' bid list is $\{s_1, s_5\}$.

- a. Describe the fair division when Chris' fair share is s_1 .

Justin = s_3 JC = s_5 Lance = s_2 Joey = s_4 Chris = s_1

- b. Describe the fair division when Lance's fair share is s_1 .

Justin = s_3 JC = s_2 Lance = s_1 Joey = s_4 Chris = s_5

10. Six players (Ash, Pikachu, Charmander, Squirtle, Bulbasaur, and Jigglypuff) are dividing a cake among themselves using the lone-divider method. Ash cuts the cake into six slices $s_1, s_2, s_3, s_4, s_5,$ and s_6 . When the chooser's bid list opened, Pikachu's bid list is $\{s_2, s_3, s_5\}$, Charmander's bid list is $\{s_1, s_5, s_6\}$, Squirtle's bid list is $\{s_3, s_5, s_6\}$, Bulbasaur's bid list is $\{s_2, s_3\}$, and Jigglypuff's bid list is $\{s_3\}$. Describe a fair division of the cake.

Ash = s_4 Pikachu = s_5 Charmander = s_1

Squirtle = s_6 Bulbasaur = s_2 Jigglypuff = s_3

11. Four partners (Hercules, Iolaus, Xena, and Gabrielle) are dividing a piece of land among themselves using the lone-divider method. Using a map, the divider divides the property into four parcels $s_1, s_2, s_3,$ and s_4 . The following table shows the relative value of the parcels (as a percentage of the total value of the land) in the eyes of each partner, but some of the entries in the table are missing.

	s_1	s_2	s_3	s_4
Hercules	<u>27%</u>	30%	21%	22%
Iolaus	35%	20%	<u>25%</u>	20%
Xena	<u>25%</u>	26%	28%	21%
Gabrielle	25%	25%	<u>25%</u>	<u>25%</u>

- a. Fill in the remainder of the table.
 b. Who was the divider?

Gabrielle

$$\frac{100}{4} = 25\%$$

- c. Describe the chooser's respective bid lists.

Hercules = s_1, s_2

Iolaus = s_1, s_3

Xena = s_1, s_2, s_3

- d. Describe the fair division of the property where Xena's fair share is s_2 .

Hercules = s_1

Iolaus = s_3

Xena = s_2

Gabrielle = s_4

12. Jared, Karla, and Lori are dividing a sub sandwich (half meatball, half vegetarian) using the lone-chooser method. Jared likes the vegetarian and meatball parts equally well, Karla is a strict vegetarian and does not eat meat at all, and Lori likes the meatball part twice as much as she likes the vegetarian part. Suppose that Karla and Jared are the dividers, and Lori is the chooser. In the first division Karla makes the first cut, and Jared chooses the share he likes best.

a. Describe the first division (exactly where Karla cuts the sandwich and who ends up with which piece).

see last page

b. Describe the second division that Jared makes on his share of the sandwich.

see last page

c. Describe the second division that Karla makes on her share of the sandwich.

see last page

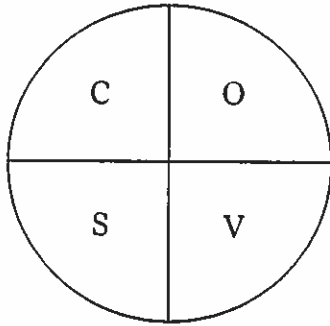
d. Describe the final fair division of the sandwich, and give the value of each share as a percentage of the total value of the sandwich in the eyes of the player receiving it.

see last page

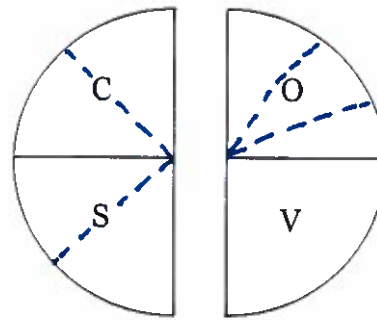
13. Justin Timberlake, Channing Tatum, and Chris Hemsworth are dividing the cake shown below using the lone-chooser method. The players value the different parts of the cake as follows:

- Justin likes chocolate and orange equally well, but hates strawberry and vanilla.
- Channing likes chocolate and strawberry equally well, but hates orange and vanilla.
- Chris likes chocolate and vanilla equally well, but hates orange and strawberry.

Suppose that Justin and Channing are the dividers, and Chris is the chooser. In the first division Justin makes the first cut as shown below, and Channing chooses the share he likes best.



Original Cake



Cake after Justin's cut

- a. Since Justin was the divider and Channing gets to choose which share he likes best, which half will Channing take?

The left piece because he likes chocolate + strawberry.

- b. Describe how Justin will divide his half of the cake (remember to consider there are three people).

Since Justin has the right piece + only likes the orange section, he will divide the orange part into thirds

- c. Describe how Channing will divide his half of the cake (remember to consider there are three people).

Of Channing's left piece, he likes chocolate + strawberry the same so he will divide his whole half into thirds

- d. Now that Justin and Channing have divided their shares, which piece will Chris take from each share?

Chris will take a full chocolate piece from the left + take the slice that has all of vanilla + a little orange from the right.

14. A cake values at \$30 is divided among five players ($P_1, P_2, P_3, P_4,$ and P_5) using the last-diminsher method. The players play in a fixed order, with P_1 first, P_2 second, and so on. In round 1, P_1 makes the first cut and makes a claim. For each of the remaining players, the value of the current piece at the time it is their turn to play is given in the following table.

	P_2	P_3	P_4	P_5
Value of the current piece	\$7.00	\$4.50	\$6.50	\$4.00

- a. Which player gets his or her share at the end of round 1?

$$\frac{30}{5} = \$6$$

P_4

- b. What is the value of the share to the player receiving it?

\$6.00

- c. Which player makes the first cut to start round 2?

P_1

15. A cake values at \$30 is divided among five players ($P_1, P_2, P_3, P_4,$ and P_5) using the last-diminsher method. The players play in a fixed order, with P_1 first, P_2 second, and so on. In round 1, P_1 makes the first cut and makes a claim. For each of the remaining players, the value of the current piece at the time it is their turn to play is given in the following table.

	P_2	P_3	P_4	P_5
Value of the current piece	\$5.00	\$5.50	\$8.50	\$7.00

- a. Which player gets his or her share at the end of round 1?

$$\frac{30}{5} = \$6$$

P_5

- b. What is the value of the share to the player receiving it?

\$7.00

- c. Which player makes the first cut to start round 2?

P_1

16. A cake is divided among six players (P_1, P_2, P_3, P_4, P_5 and P_6) using the last-diminisher method. The players play in a fixed order, with P_1 first, P_2 second, and so on. In round 1, after P_1 makes the first cut, $P_2, P_5,$ and P_6 are the only diminishers. In round 2, after the first claim is made, there are no diminishers. In round 3, after the first claim is made, each successive player is a diminisher.

a. Which player gets his or her share at the end of round 1?

P_6

b. Which player makes the first cut to start round 2?

P_1

c. Which player gets his or her share at the end of round 2?

P_1

d. Which player gets his or her share at the end of round 3?

P_5

e. Which player makes the first cut to start round 4?

P_2

17. A cake is divided among 12 players ($P_1, P_2, P_3, \dots, P_{12}$) using the last-diminisher method. The players play in a fixed order, with P_1 first, P_2 second, and so on. In round 1, after P_1 makes the first cut, $P_3, P_7,$ and P_9 are the only diminishers. In round 2, after the first claim is made, P_5 is the only diminisher. In round 3, after the first claim is made, all the other players pass.

a. Which player gets his or her share at the end of round 1?

P_9

b. Which player makes the first cut to start round 2?

P_1

c. Which player gets his or her share at the end of round 2?

P_5

d. Which player gets his or her share at the end of round 3?

P_1

e. Which player makes the first cut to start round 4?

P_2

18. Mario and Luigi are dividing an estate consisting of a cabin and a classic car using the method of sealed bids.

	Mario	Luigi
Cabin	\$60,900	\$65,300
Classic Car	\$29,200	\$33,200
Total Bids	\$90,100	\$98,500
Fair Share	\$45,050	\$49,250
Item Value	\$0	\$98,500
Cash Settlement	- \$45,050 receive	\$49,250 owe
Surplus	\$2100	\$2100
Total Estate	\$47,150 receive	\$47,150 owe

Mario = gets \$47,150 cash

Luigi = gets cabin + car + pays \$47,150

19. Five heirs (A, B, C, D, and E) are dividing an estate consisting of six items using the method of sealed bids. The heir's bids on each of the items are given in the following table.

	A	B	C	D	E
Item 1	\$352	\$295	\$395	\$368	\$324
Item 2	\$98	\$102	\$98	\$95	\$105
Item 3	\$460	\$449	\$510	\$501	\$476
Item 4	\$852	\$825	\$832	\$817	\$843
Item 5	\$513	\$501	\$505	\$505	\$491
Item 6	\$725	\$738	\$750	\$744	\$761
Total Bids	\$3000	\$2910	\$3090	\$3030	\$3000
Fair Share	\$600	\$582	\$618	\$606	\$600
Item Value	\$1365	\$0	\$905	\$0	\$866
Cash Settlement	\$765 owe	-\$582 receive	\$287 owe	-\$606 receive	\$266 owe
Surplus	\$26	\$26	\$26	\$26	\$26
Total Estate	\$739 owe	\$608 receive	\$261 owe	\$632 receive	\$240 owe

A = gets item 4 + item 5 + pays \$739

B = gets \$608 in cash

C = gets item 1 + item 3 + pays \$261

D = gets \$632 in cash

E = gets item 2 + item 6 + pays \$240

20. Alan, Bly, and Claire are dividing five items using the method of sealed bids. Their bids on each of the items are given in the following table.

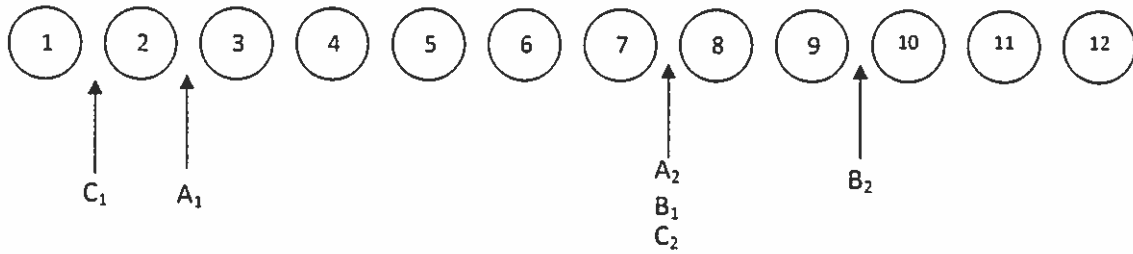
	Alan	Bly	Claire
Item 1	\$14,000	\$12,000	\$22,000
Item 2	\$24,000	\$15,000	\$33,000
Item 3	\$16,000	\$18,000	\$14,000
Item 4	\$16,000	\$16,000	\$18,000
Item 5	\$18,000	\$24,000	\$20,000
Total Bids	\$88,000	\$85,000	\$107,000
Fair Share	\$29,333.33	\$28,333.33	\$35,666.67
Item Value	\$0	\$42,000	\$73,000
Cash Settlement	-\$29,333.33 receive	\$13,666.67 owe	\$37,333.33 owe
Surplus	\$7,222.22	\$7,222.22	\$7,222.22
Total Estate	\$36,555.55 receive	\$6,444.45 owe	\$30,111.11 owe

Alan = gets \$36,555.55 in cash

Bly = gets item 3 + item 5 + pays \$6,444.45

Claire = gets item 1, item 2, item 4 + pays \$30,111.11

21. Three children (*A*, *B*, and *C*) are dividing the array of 12 candy pieces shown in the following figure using the method of markers. The players' bids are indicated in the figure.



- a. Describe the allocation of candy to each child.

$A = \underline{3, 4, 5, 6, 7}$

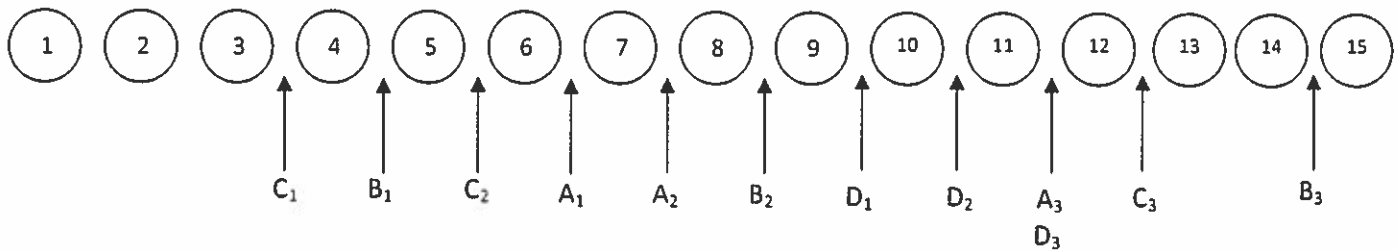
$B = \underline{10, 11, 12}$

$C = \underline{1}$

- b. Which items are left over?

$\underline{2, 8, 9}$

22. Four players (*A*, *B*, *C*, and *D*) are dividing the array of 15 items shown in the following figure using the method of markers. The players' bids are indicated in the figure.



- a. Describe the allocation of items to each player.

$A = \underline{7}$

$B = \underline{15}$

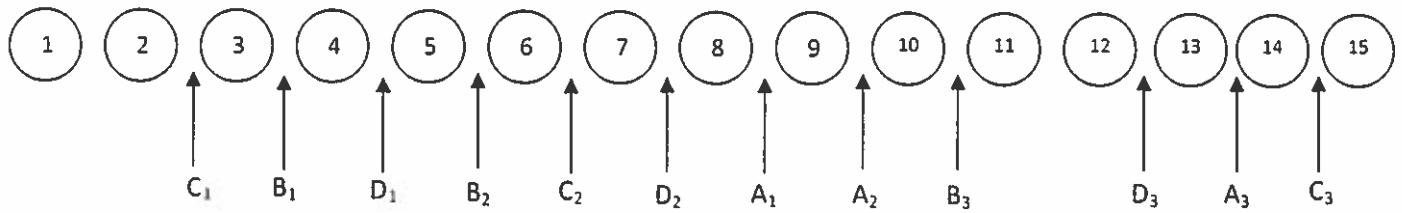
$C = \underline{1, 2, 3}$

$D = \underline{11}$

- b. Which items are left over?

$\underline{4, 5, 6, 8, 9, 10, 12, 13, 14}$

23. Use the following set of markers to answer the following questions:



- a. What pieces does *A* get? 14, 15
- b. What pieces does *B* get? 4, 5
- c. What pieces does *C* get? 1, 2
- d. What pieces does *D* get? 8, 9, 10, 11, 12
- e. What is leftover? 3, 6, 7, 13

24. What are the essential elements for fair division problems?

The goods, the players, the value systems

25. What are the four characteristics of fair division schemes?

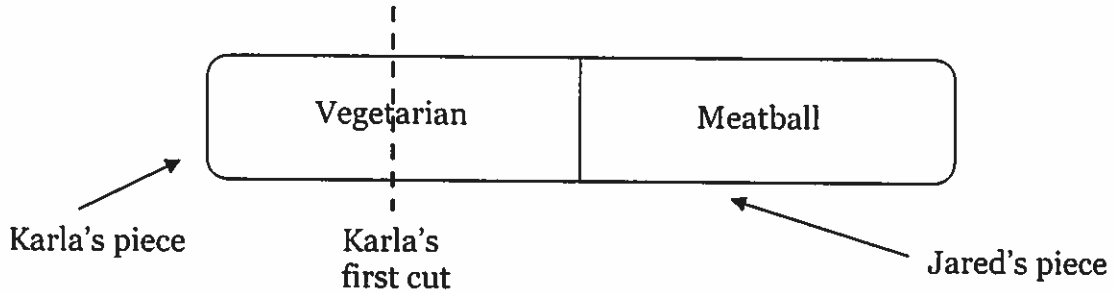
Rationality, cooperation, privacy, symmetry

26. What are the three different fair-division methods and what do they mean?

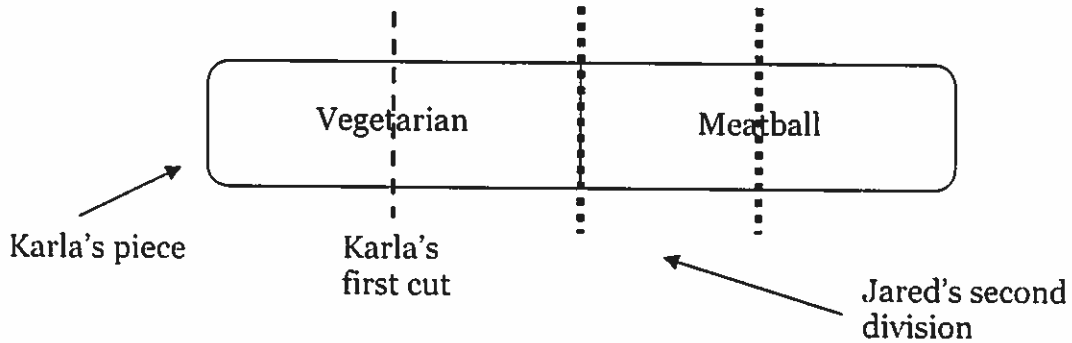
Continuous (divisible goods), discrete (indivisible goods), mixed (divisible + indivisible goods)

Problem #12

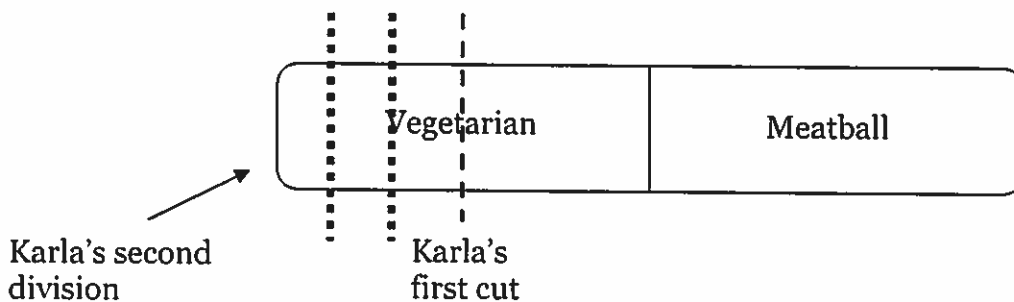
- a. Karla would divide the sandwich into one piece consisting of half of the vegetarian part ($1/4$ of the entire size of the sandwich) and another piece consisting of half of the vegetarian part and the entire meatball part ($3/4$ of the entire size of the sandwich). Jared would choose the larger slice containing the meatball part and half of the vegetarian part.



- b. Jared would divide his piece in three equally sized pieces – two pieces would be all meat (each $1/4$ of the total sandwich) and the other would be all vegetarian ($1/4$ of the total sandwich).



- c. Karla would divide her piece into three equally sized pieces – each piece would be all vegetarian (each $1/3$ of $1/4$ of the total sandwich)



- d. Since Lori, the chooser, likes the meatball part twice as much as the vegetarian part, she will select one of Jared's meatball parts and one of Karla's vegetarian parts.