Key

IB Math Standard

REVIEW

Test on Applications of Integration [CALCULATOR PROHIBITED]

Unless otherwise indicated, all answers should be given exactly or to three significant figures.

1 & 2. Evaluate these definite integrals. [4 marks each]

a) $\int_{-2}^{2}(6x^2+1)\,dx$

$= 2x^3 + x \Big|_{-2}^{2}$

$= 2(2)^3 + 2 - (2(-2)^3 + (-2))$

$= 18 - (-18)$

$= 36$

b) $\int_{1}^{4}\frac{10}{\sqrt{x}}\,dx$

$= \int_1^4 10x^{-1/2}\,dx$

$= 20x^{1/2}\Big|_1^4$

$= 20(4)^{1/2} - 20(1)^{1/2}$

$= 40 - 20 = 20$

c) $\int_{0}^{\pi}(1+2\sin x)dx$

$= x - 2\cos x \Big|_0^{\pi}$

$= \pi - 2\cos\pi - (0 - 2\cos(0))$

$= \pi - 2(-1) + 2(1)$

$= \pi + 2 + 2$

$= 4 + \pi$

3. Let g be a function such that $\int_1^3 g(x)dx = 10$.

a) Deduce the value of

$\int_1^3 \frac{1}{2} g(x)dx$ $= 5$

$\int_1^3 [g(x)+4]dx$ $= 10 + 4x\Big|_1^3$

$= 10 + (12-4) = 18$

b) If $\int_c^d g(x+2)dx = 10$, write down the values of c and of d.

$c = -1$

$d = 5$

4. Evaluate $\int_0^1 16x(x^2+5)^3\, dx$ using an appropriate u-substitution.

$u = x^2+5$

$8\, du = 2x\, dx \cdot 8$

$8\, du = 16x\, dx$

$= 8\int_0^1 u^3\, du$

$= 8\left(\frac{1}{4}u^4\right)\Big|_0^1$

$= 2(x^2+5)^4\Big|_0^1$

$= 2\left[(1^2+5)^4 - (0^2+5)^4\right]$

$= 2(1296 - 625) = 1342$

5. For the first few seconds of a race, a horse's acceleration is modeled by the equation $a = 6 - 1.2t$, where t is the time in seconds from a standing start. [At time t = 0, v = 0 and s = 0.]

Find the horse's acceleration, velocity, and position 5 seconds after the start of the race. [9]

$a(5) = 6 - 1.2(5)$

$\boxed{a(5) = 0}$

$v(t) = \int 6 - 1.2t \, dt$

$v(t) = 6t - 0.6t^2$

$v(5) = 30 - 0.6(5)^2$

$= 30 - 15$

$\boxed{v(5) = 15}$

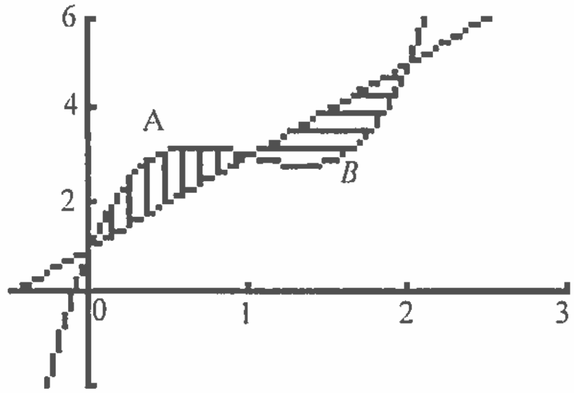
$s(t) = 3t^2 - 0.2t^3$

$s(5) = 3(5)^2 - 0.2(5)^3$

$s(5) = 75 - 25$

$\boxed{s(5) = 50}$

6. The diagram shows parts of the graphs with equations $f(x) = 2x + 1$ and $g(x) = 3x^3 - 9x^2 + 8x + 1$, and the two shaded regions A and B which are enclosed by the graphs.



a) Show that the graphs intersect at x = 0, x = 1, and x = 2. [3]

b) Show that the area of the shaded region A is $\frac{3}{4}$. [3]

c) Find the area of shaded region B. [3]

a) $2x+1 = 3x^3 - 9x^2 + 8x + 1$

$0 = 3x^3 - 9x^2 + 6x$

$0 = 3x(x^2 - 3x + 2)$

$0 = 3x(x-2)(x-1)$

$x = 0, 2, 1$

b) $\int_0^1 3x^3 - 9x^2 + 8x + 1 - (2x+1)\,dx$

$= \int_0^1 3x^3 - 9x^2 + 6x\,dx$

$= \frac{3}{4}x^4 - 3x^3 + 3x^2 \Big|_0^1$

$= \frac{3}{4} - 3 + 3 = \frac{3}{4}$ ✓

c) $\int_1^2 2x+1-(3x^3-9x^2+8x+1)$

$= \int_1^2 -3x^3 + 9x^2 - 6x\,dx$

$= -\frac{3}{4}x^4 + 3x^3 - 3x^2 \Big|_1^2$

$= -\frac{3}{4}(2)^4 + 3(2)^3 - 3(2)^2 - \left(-\frac{3}{4}(1)^4 + 3(1)^3 - 3(1)^2\right)$

$= -12 + 24 - 12 - \left(-\frac{3}{4} + 3 - 3\right)$

$= \frac{3}{4}$

IB Math Standard

REVIEW

Test on Applications of Integration [CALCULATOR PERMITTED]

Unless otherwise indicated, all answers should be given exactly or to three significant figures.

7. a) Solve the equation $(2x+1)^3 = 0$.

$x = -\frac{1}{2}$

b) Calculate the area between the curve $y = (2x+1)^3$ and the x axis from $x = -2$ to $x = 2$.

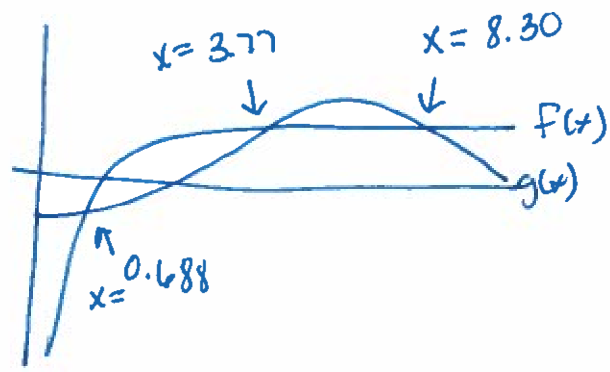
$$A = \int_{-2}^{2} \left| (2x+1)^3 \right| dx$$

$$= 88.25$$

8. Let A be the area of the region enclosed by $f(x) = \ln(3x-2)+1$ and $g(x) = -4\cos(0.5x)+2$ for $0 \le x \le 10$.

a) Find an expression for A.

b) Calculate the value of A.



a) $A = \int_{0.688}^{3.77} f(x) - g(x)\,dx + \int_{3.77}^{8.30} g(x) - f(x)\,dx$

b) $A = 5.73 + 6.46$

$= 12.19$

9. Find the volume of the solid formed when the area between the curve $y = x^2 + 2$ and the x-axis from x = 1 to x = 3 is rotated around the x-axis.

$$V = \pi \int_1^3 (x^2+2)^2 \, dx$$ ← put in calc

Vol. = 286

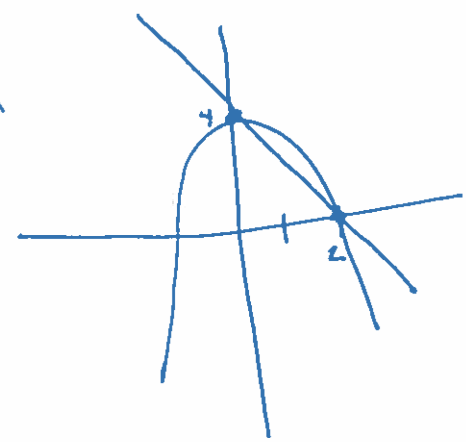
$$\cancel{= \pi \int_1^3 x^4 + 4x^2 + 4 \, dx}$$

$$\cancel{= \pi \left(\frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x \right) \Big|_1^3}$$

10. The area enclosed between the curve $y = 4 - x^2$ and the line $y = 4 - 2x$ is rotated around the x-axis. Find the volume of the solid generated.

$$V = \pi \int_0^2 \left[4 - x^2 - (4 - 2x) \right]^2 dx$$

$$= 4.19$$



11. Show that $\int_0^1 e^{-kx}\,dx = \frac{1}{k}(1-e^{-k})$. [5]

$u = -kx$

$-\frac{1}{k}\,du = \frac{-k}{-k}\,dx$

$= -\frac{1}{k}\int_0^1 e^u\,du$

$= -\frac{1}{k} e^u \Big|_0^1$

$= -\frac{1}{k} e^{-kx} \Big|_0^1$

$= -\frac{1}{k}\left[e^{-k(1)} - e^{-k(0)}\right]$

$= \frac{-1}{k}\left[e^{-k} - 1\right]$

$= \frac{1}{k}(1-e^{-k})$ ✓

12. If $\int_1^k \frac{3}{x}\,dx = 6$, find the value of k. [4]

$\int_1^k \frac{3}{x}\,dx = 6$

$3\ln x \Big|_1^k = 6$

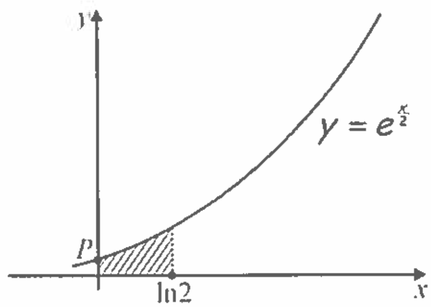
$3\ln k - \cancel{3\ln 1}^{\,0} = 6$

$3\ln k = 6$

$\ln k = 2$

$k = e^2$

13. The diagram shows part of the graph of $y = e^{\frac{x}{2}}$.



(a) Find the coordinates of the point P, where the graph meets the y-axis. [1]

$y = e^{\frac{0}{2}} = 1 \qquad (0, 1)$

The shaded region between the graph and the x-axis, bounded by x = 0 and x = ln 2, is rotated through 360° about the x-axis.

(b) Write down an integral which represents the volume of the solid obtained. [4]

$$V = \pi \int_0^{\ln 2} \left(e^{\frac{x}{2}}\right)^2 dx$$

(c) Show that this volume is π. [4]

$$V = \pi \int_0^{\ln 2} e^x \, dx$$

$$= \pi \left[e^x \Big|_0^{\ln 2} \right]$$

$$= \pi \left[e^{\ln 2} - e^0 \right]$$

$$= \pi [2 - 1]$$

$$= \pi (1)$$

$$= \pi$$