Practice Test on Chapter 7/8:Analytic Trigonometry [CALCULATOR PROHIBITED]

1. Simplify:

a) $\cot\theta \sec\theta \sin\theta$

$$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \frac{\sin\theta}{1} = 1$$

b) $\cos^3 y + \cos y \sin^2 y$.

$$= \cos y (\cos^2 y + \sin^2 y)$$

$$= \cos y (1)$$

$$= \cos y$$

2. Verify that $\sec\theta \csc\theta - \tan\theta = \cot\theta$.

$$= \frac{1}{\cos\theta \sin\theta} - \frac{\sin\theta}{\cos\theta}\left(\frac{\sin\theta}{\sin\theta}\right)$$

$$= \frac{1 - \sin^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

3. Verify that $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$.

$$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

$$= \frac{\sin^2 x - \cos^2 x \sin^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$

$$= \frac{\sin^2 x \sin^2 x}{\cos^2 x}$$

$$= \tan^2 x \sin^2 x$$

4. Verify that $\sin 2B(\cot B + \tan B) = 2$.

$= 2\sin B\cos B\left(\frac{\cos B}{\sin B} + \frac{\sin B}{\cos B}\right)$

$= 2\cos^2 B + 2\sin^2 B$

$= 2(\cos^2 B + \sin^2 B)$

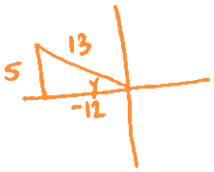
$= 2(1)$

$= 2$

5. Given that $\cos y = -\frac{12}{13}$ and y is in QII, find

a) $\sin y$

$= \frac{5}{13}$



$\sqrt{(13)^2-(12)^2} = 5$

b) $\cos 2y$

$= \cos^2 y - \sin^2 y$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169}$

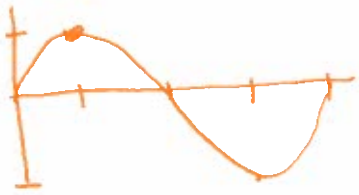
$= \frac{119}{169}$

6. If $\cot x \sin^2 x \sec x = 1$, find the value of x.

$\frac{\cos x}{\sin x} \cdot \frac{\sin^2 x}{1} \cdot \frac{1}{\cos x} = 1$

$\sin x = 1$

$x = \frac{\pi}{2}, 90°$



7. a) Factor $2\sin^2 x - \sin x - 1$.

$(2\sin x + 1)(\sin x - 1)$

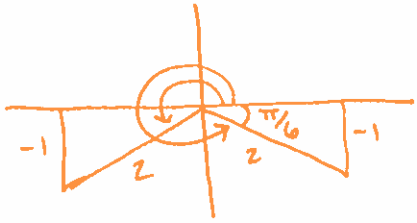
b) Hence, solve $2\sin^2 x - \sin x - 1 = 0$ for $0 \le x \le 2\pi$.

$(2\sin x + 1)(\sin x - 1) = 0$

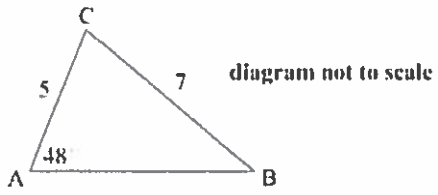
$2\sin x + 1 = 0 \qquad \sin x - 1 = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = 1$

$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$



8. a) In triangle ABC, AC = 5, BC = 7, $\angle A = 48°$, as shown in the diagram.



Find $\angle B$, giving your answer correct to the nearest degree. [3]

$\frac{\sin B}{5} = \frac{\sin 48}{7}$

$\angle B = 32.1°$

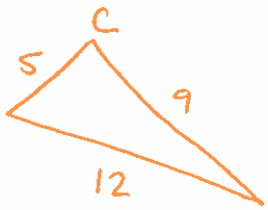
b) Explain why there is no triangle for which a = 15, b = 25, and $A = 85°$ [3]

$\frac{\sin 85}{15} = \frac{\sin B}{25}$

$\sin B = 1.66$

not possible.

9. A triangle has sides of 5 cm, 9 cm, and 12 cm.



a) Find the size of the largest angle, in degrees.

$$\cos C = \frac{5^2 + 9^2 - 12^2}{2(5)(9)}$$

$$\angle C = 115^\circ$$

b) Find the area of the triangle.

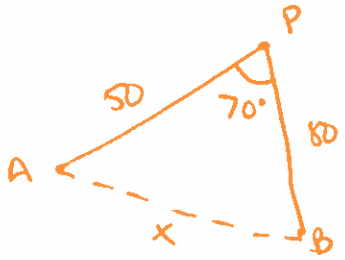
$$A = \frac{1}{2}(5)(9)\sin 115$$

$$= 20.4 \text{ cm}^2$$

10. Two boats A and B start moving from the same point P. Boat A moves in a straight line at 20 km/hr and boat B moves in a straight line at 32 km/hr. The angle between their paths is 70°.

Find the distance between the boats after 2.5 hours.

A: $20(2.5) = 50$
B: $32(2.5) = 80$



$$x^2 = 50^2 + 80^2 - 2(50)(80)\cos 70$$

$$x = 78.5 \text{ km}$$

11. Find all solutions to $2\cos^2 x - \cos x = 3$ for $0 \le x \le 2\pi$.

$\cos x(2\cos x - 1) = 3$

$\cos x = 3$ or $2\cos x - 1 =$

$2\cos^2 x - \cos x - 3 = 0$

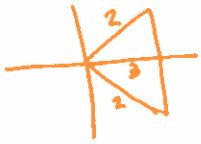
$(2\cos x - 3)(\cos x + 1) = 0$

$2\cos x - 3 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{3}{2}$ (A/H)

$\cos x = -1$

$x = \pi$

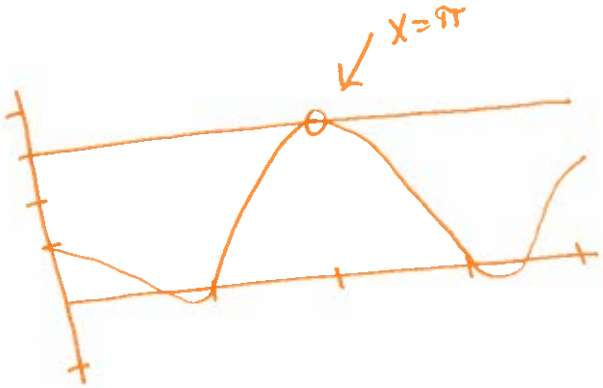


$\emptyset$

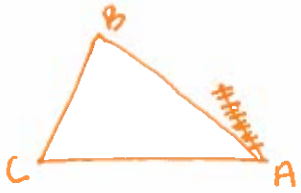
not possible

— OR —

graph...



12. Find the <u>two</u> triangles for which a = 12 meters, b = 31 meters, and $A = 20.5^\circ$.



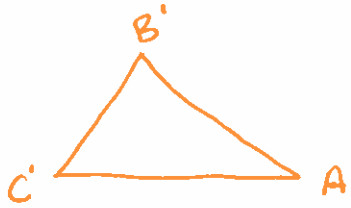
$$\frac{\sin 20.5}{12} = \frac{\sin B}{31}$$

$$\angle B = 64.8^\circ$$

$$\angle C = 94.7^\circ$$

$$\frac{\sin 20.5}{12} = \frac{\sin 94.7}{c}$$

$$c = 34.2 \text{ m}$$



$$\angle B' = 180 - 64.8$$
$$= 115.2^\circ$$

$$\angle C' = 44.3^\circ$$

$$\frac{\sin 20.5}{12} = \frac{\sin 44.3}{c}$$

$$c = 23.9 \text{ m}$$

13. The points P, Q, R are three markers on level ground, joined by straight paths PQ, QR, PR as shown in the diagram. QR = 9 km, $P\hat{Q}R = 35°$, $P\hat{R}Q = 25°$.

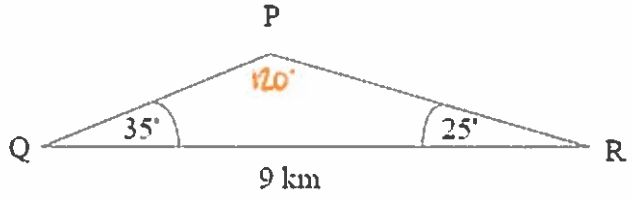


diagram not to scale

a) Find the length PR. [3]

$$\frac{9}{\sin 120} = \frac{PR}{\sin 35}$$

$$PR = 5.96$$

b) Tom sets out to walk from Q to P at a steady speed of 8 km/h. At the same time, Alan sets out to jog from R to P at a steady speed of a km/h. They reach P at the same time.

Calculate the value of a. [7]

$d = rt$

$t = \frac{d}{r}$

$$\frac{9}{\sin 120} = \frac{QP}{\sin 25}$$

$$QP = 4.39$$

$$t_{QP} = \frac{4.39}{8} = 0.549 \text{ hr.}$$

$$t_{PR} = \frac{5.96}{a} = 0.549$$

$$a = 10.9 \text{ km/hr.}$$