

Test Review #1

NO CALCULATOR!

1. [Maximum mark: 8]

Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.(a) (i) Find $g(0)$.(ii) Find $(f \circ g)(0)$.

[5 marks]

(b) Find $f^{-1}(x)$.

[3 marks]

$$a) (i) \quad g(0) = e^{3(0)} - 2 = e^0 - 2 = 1 - 2 = -1$$

$$(ii) \quad f \circ g(0) = f(-1) = 2(-1)^3 + 3 = -2 + 3 = 1$$

$$b) \quad x = 2y^3 + 3$$

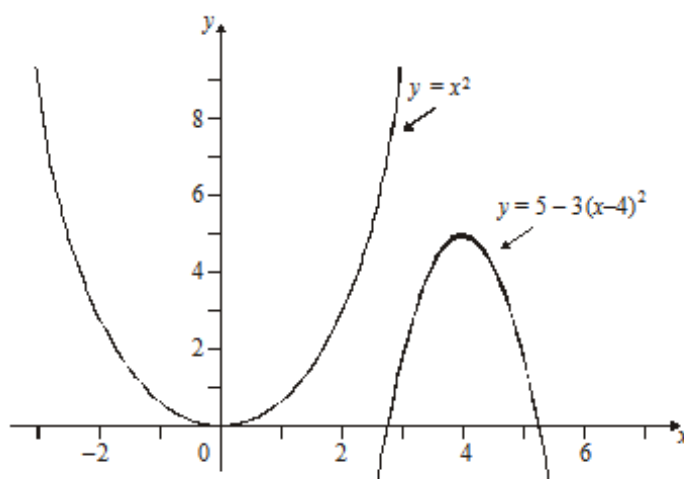
$$x - 3 = 2y^3$$

$$\frac{x-3}{2} = y^3$$

$$\sqrt[3]{\frac{x-3}{2}} = y$$

NO CALCULATOR!

The diagram shows parts of the graphs of $y = x^2$ and $y = 5 - 3(x - 4)^2$.



The graph of $y = x^2$ may be transformed into the graph of $y = 5 - 3(x - 4)^2$ by four transformations.

Fully describe these four transformations.

- a) reflected over x-axis b) translated up 5
 c) vertical stretch by 3 d) translated rt. 4

NO CALCULATOR!

- a) Express the function $h(x) = \sqrt{2x + 3}$ as the composition of two functions, $f(g(x))$.

$$f(x) = \underline{\sqrt{x}} \qquad g(x) = \underline{2x+3}$$

- b) Find the equation for $h^{-1}(x)$.

$$\begin{aligned}x^2 &= \sqrt{2y+3}^2 \\x^2 &= 2y+3 \\ \frac{x^2-3}{2} &= \frac{2y}{2}\end{aligned}$$

$$h^{-1}(x) = \frac{x^2-3}{2}$$

NO CALCULATOR!

Find an equation of the line passing through $(2, -1)$ and perpendicular to $2x - 3y = 5$. Write your answer in the form $Ax + By + C = 0$, where A , B , and C are integers.

$$-3y = 5 - 2x$$

$$y = \frac{5 - 2x}{-3}$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

$$m = -\frac{3}{2} \quad (2, -1)$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -\frac{3}{2}(x - 2)$$

$$y + 1 = -\frac{3}{2}x + 3$$

$$2(y + 1) = -3x + 6$$

$$2y + 2 = -3x + 6$$

$$3x + 2y - 4 = 0$$

$$2y = -3x + 4$$

NO CALCULATOR!

Given the points (5 , -3) and (9 , 3), find

- a) the distance between them
- b) the midpoint between them
- c) the slope between them

$$a) d = \sqrt{(9-5)^2 + (3+3)^2}$$

$$= \sqrt{16 + 36} = \sqrt{52}$$

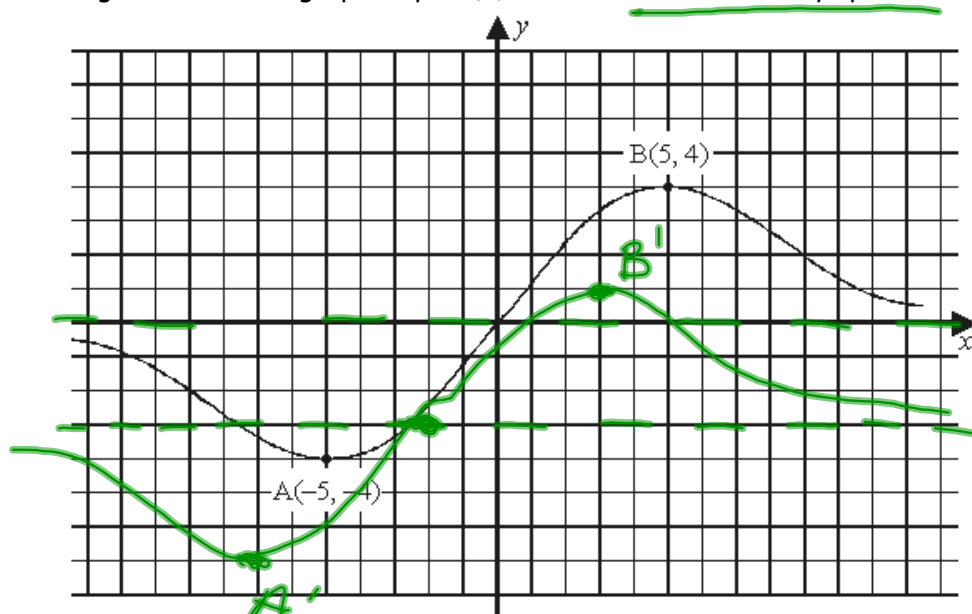
$$\begin{array}{r} 16 \\ +36 \\ \hline 52 \end{array}$$

$$b) \left(\frac{9+5}{2}, \frac{3+(-3)}{2} \right) = (7, 0)$$

$$c) \frac{3+3}{9-5} = \frac{6}{4} = \frac{3}{2}$$

NO CALCULATOR!

The diagram shows the graph of $y = f(x)$, with the x-axis as an asymptote.



(a) On the same axes, draw the graph of $y = f(x + 2) - 3$, clearly indicating the coordinates of the new points A' and B' .

(b) Write down the equation of the asymptote to the graph of $y = f(x + 2) - 3$.

$y = -3$

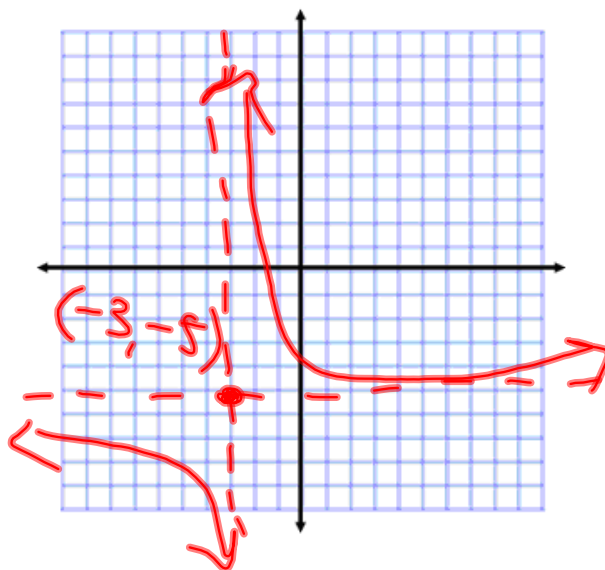
CALCULATOR ALLOWED!

1. The graph of the rational function $f(x) = \frac{1}{x}$ is transformed by translating the graph down 5 and left 3.

a) Write an equation of the translated graph.

$$f(x) = \frac{1}{x+3} - 5$$

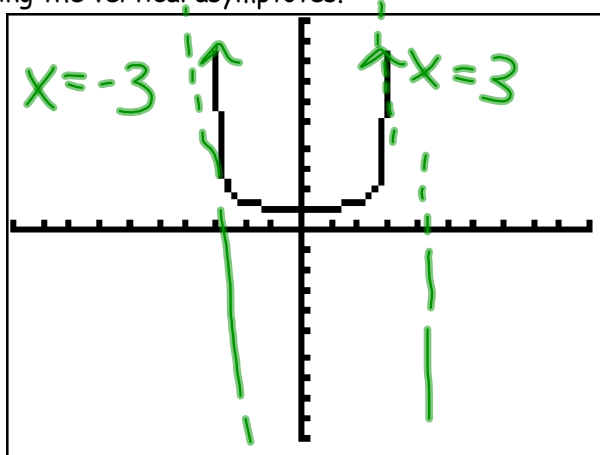
b) Sketch the new graph, showing the point where the asymptotes intersect.



CALCULATOR ALLOWED!

The function $f(x)$ is defined by $f(x) = \frac{3}{\sqrt{9-x^2}}$.

- a) Using your calculator and the grid below, sketch an accurate graph of $f(x)$, correctly showing the vertical asymptotes.



- b) Write down the equation of each vertical asymptote.

$$x = 3, x = -3$$

- c) Write down the range of the function f .

$$[1, \infty) / y \geq 1$$

CALCULATOR ALLOWED!

Describe the following features of $f(x) = x^3 - 4x^2 + x - 4$ Domain: $x \in \mathbb{R}$ Range: $y \in \mathbb{R}$ Local Max (x,y): $(0.131, -3.94)$ Local Min (x,y): $(2.54, -10.9)$ Increasing on x-interval(s): $(-\infty, 0.131) \cup (2.54, \infty)$ Decreasing on x-interval(s): $(0.131, 2.54)$ 

CALCULATOR ALLOWED!

The functions f and g are defined by $f(x) = 3x$ or $g(x) = x + 2$.

(a) Find expressions for the inverses $f^{-1}(x)$ and $g^{-1}(x)$

$$\begin{aligned} f^{-1}(x) \quad x &= 3y \\ \frac{x}{3} &= y \end{aligned}$$

$$\begin{aligned} g^{-1}(x) \quad x &= y + 2 \\ y &= x - 2 \end{aligned}$$

(b) Show that $f^{-1}(18) + g^{-1}(18) = 22$

$$\frac{18}{3} + (18 - 2) = 22$$

$$6 + (16) = 22$$

$$22 = 22 \checkmark$$

CALCULATOR ALLOWED!

Expand the binomial: $(4x - 3y)^2$

$$(4x - 3y)^2$$

$$= (4x - 3y)(4x - 3y)$$

$$= 16x^2 - 12xy - 12xy + 9y^2$$

$$= 16x^2 - 24xy + 9y^2$$

Simplify Completely: $\frac{10 - 15x}{20 + 5x}$

$$\frac{\cancel{5}(2 - 3x)}{\cancel{5}(4 + x)} = \frac{2 - 3x}{4 + x}$$