IB Math Standard Yr2 [40 marks - 45 minutes]

Test REVIEW on Applications of Differentiation

1. The displacement s metres of a car, t seconds after leaving a fixed point A, is given by $s = 10t - 0.5t^2$.

a) Calculate the velocity when $t = 0$. [3]

$v(t) = 10 - 1t$

$v(0) = 10$ m/s

b) Calculate the value of t when the velocity is zero. [2]

$10 - t = 0$

$t = 10$

c) Calculate the displacement of the car from A when the velocity is zero. [1]

$S(10) = 10(10) - 0.5(10)^2$

$= 100 - 50$

$= 50$ m.

2. Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	−14	−18
$f'(x)$	1	1
$g'(x)$	−5	−3
$h''(x)$	−6	0

a) Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. [3]

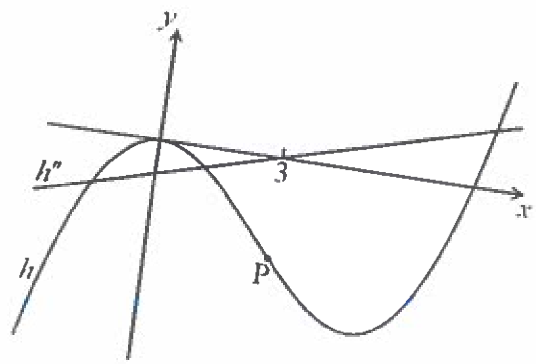
$g(3) = -18$

$f'(3) = 1$

$h''(2) = -6$

The following diagram shows parts of the graphs of h and h''. There is a point of inflexion on the graph of h at P, when $x = 3$.

b) Fully explain why P is a point of inflexion. [2]



$h''(3) = 0$

2nd derivative equals 0 which means $x=3$ is a point of inflection for h, to left of $x=3$ h'' is negative and to the right h'' is positive.

c) Given that $h(x) = f(x) \times g(x)$, find the y-coordinate of P. [2]

$x = 3$

$h(3) = f(3) \times g(3)$

$= 3(-18)$

$= -54$

3. Consider $f(x) = \ln(x^4 + 1)$.
The second derivative is given by $f''(x) = \frac{4x^2(3-x^4)}{(x^4+1)^2}$.

a) Find the value of $f(0)$. [2]

$f(0) = \ln(0^4+1) = \ln 1$
$= 0$

The equation $f''(x) = 0$ has only three solutions, when $x = 0,\ \pm\sqrt[4]{3}\ (\pm 1.316\ \ldots)$.

b) i) Find the value of $f''(1)$. [2]

$f''(1) = \dfrac{4(1)^2(3-1^4)}{(1^4+1)^2}$

$= \dfrac{4(2)}{2^2} = \dfrac{8}{4} = 2$

ii) Hence, show that there is no inflection point on the graph of $f(x)$ at $x = 0$. [3]

$f''(x) = 0$ at inflection points

$4x^2(3-x^4) = 0$
$x = 0$

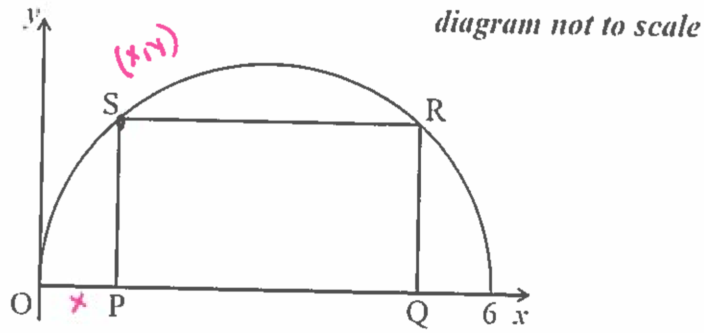
x: $-\infty$ ... 0 ... ∞
$+$ ... 0 ... $+$

$f''(1) = +2$

$f''(-1) = +2$

2nd derivative goes from concave up to zero to concave up – concavity does not chang

4. Consider the graph of the semicircle given by $f(x) = \sqrt{6x - x^2}$, for $0 \le x \le 6$. A rectangle $PQRS$ is drawn with upper vertices R and S on the graph of f, and PQ on the x-axis, as shown in the following diagram.



a) Let $OP = x$.

i) Find an expression for PQ, giving your answer in terms of x. [2]

$PQ = 6 - 2x$

ii) Hence, write down an expression for the area of the rectangle, giving your answer in terms of x. [1]

$A = (6-2x)(\sqrt{6x-x^2})$

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b) i) Find the rate of change of area when $x = 2$. [2]

Use calculator

*math -8!

$$A' = (-2)(\sqrt{6x-x^2}) + (6-2x)\tfrac{1}{2}(6x-x^2)^{-1/2}(-2x+6)$$

$$= -2\sqrt{6x-x^2} + \frac{(6-2x)^2}{2\sqrt{6x-x^2}}$$

$$A'(2) = -2\sqrt{6(2)-2^2} + \frac{(6-2(2))^2}{2\sqrt{6(2)-2^2}}$$

$$= -2\sqrt{8} + \frac{4}{2\sqrt{8}} = -2\sqrt{8} + \frac{2}{\sqrt{8}}$$

$$\boxed{x \approx -4.95}$$

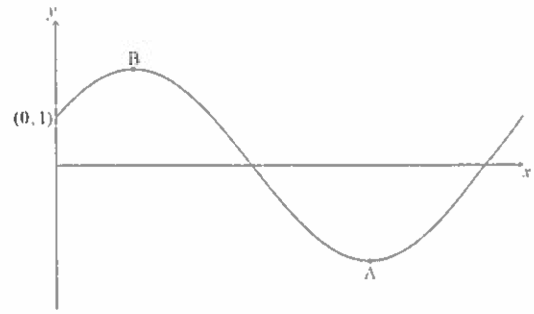
ii) The area is decreasing for $a < x < b$.
Find the value of a and of b. [2]

Use graph of area + look for negative slope.

$0.879 < x < 3$

5. Let $f(x) = \cos x + \sqrt{3}\sin x, 0 \le x \le 2\pi$.
The diagram shows the graph of f.

The y-intercept is at $(0,1)$, there is a minimum point at $A(p,q)$ and a maximum point at B.



a) Find $f'(x)$. [2]

$= -\sin x + \sqrt{3}\cos x$

b) Hence, using your answer from a),

i) Show that $q = -2$. [7]

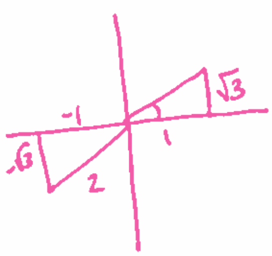
$0 = -\sin x + \sqrt{3}\cos x$

$\sin x = \sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = \sqrt{3}$

$\tan x = \sqrt{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$



$f\left(\frac{4\pi}{3}\right) = \cos\frac{4\pi}{3} + \sqrt{3}\sin\frac{4\pi}{3}$

$= \frac{-1}{2} + \sqrt{3}\left(\frac{-\sqrt{3}}{2}\right)$

$= -\frac{1}{2} - \frac{3}{2} = -\frac{4}{2}$

$= -2$

ii) Verify that A is the minimum point. [3]

$f'(x) = -\sin\frac{4\pi}{3} + \sqrt{3}\cos\frac{4\pi}{3}$

$= -\left(-\frac{\sqrt{3}}{2}\right) + \sqrt{3}\left(\frac{-1}{2}\right)$

$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$ min.

$f''(x) = -\cos\frac{4\pi}{3} + -\sqrt{3}\sin\frac{4\pi}{3}$

$f''\left(\frac{4\pi}{3}\right) = \frac{1}{2} - \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) = \frac{4}{2} = +$

1st derivative $= 0$

2nd derivative $= +$ (CC ↑)

c) Write down the maximum value of $f(x)$. [1]

$\to x = \frac{\pi}{3}$

$f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} + \sqrt{3}\sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2} + \sqrt{3}\left(\frac{1}{2}\right) = \frac{2\sqrt{3}}{2} = \sqrt{3}$