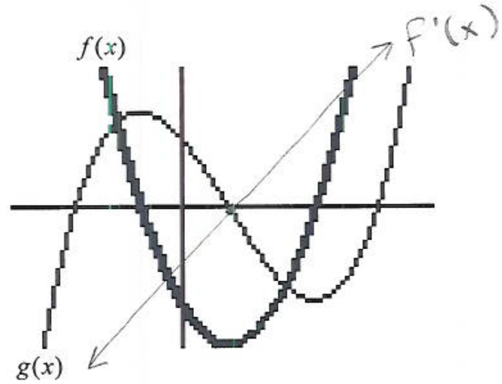
Key

IB Math Standard
Calculus Test #1 REVIEW
Basic Differentiation: 48 marks [Calculator prohibited]

Unless otherwise specified, all answers must be given exactly or to three significant figures.

1. Two functions are shown on the diagram. One function is the derivative of the other function.



a) It appears that $f(x) = g'(x)$. Justify this conclusion in two different ways.

1) where $g(x)$ is increasing, $f(x)$ is positive
where $g(x)$ is decreasing, $f(x)$ is negative

2) minimum of $f(x)$ = zero of $g(x)$

b) On the diagram, draw a possible graph of $f'(x)$.
Explain your reasoning.

$f'(x)$ must be negative where $f(x)$ is decreasing
$f'(x)$ " " positive where $f(x)$ is increasing

2. Differentiate with respect to x:

a) $y = -2x^5 + 4x^3 + x - 8$

$y' = -10x^4 + 12x^2 + 1$

b) $y = \frac{3}{x} - \frac{5}{x^3}$

$y = 3x^{-1} - 5x^{-3}$

$y' = -3x^{-2} + 3(5)x^{-4}$

$= \frac{-3}{x^2} + \frac{15}{x^4}$

3. Differentiate with respect to x:

a) $y = -\cos x \sin x$

$y' = -\cos x(\cos x) + (\sin x)(\sin x)$

$= -\cos^2 x + \sin^2 x$

b) $y = \frac{x^2 + 1}{3x - 5}$

$y' = \frac{(3x-5)(2x) - (x^2+1)(3)}{(3x-5)^2}$

$y' = \frac{6x^2 - 10x - 3x^2 - 3}{(3x-5)^2}$

$= \frac{3x^2 - 10x - 3}{(3x-5)^2}$

4. Differentiate with respect to x:

a) $y = (2x^3 + 1)^5$

$y' = 5(2x^3+1)^4(6x^2)$

$= 30x^2(2x^3+1)^4$

b) $y = \sqrt{-9-5x}$

$y = (-9-5x)^{1/2}$

$y' = \frac{1}{2}(-9-5x)^{-1/2}(-5)$

$= \dfrac{-5}{2\sqrt{-9-5x}}$

5. Differentiate with respect to x:

a) $y = \ln(\cos x)$

$y' = \dfrac{-\sin x}{\cos x} = -\tan x$

b) $y = \tan^2 3x$

$y = [\tan(3x)]^2$

$= 2\tan(3x) \cdot \dfrac{1}{\cos^2 3x} \cdot 3$

$= \dfrac{6\tan(3x)}{\cos^2(3x)}$

6. Let $f(x) = e^{2x}\cos x,\ -1 \le x \le 2$.

a) Show that $f'(x) = e^{2x}(2\cos x - \sin x)$ [3]

$$f'(x) = e^{2x}(2)(\cos x) + e^{2x}(-\sin x)$$

*Factor out e^{2x}

$$= e^{2x}(2\cos x - \sin x)$$

b) Let the line L be the tangent to the curve of f at x = 0.
Find the equation of L. [5]

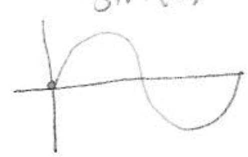
$$m = e^{2(0)}(2\cos(0) - \sin(0))$$

$\cos(0) = 1$

(@ $x=0$)

$$= (1)(2(1) - 0)$$

$\sin(0) = 0$



$$m = 2$$

$x = 0$

$$F(0) = e^{2(0)}\cos(0)$$

$$= (1)(1) = 1$$

$m=2$

pt: $(0,1)$

$$y - 1 = 2(x - 0)$$

$$\boxed{y = 2x + 1}$$

7. A rock thrown vertically upward from the surface of the moon is modeled by the equation $s(t) = 24t - 0.8t^2$.

a) Find an equation for v(t). $= s'(t)$ [2]

$v(t) = 24 - 1.6t$

b) What is the initial velocity of the object? [2]

$v(0) = 24$

c) When will the object reach its maximum height? [2]

$v(t) = 0$

$0 = 24 - 1.6t$

$1.6t = 24$

$t = 15$

d) When will the object hit the ground? [2]

$s(t) = 0$

$0 = 24t - 0.8t^2$

$0 = t(24 - 0.8t)$

$t = 0$ or $0 = 24 - 0.8t$

$\boxed{t = 30}$

e) Find the acceleration due to gravity of an object on the moon. [2]

$v'(t)$

$a(t) = -1.6$

IB Math Standard
Calculus Test #1 REVIEW
Basic Differentiation: 32 marks [Calculator Permitted]

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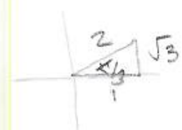
8. Consider the function $f(x) = k\sin x + 3x$, where k is a constant.

a) Find $f'(x)$.

$$f'(x) = k\cos x + 3$$

b) When $x = \frac{\pi}{3}$, the gradient of the curve of $f(x)$ is 8.
Find the value of k.



$$8 = k\cos\left(\frac{\pi}{3}\right) + 3$$

$$8 = \frac{1}{2}k + 3$$

$$5 = \frac{1}{2}k$$

$$\boxed{10 = k}$$

9. Find an equation for the normal [perpendicular] to the curve $y = x + \frac{2}{x}$ at the point (1 , 3).

$y = x + 2x^{-1}$

$$y' = 1 - 2x^{-2}$$

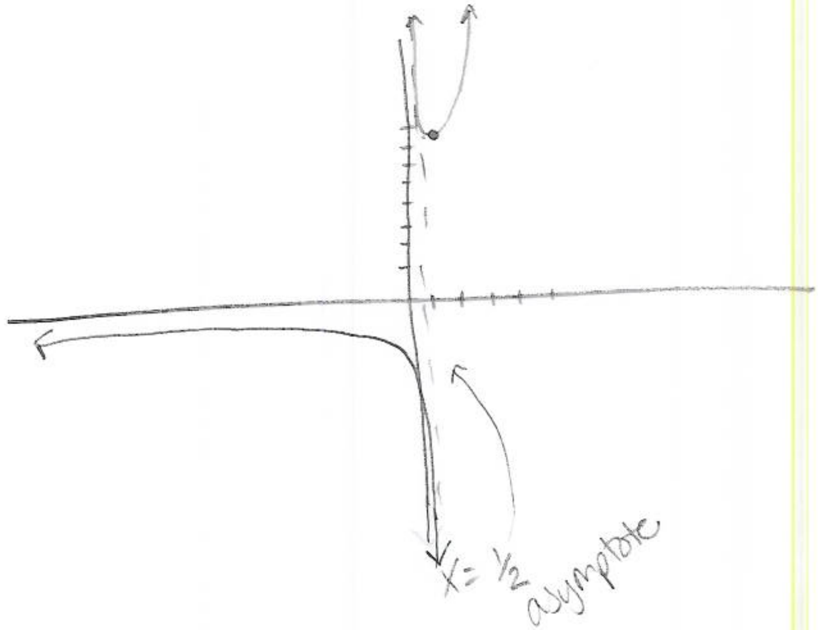
$$@1 \quad y' = 1 - 2(1)^{-2} = \overset{\text{tangent}}{\underset{}{-1}} \ m \qquad \boxed{m = 1} \text{ for normal}$$

$$y - 3 = 1(x - 1)$$

$$y = x + 2$$

10. Consider the function $f(x) = e^{(2x-1)} + \dfrac{5}{(2x-1)}$.

a) Sketch the curve of f for $-2 \le x \le 2$, including any asymptotes.



b) Find $f'(x)$.

$5(2x-1)^{-1}$

$$F'(x) = e^{(2x-1)}(2) + -5(2x-1)^{-2}(2)$$

$$= 2e^{(2x-1)} - \frac{10}{(2x-1)^2}$$

11. A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height H metres of the rock-climber after t seconds of the fall is given by:

$$H = 50 - 5t^2, \qquad 0 \le t \le 2$$

$$H = 90 - 40t + 5t^2, \qquad 2 \le t \le 5$$

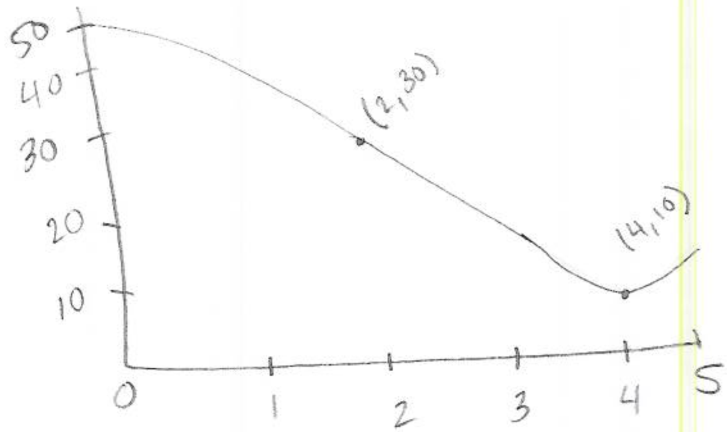
(a) Find the height of the rock-climber when t = 2. [1]

$50 - 5(2)^2 = 30$

$90 - 40(2) + 5(2)^2 = 30$

$\boxed{30}$ m

(b) Sketch a graph of H against t for $0 \le t \le 5$. [4]



(c) Find $\frac{dH}{dt}$ for:

(i) $0 \le t \le 2$ $\quad -10t$ [1]

(ii) $2 \le t \le 5$ $\quad -40 + 10t$ [1]

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(d) Find the velocity of the rock-climber when $t = 2$. [2]

$v(2) = -20$

(e) Find the times when the velocity of the rock-climber is zero. [3]

$0 = -10t$ $\qquad$ $0 = -40 + 10t$

$\boxed{t=0}$ $\qquad$ $\boxed{t = 4}$

(f) How close does the rock climber get to the ground? [2]

@ $t=4$, $H = \boxed{10m}$