International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

# Mathematics

# Standard level

## Specimen papers 1 and 2

### For first examinations in 2014

# CONTENTS

International Baccalaureate®
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**MATHEMATICS**
**STANDARD LEVEL**
**PAPER 1**

SPECIMEN

1 hour 30 minutes

Candidate session number

| 0 | 0 | | | | | | | |
|---|---|---|---|---|---|---|---|---|

Examination code

| X | X | X | X | – | X | X | X | X |
|---|---|---|---|---|---|---|---|---|

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

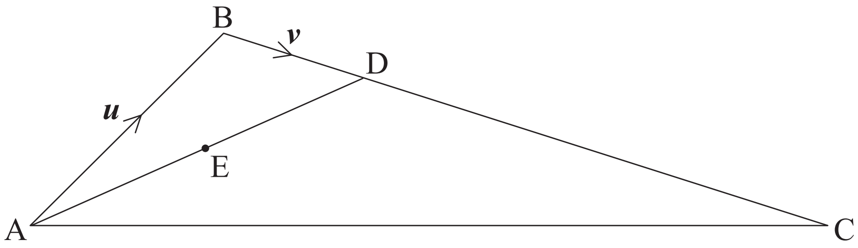
*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

**SECTION A (44 Marks)**

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

**1.** *[Maximum mark: 7]*

In the following diagram, $\boldsymbol{u} = \overrightarrow{AB}$ and $\boldsymbol{v} = \overrightarrow{BD}$.




The midpoint of $\overrightarrow{AD}$ is E and $\frac{BD}{DC} = \frac{1}{3}$.

Express each of the following vectors in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$.

(a) $\overrightarrow{AE}$ *[3 marks]*

(b) $\overrightarrow{EC}$ *[4 marks]*

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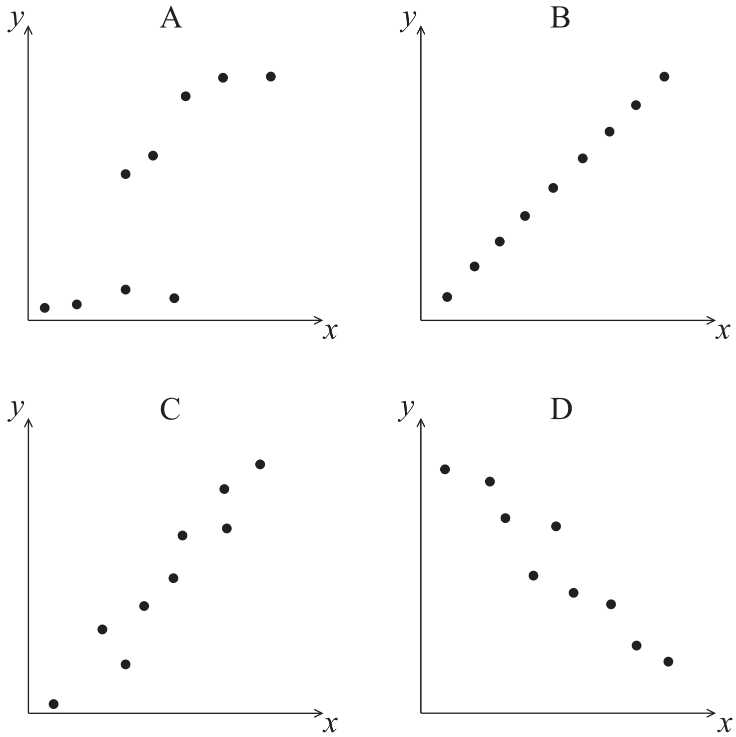
**2.** *[Maximum mark: 5]*

There are nine books on a shelf. For each book, $x$ is the number of pages, and $y$ is the selling price in pounds (£). Let $r$ be the correlation coefficient.

(a) Write down the possible minimum and maximum values of $r$. *[2 marks]*

(b) Given that $r = 0.95$, which of the following diagrams best represents the data. *[1 mark]*




(c) For the data in diagram D, which **two** of the following expressions describe the correlation between $x$ and $y$?

perfect, zero, linear, strong positive, strong negative,
weak positive, weak negative *[2 marks]*

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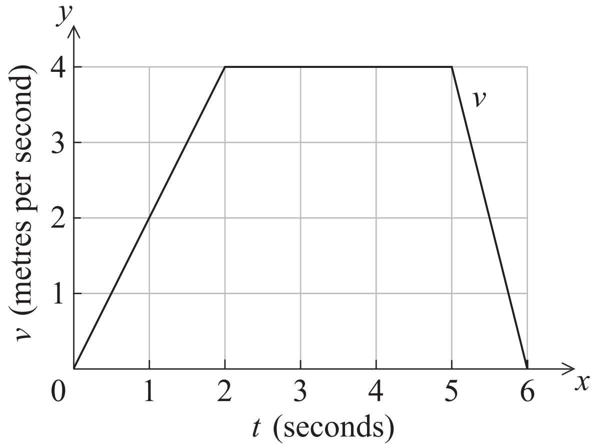
**3.** *[Maximum mark: 6]*

A toy car travels with velocity $v\ \mathrm{m\,s^{-1}}$ for six seconds. This is shown in the graph below.




(a) Write down the car's velocity at $t = 3$. *[1 mark]*

(b) Find the car's acceleration at $t = 1.5$. *[2 marks]*

(c) Find the total distance travelled. *[3 marks]*

**4.** *[Maximum mark: 5]*

A data set has a mean of 20 and a standard deviation of 6.

(a) Each value in the data set has 10 added to it. Write down the value of

(i) the new mean;

(ii) the new standard deviation. *[2 marks]*

(b) Each value in the original data set is multiplied by 10.

(i) Write down the value of the new mean.

(ii) Find the value of the new variance. *[3 marks]*

**5.** *[Maximum mark: 7]*

(a) Find $\int \frac{e^x}{1+e^x}\,dx$. *[3 marks]*

(b) Find $\int \sin 3x \cos 3x\,dx$. *[4 marks]*

**6.** *[Maximum mark: 7]*

The expression $6\sin x\cos x$ can be expressed in the form $a\sin bx$ .

(a) Find the value of $a$ and of $b$. *[3 marks]*

(b) Hence or otherwise, solve the equation $6\sin x\cos x = \frac{3}{2}$, for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$. *[4 marks]*

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

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**7.** *[Maximum mark: 7]*

Given that $f(x) = \dfrac{1}{x}$, answer the following.

(a) Find the first four derivatives of $f(x)$. *[4 marks]*

(b) Write an expression for $f^{(n)}(x)$ in terms of $x$ and $n$. *[3 marks]*

*Do **NOT** write solutions on this page.*

**SECTION B (46 Marks)**

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

**8.** *[Maximum mark: 15]*

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. *[2 marks]*

(b) For the graph of $f$

(i) write down the coordinates of the vertex;

(ii) write down the $y$-intercept;

(iii) find both $x$-intercepts. *[7 marks]*

(c) **Hence** sketch the graph of $f$. *[3 marks]*

(d) Let $g(x) = x^2$. The graph of $f$ may be obtained from the graph of $g$ by the following two transformations

a stretch of scale factor $t$ in the $y$-direction,

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of $t$. *[3 marks]*

*Do **NOT** write solutions on this page.*

**9.** *[Maximum mark: 14]*

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

score on second die

| score on first die | | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| | 1 | • | • | • | • | • | • |
| | 2 | • | • | • | • | • | • |
| | 3 | • | • | • | • | • | • |
| | 4 | • | • | • | • | • | • |
| | 5 | • | • | • | • | • | • |
| | 6 | • | • | • | • | • | • |

Let $X$ be the sum of the scores on the two dice.

(a) (i) Find $\mathrm{P}(X = 6)$.

(ii) Find $\mathrm{P}(X > 6)$.

(iii) Find $\mathrm{P}(X = 7 \mid X > 6)$. *[6 marks]*

(b) Elena plays a game where she tosses two dice.

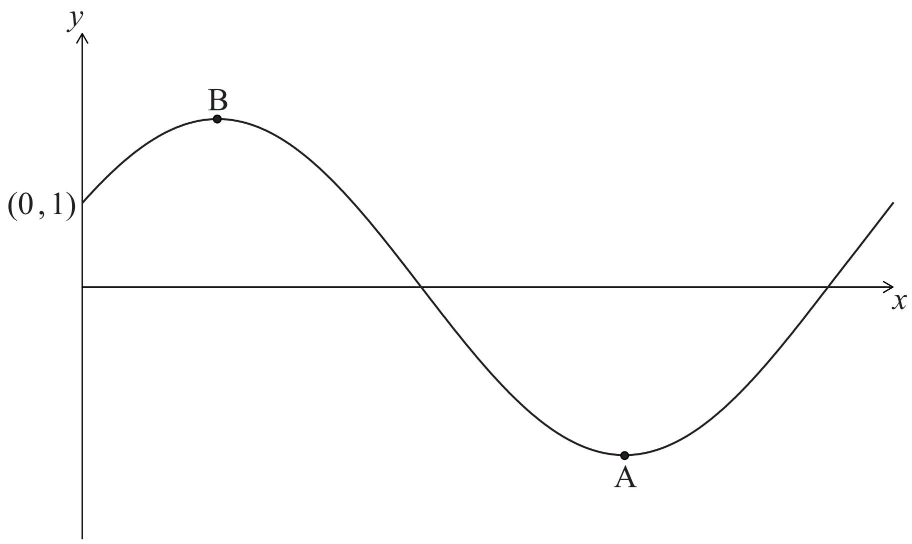
If the sum is 6, she **wins** 3 points.
If the sum is greater than 6, she **wins** 1 point.
If the sum is less than 6, she **loses** $k$ points.

Find the value of $k$ for which the game is fair. *[8 marks]*

*Do **NOT** write solutions on this page.*

**10.** *[Maximum mark: 17]*

Let $f(x) = \cos x + \sqrt{3}\sin x$, $0 \le x \le 2\pi$. The following diagram shows the graph of $f$.




The $y$-intercept is at $(0, 1)$, there is a minimum point at A $(p, q)$ and a maximum point at B.

(a) Find $f'(x)$. *[2 marks]*

(b) Hence

(i) show that $q = -2$;

(ii) verify that A is a minimum point. *[10 marks]*

(c) Find the maximum value of $f(x)$. *[3 marks]*

The function $f(x)$ can be written in the form $r\cos(x - a)$.

(d) Write down the value of $r$ and of $a$. *[2 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

SPEC/5/MATME/SP1/ENG/TZ0/XX/M

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# MARKSCHEME

## SPECIMEN

## MATHEMATICS

## Standard Level

## Paper 1

11 pages

## Instructions to Examiners

### Abbreviations

***M*** Marks awarded for attempting to use a correct **Method**.

***A*** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

***R*** Marks awarded for clear **Reasoning**.

***N*** Marks awarded for **correct** answers if **no** working shown.

***AG*** Answer given in the question and so no marks are awarded.

### Using the markscheme

**1 General**

*Award marks using the annotations as noted in the markscheme eg* ***M1****,* ***A2****.*

**2 Method and Answer/Accuracy marks**

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

**3 *N* marks**

*If* ***no*** *working shown, award* ***N*** *marks for* ***correct*** *answers. In this case, ignore mark breakdown (****M****,* ***A****,* ***R****).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

**4 Follow through marks (only applied after an error is made)**

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

**5 Mis-read**

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

**6 Alternative methods**

*Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme*

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

**7 Alternative forms**

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

**8 Accuracy of Answers**

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.*

There are 2 types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

**9 Calculators**

*No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded.*

**10 Style**

*The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

## SECTION A

**1.** (a) $\vec{AE} = \frac{1}{2}\vec{AD}$ ***A1***

attempt to find $\vec{AD}$ ***M1***

*e.g.* $\vec{AB} + \vec{BD}$, $\boldsymbol{u} + \boldsymbol{v}$

$\vec{AE} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})$ $\left( = \frac{1}{2}\boldsymbol{u} + \frac{1}{2}\boldsymbol{v} \right)$ ***A1*** ***N2***

***[3 marks]***

(b) $\vec{ED} = \vec{AE} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})$ ***A1***

$\vec{DC} = 3\boldsymbol{v}$ ***A1***

attempt to find $\vec{EC}$ ***M1***

*e.g.* $\vec{ED} + \vec{DC}$, $\frac{1}{2}(\boldsymbol{u} + \boldsymbol{v}) + 3\boldsymbol{v}$

$\vec{EC} = \frac{1}{2}\boldsymbol{u} + \frac{7}{2}\boldsymbol{v}$ $\left( = \frac{1}{2}(\boldsymbol{u} + 7\boldsymbol{v}) \right)$ ***A1*** ***N2***

***[4 marks]***

***Total [7 marks]***

**2.** (a) min value of $r$ is $-1$, max value of $r$ is 1 ***A1A1*** ***N2***

***[2 marks]***

(b) C ***A1*** ***N1***

***[1 mark]***

(c) linear, strong negative ***A1A1*** ***N2***

***[2 marks]***

***Total [5 marks]***

**3.** (a) $4\ (\mathrm{m\,s^{-1}})$ ***A1*** ***N1***

***[1 mark]***

(b) recognising that acceleration is the gradient ***M1***

*e.g.* $a(1.5) = \dfrac{4-0}{2-0}$

$a = 2\ (\mathrm{m\,s^{-2}})$ ***A1*** ***N1***

***[2 marks]***

(c) recognizing area under curve ***M1***

*e.g.* trapezium, triangles, integration

correct substitution ***A1***

*e.g.* $\dfrac{1}{2}(3+6)4,\ \int_0^6 |v(t)|\,\mathrm{d}t$

distance $=18$ (m) ***A1*** ***N2***

***[3 marks]***

***Total [6 marks]***

**4.** (a) (i) new mean is $20+10=30$ ***A1*** ***N1***

(ii) new sd is 6 ***A1*** ***N1***

***[2 marks]***

(b) (i) new mean is $20\times 10=200$ ***A1*** ***N1***

(ii) **METHOD 1**

variance is 36 ***A1***

new variance is $36\times 100=3600$ ***A1*** ***N2***

**METHOD 2**

new sd is 60 ***A1***

new variance is $60^2=3600$ ***A1*** ***N2***

***[3 marks]***

***Total [5 marks]***

**5.** (a) attempt to use substitution or inspection *M1*

*e.g.* $u = 1 + e^x$ so $\frac{du}{dx} = e^x$

correct working *A1*

*e.g.* $\int \frac{du}{u} = \ln u$

$\ln(1 + e^x) + C$ *A1* *N3*

***[3 marks]***

(b) **METHOD 1**

attempt to use substitution or inspection *M1*
*e.g.* let $u = \sin 3x$

$\frac{du}{dx} = 3\cos 3x$ *A1*

$\frac{1}{3}\int u\,du = \frac{1}{3} \times \frac{u^2}{2} + C$ *A1*

$\int \sin 3x \cos 3x dx = \frac{\sin^2 3x}{6} + C$ *A1* *N2*

***[4 marks]***

**METHOD 2**

attempt to use substitution or inspection *M1*
*e.g.* let $u = \cos 3x$

$\frac{du}{dx} = -3\sin 3x$ *A1*

$-\frac{1}{3}\int u\,du = -\frac{1}{3} \times \frac{u^2}{2} + C$ *A1*

$\int \sin 3x \cos 3x dx = \frac{\cos^2 3x}{6} + C$ *A1* *N2*

***[4 marks]***

**METHOD 3**

recognizing double angle *M1*
correct working *A1*

*e.g.* $\frac{1}{2}\sin 6x$

$\int \sin 6x dx = \frac{-\cos 6x}{6} + C$ *A1*

$\int \frac{1}{2}\sin 6x dx = -\frac{\cos 6x}{12} + C$ *A1* *N2*

***[4 marks]***

***Total [7 marks]***

**6.** (a) recognizing double angle ***M1***

*e.g.* $3\times 2\sin x\cos x$, $3\sin 2x$

$a = 3$, $b = 2$ ***A1A1*** ***N3***

***[3 marks]***

(b) substitution $3\sin 2x = \frac{3}{2}$ ***M1***

$\sin 2x = \frac{1}{2}$ ***A1***

finding the angle ***A1***

*e.g.* $\frac{\pi}{6}$, $2x = \frac{5\pi}{6}$

$x = \frac{5\pi}{12}$ ***A1*** ***N2***

**Note:** Award ***A0*** if other values are included.

***[4 marks]***

***Total [7 marks]***

**7.** (a) $f'(x) = -x^{-2}$ $\left(\text{or } -\frac{1}{x^2}\right)$ ***A1*** ***N1***

$f''(x) = 2x^{-3}$ $\left(\text{or } \frac{2}{x^3}\right)$ ***A1*** ***N1***

$f'''(x) = -6x^{-4}$ $\left(\text{or } -\frac{6}{x^4}\right)$ ***A1*** ***N1***

$f^{(4)}(x) = 24x^{-5}$ $\left(\text{or } \frac{24}{x^5}\right)$ ***A1*** ***N1***

***[4 marks]***

(b) $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}$ or $(-1)^n n!\left(x^{-(n+1)}\right)$ ***A1A1A1*** ***N3***

***[3 marks]***

***Total [7 marks]***

**SECTION B**

**8.** (a) $f(x) = 3(x^2 + 2x + 1) - 12$ ***A1***

$= 3x^2 + 6x + 3 - 12$ ***A1***

$= 3x^2 + 6x - 9$ ***AG*** ***N0***

***[2 marks]***

(b) (i) vertex is $(-1, -12)$ ***A1A1*** ***N2***

(ii) $y = -9$, or $(0, -9)$ ***A1*** ***N1***

(iii) evidence of solving $f(x) = 0$ ***M1***

*e.g.* factorizing, formula

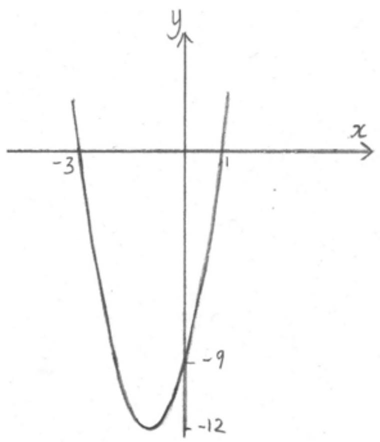
correct working ***A1***

*e.g.* $3(x+3)(x-1) = 0$, $x = \frac{-6 \pm \sqrt{36+108}}{6}$

$x = -3$, $x = 1$, or $(-3, 0)$, $(1, 0)$ ***A1A1*** ***N2***

***[7 marks]***

(c)




***A1A1A1*** ***N3***

> **Note:** Award ***A1*** for a parabola opening upward,
> ***A1*** for vertex in approximately correct position,
> ***A1*** for intercepts in approximately correct positions.
> Scale and labelling not required.

***[3 marks]***

(d) $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t = 3$ ***A1A1A1*** ***N3***

***[3 marks]***

***Total [15 marks]***

**9.** (a) (i) number of ways of getting $X = 6$ is 5 ***A1***

$\mathrm{P}(X = 6) = \frac{5}{36}$ ***A1*** ***N2***

(ii) number of ways of getting $X > 6$ is 21 ***A1***

$\mathrm{P}(X > 6) = \frac{21}{36}\ \left(= \frac{7}{12}\right)$ ***A1*** ***N2***

(iii) $\mathrm{P}(X = 7 \mid X > 6) = \frac{6}{21}\ \left(= \frac{2}{7}\right)$ ***A2*** ***N2***

***[6 marks]***

(b) attempt to find $\mathrm{P}(X < 6)$ ***M1***

*e.g.* $1 - \frac{5}{36} - \frac{21}{36}$

$\mathrm{P}(X < 6) = \frac{10}{36}$ ***A1***

fair game if $\mathrm{E}(W) = 0$ (may be seen anywhere) ***R1***

attempt to substitute into $\mathrm{E}(X)$ formula ***M1***

*e.g.* $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right)$

correct substitution into $\mathrm{E}(W) = 0$ ***A1***

*e.g.* $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right) = 0$

work towards solving ***M1***

*e.g.* $15 + 21 - 10k = 0$

$36 = 10k$ ***A1***

$k = \frac{36}{10}\ (= 3.6)$ ***A1*** ***N4***

***[8 marks]***

***Total [14 marks]***

**10.** (a) $f'(x) = -\sin x + \sqrt{3}\cos x$ ***A1A1*** ***N2***

***[2 marks]***

(b) (i) at A, $f'(x) = 0$ ***R1***

correct working ***A1***

*e.g.* $\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$ ***A1***

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ ***A1***

attempt to substitute **their** $x$ into $f(x)$ ***M1***

*e.g.* $\cos\left(\frac{4\pi}{3}\right) + \sqrt{3}\sin\left(\frac{4\pi}{3}\right)$

correct substitution ***A1***

*e.g.* $-\frac{1}{2} + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)$

correct working that clearly leads to $-2$ ***A1***

*e.g.* $-\frac{1}{2} - \frac{3}{2}$

$q = -2$ ***AG*** ***N0***

(ii) correct calculations to find $f'(x)$ either side of $x = \frac{4\pi}{3}$ ***A1A1***

*e.g.* $f'(\pi) = 0 - \sqrt{3},\ f'(2\pi) = 0 + \sqrt{3}$

$f'(x)$ changes sign from negative to positive ***R1***

so A is a minimum ***AG*** ***N0***

***[10 marks]***

(c) max when $x = \frac{\pi}{3}$ ***R1***

correctly substituting $x = \frac{\pi}{3}$ into $f(x)$ ***A1***

*e.g.* $\frac{1}{2} + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$

max value is 2 ***A1*** ***N1***

***[3 marks]***

(d) $r = 2,\ a = \frac{\pi}{3}$ ***A1A1*** ***N2***

***[2 marks]***

***Total [17 marks]***

International Baccalaureate®
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**MATHEMATICS**
**STANDARD LEVEL**
**PAPER 2**

SPECIMEN

1 hour 30 minutes

Candidate session number

| 0 | 0 | | | | | | | |
|---|---|---|---|---|---|---|---|---|

Examination code

| X | X | X | X | – | X | X | X | X |
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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

**SECTION A (47 Marks)**

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

**1.** *[Maximum mark: 5]*

In an arithmetic series, the first term is $-7$ and the sum of the first 20 terms is 620.

(a) Find the common difference. *[3 marks]*

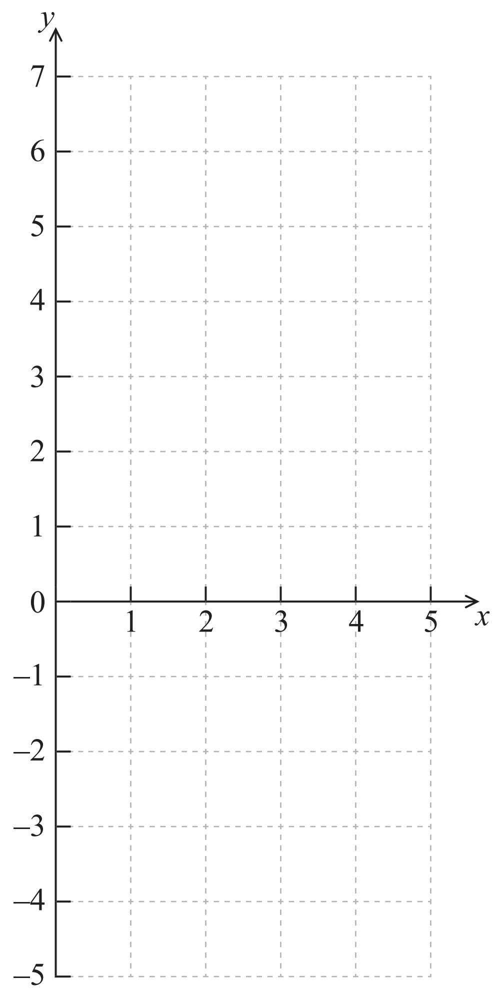
(b) Find the value of the 78th term. *[2 marks]*

**2.** *[Maximum mark: 7]*

Let $f(x) = 4x - \mathrm{e}^{x-2} - 3$, for $0 \le x \le 5$.

(a) Find the $x$-intercepts of the graph of $f$. *[3 marks]*

(b) On the grid below, sketch the graph of $f$. *[3 marks]*




(c) Write down the gradient of the graph of $f$ at $x = 3$. *[1 mark]*

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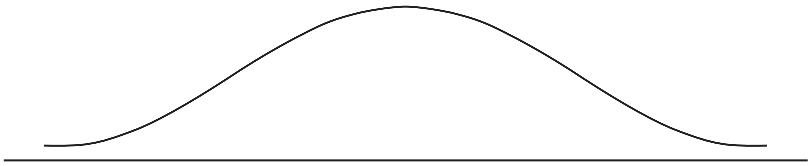
**3.** *[Maximum mark: 6]*

A random variable $X$ is distributed normally with mean 450. It is known that $\mathrm{P}(X > a) = 0.27$.

(a) Represent all this information on the following diagram. *[3 marks]*



(b) Given that the standard deviation is 20, find $a$. Give your answer correct to the nearest whole number. *[3 marks]*

**4.** *[Maximum mark: 7]*

Consider the lines $L_1$, $L_2$, $L_3$, and $L_4$, with respective equations.

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \qquad L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + p\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$L_3: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix} \qquad L_4: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = q\begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$$

(a) Write down the line which is parallel to $L_4$. *[1 mark]*

(b) Write down the position vector of the point of intersection of $L_1$ and $L_2$. *[1 mark]*

(c) Given that $L_1$ is perpendicular to $L_3$, find the value of $a$. *[5 marks]*

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**5.** *[Maximum mark: 7]*

The probability of obtaining heads on a biased coin is 0.4. The coin is tossed 600 times.

(a) (i) Write down the mean number of heads.

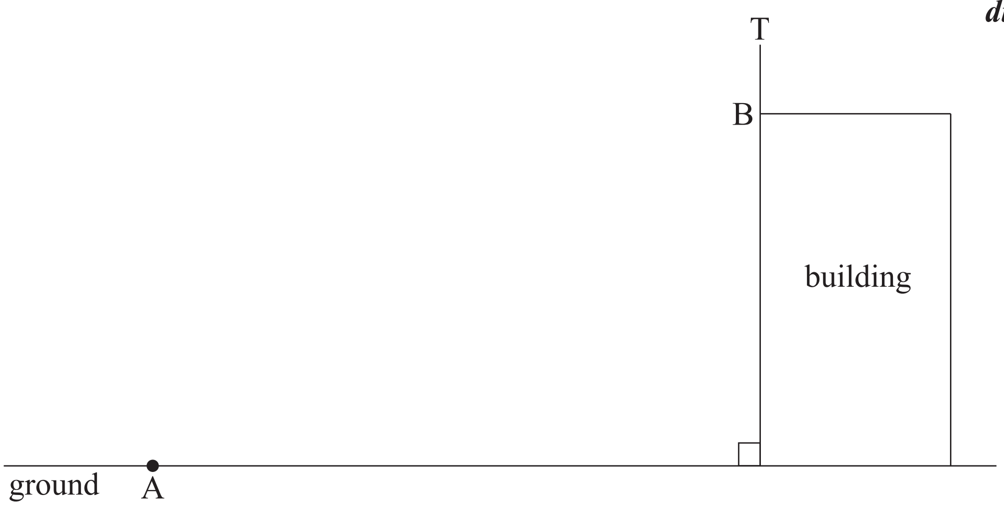
(ii) Find the standard deviation of the number of heads. *[4 marks]*

(b) Find the probability that the number of heads obtained is less than one standard deviation away from the mean. *[3 marks]*

**6.** *[Maximum mark: 7]*

The following diagram shows a pole BT 1.6 m tall on the roof of a vertical building. The angle of depression from T to a point A on the horizontal ground is $35^\circ$. The angle of elevation of the top of the building from A is $30^\circ$.

***diagram not to scale***




Find the height of the building.

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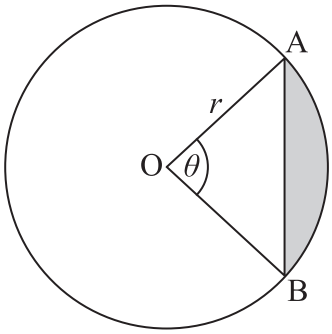
**7.** *[Maximum mark: 8]*

A circle centre O and radius $r$ is shown below. The chord [AB] divides the area of the circle into two parts. Angle AOB is $\theta$.




(a) Find an expression for the area of the shaded region. *[3 marks]*

(b) The chord [AB] divides the area of the circle in the ratio 1:7. Find the value of $\theta$. *[5 marks]*

*Do **NOT** write solutions on this page.*

**SECTION B (43 Marks)**

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

**8.** *[Maximum mark: 13]*

Each day, a factory recorded the number ($x$) of boxes it produces and the total production cost ($y$) dollars. The results for nine days are shown in the following table.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $x$ | 26 | 44 | 65 | 43 | 50 | 31 | 68 | 46 | 57 |
| $y$ | 400 | 582 | 784 | 625 | 699 | 448 | 870 | 537 | 724 |

(a) Write down the equation of the regression line of $y$ on $x$. *[2 marks]*

Use your regression line as a model to answer the following.

(b) Interpret the meaning of

(i) the gradient;

(ii) the $y$-intercept. *[2 marks]*

(c) Estimate the cost of producing 60 boxes. *[2 marks]*

(d) The factory sells the boxes for $19.99 each. Find the least number of boxes that the factory should produce in one day in order to make a profit. *[3 marks]*

(e) Comment on the appropriateness of using your model to

(i) estimate the cost of producing 5000 boxes;

(ii) estimate the number of boxes produced when the total production cost is $540. *[4 marks]*

*Do **NOT** write solutions on this page.*

**9.** *[Maximum mark: 16]*

Let $h(x) = \dfrac{2x-1}{x+1}$, $x \neq -1$.

(a) Find $h^{-1}(x)$. *[4 marks]*

(b) (i) Sketch the graph of $h$ for $-4 \leq x \leq 4$ and $-5 \leq y \leq 8$, including any asymptotes.

(ii) Write down the equations of the asymptotes.

(iii) Write down the $x$-intercept of the graph of $h$. *[7 marks]*

(c) Let $R$ be the region in the first quadrant enclosed by the graph of $h$, the $x$-axis and the line $x = 3$.

(i) Find the area of $R$.

(ii) Write down an expression for the volume obtained when $R$ is revolved through $360^\circ$ about the $x$-axis. *[5 marks]*

*Do **NOT** write solutions on this page.*

**10.** *[Maximum mark: 14]*

A rock falls off the top of a cliff. Let $h$ be its height above ground in metres, after $t$ seconds.

The table below gives values of $h$ and $t$.

| $t$ (seconds) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $h$ (metres) | 105 | 98 | 84 | 60 | 26 |

(a) Jane thinks that the function $f(t) = -0.25t^3 - 2.32t^2 + 1.93t + 106$ is a suitable model for the data. Use Jane's model to

(i) write down the height of the cliff;

(ii) find the height of the rock after 4.5 seconds;

(iii) find after how many seconds the height of the rock is 30 m. *[5 marks]*

(b) Kevin thinks that the function $g(t) = -5.2t^2 + 9.5t + 100$ is a better model for the data. Use Kevin's model to find when the rock hits the ground. *[3 marks]*

(c) (i) On graph paper, using a scale of 1 cm to 1 second, and 1 cm to 10 m, plot the data given in the table.

(ii) By comparing the graphs of $f$ and $g$ with the plotted data, explain which function is a better model for the height of the falling rock. *[6 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

SPEC/5/MATME/SP2/ENG/TZ0/XX/M

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# MARKSCHEME

## SPECIMEN

## MATHEMATICS

## Standard Level

## Paper 2

15 pages

## Instructions to Examiners

### Abbreviations

***M*** Marks awarded for attempting to use a correct **Method**.

***A*** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

***R*** Marks awarded for clear **Reasoning**.

***N*** Marks awarded for **correct** answers if **no** working shown.

***AG*** Answer given in the question and so no marks are awarded.

### Using the markscheme

### 1 General

*Award marks using the annotations as noted in the markscheme eg* ***M1****,* ***A2.***

### 2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

### 3 *N* marks

*If* ***no*** *working shown, award* ***N*** *marks for* ***correct*** *answers. In this case, ignore mark breakdown (****M****,* ***A****,* ***R****).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

**4 Follow through marks (only applied after an error is made)**

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

**5 Mis-read**

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

**6 Alternative methods**

*Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme*

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

## 7 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

## 8 Accuracy of Answers

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.*

There are 2 types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

## 9 Calculators

*A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.*

**Calculator notation**
The Mathematics SL guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

## 10 Style

*The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

**SECTION A**

**1.** (a) attempt to substitute into sum formula for AP ***M1***

*e.g.* $S_{20} = \frac{20}{2}\left(2(-7)+19d\right),\ \frac{20}{2}(-7+u_{20})$

setting up correct equation using sum formula ***A1***

*e.g.* $\frac{20}{2}\left(2(-7)+19d\right)=620$

$d=4$ ***A1*** ***N2***

***[3 marks]***

(b) correct substitution $-7+77(4)$ ***A1***

$u_{78}=301$ ***A1*** ***N2***

***[2 marks]***

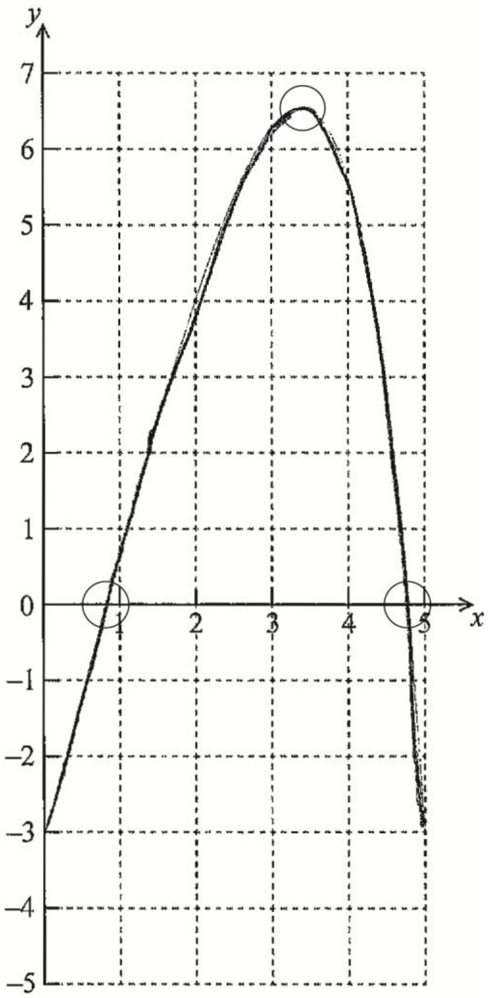
***Total [5 marks]***

**2.** (a) intercepts when $f(x) = 0$ ***M1***

$(0.827, 0)$ $(4.78, 0)$ (accept $x = 0.827$ $x = 4.78$) ***A1A1*** ***N3***

***[3 marks]***

(b)




***A1A1A1*** ***N3***

> **Note:** Award ***A1*** for maximum point in circle, ***A1*** for $x$-intercepts in circles, ***A1*** for correct shape ($y$ approximately greater than $-3.14$).

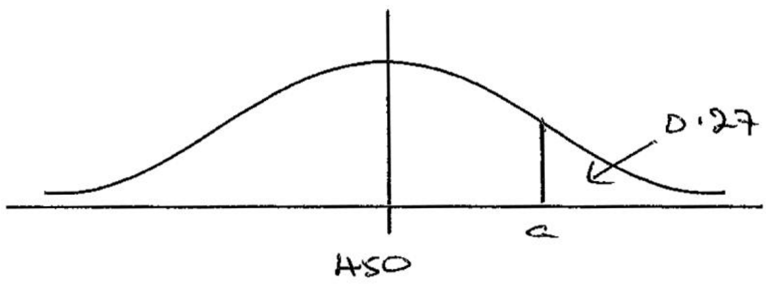
***[3 marks]***

(c) gradient is 1.28 ***A1*** ***N1***

***[1 mark]***

***Total [7 marks]***

**3.** (a)




***A1A1A1*** ***N3***

**Note:** Award ***A1*** for $450$, ***A1*** for $a$ to the right of the mean, ***A1*** for area $0.27$.

***[3 marks]***

(b) valid approach ***M1***

*e.g.* $\mathrm{P}(X < a) = 1 - \mathrm{P}(X > a),\ 0.73$

$a = 462.256\ldots$ ***A1***

$a = 462$ ***A1*** ***N3***

***[3 marks]***

***Total [6 marks]***

**4.** (a) $L_1$ ***A1*** ***N1***

***[1 mark]***

(b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ***A1*** ***N1***

***[1 mark]***

(c) choosing correct direction vectors ***A1A1***

*e.g.* $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix}$

recognizing that $\boldsymbol{a} \bullet \boldsymbol{b} = 0$ ***M1***

correct substitution ***A1***

*e.g.* $-3 - 4 - a = 0$

$a = -7$ ***A1*** ***N3***

***[5 marks]***

***Total [7 marks]***

**5.** (a) (i) recognizing binomial with $n = 600$, $p = 0.4$ ***M1***

$\mathrm{E}(X) = 240$ ***A1*** ***N2***

(ii) correct substitution into formula for variance or standard deviation ***A1***

*e.g.* $144$, $\sqrt{600 \times 0.4 \times 0.6}$

$\mathrm{sd} = 12$ ***A1*** ***N1***

***[4 marks]***

(b) attempt to find range of values ***M1***

*e.g.* $240 \pm 12$ $228 < X < 252$

evidence of correct approach ***A1***

*e.g.* $\mathrm{P}(X \le 251) - \mathrm{P}(X \le 228)$

$\mathrm{P}(228 < X < 252) = 0.662$ ***A1*** ***N2***

***[3 marks]***

***Total [7 marks]***

**6.** **METHOD 1**

appropriate approach *M1*
*e.g.* completed diagram

attempt at set up *A1*
*e.g.* correct placement of one angle

$\tan 30 = \frac{h}{x}$, $\tan 35 = \frac{h+1.6}{x}$ *A1A1*

attempt to set up equation *M1*
*e.g.* isolate $x$

correct equation *A1*
*e.g.* $\frac{h}{\tan 30} = \frac{h+1.6}{\tan 35}$

$h = 7.52$ *A1* ***N3***
***[7 marks]***

**METHOD 2**

$\sin 30 = \frac{h}{l}$ *A1*

in triangle ATB, $\hat{A} = 5^\circ$, $\hat{T} = 55^\circ$ *A1A1*
choosing sine rule *M1*

correct substitution
*e.g.* $\frac{h / \sin 30}{\sin 55} = \frac{1.6}{\sin 5}$ *A1*

$h = \frac{1.6 \times \sin 30 \times \sin 55}{\sin 5}$ *A1*
$h = 7.52$ *A1* ***N3***
***[7 marks]***

**7.** (a) substitution into formula for area of triangle ***A1***

*e.g.* $\frac{1}{2}r \times r \sin\theta$

evidence of subtraction ***M1***

correct expression ***A1*** ***N2***

*e.g.* $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta,\ \frac{1}{2}r^2(\theta - \sin\theta)$

***[3 marks]***

(b) evidence of recognizing that shaded area is $\frac{1}{8}$ of area of circle ***M1***

*e.g.* $\frac{1}{8}$ seen anywhere

setting up correct equation ***A1***

*e.g.* $\frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{8}\pi r^2$

eliminating 1 variable ***M1***

*e.g.* $\frac{1}{2}(\theta - \sin\theta) = \frac{1}{8}\pi,\ \theta - \sin\theta = \frac{\pi}{4}$

attempt to solve ***M1***

*e.g.* a sketch, writing $\sin x - x + \frac{\pi}{4} = 0$

$\theta = 1.77$ (do not accept degrees) ***A1*** ***N1***

***[5 marks]***

***Total [8 marks]***

**SECTION B**

**8.** (a) $y = 10.7x + 121$ ***A1A1*** ***N2***

***[2 marks]***

(b) (i) additional cost per box (unit cost) ***A1*** ***N1***

(ii) fixed costs ***A1*** ***N1***

***[2 marks]***

(c) attempt to substitute into regression equation ***M1***

*e.g.* $y = 10.7 \times 60 + 121$, $y = 760.12\ldots$

cost $= \$760$ (accept \$763 from 3 s.f. values) ***A1*** ***N2***

***[2 marks]***

(d) setting up inequality (accept equation) ***M1***

*e.g.* $19.99x > 10.7x + 121$

$x > 12.94\ldots$ ***A1***

13 boxes (accept 14 from $x > 13.02$, using 3 s.f. values) ***A1*** ***N2***

| |
|---|
| **Note:** Exception to the ***FT*** rule: if working shown, award the final ***A1*** for a correct integer solution for **their** value of *x*. |

***[3 marks]***

(e) (i) this would be extrapolation, not appropriate ***R1R1*** ***N2***

(ii) this regression line cannot predict *x* from *y*, not appropriate ***R1R1*** ***N2***

***[4 marks]***

***Total [13 marks]***

**9.** (a) $y = \dfrac{2x-1}{x+1}$

interchanging *x* and *y* (seen anywhere) ***M1***

*e.g.* $x = \dfrac{2y-1}{y+1}$

correct working ***A1***

*e.g.* $xy + x = 2y - 1$

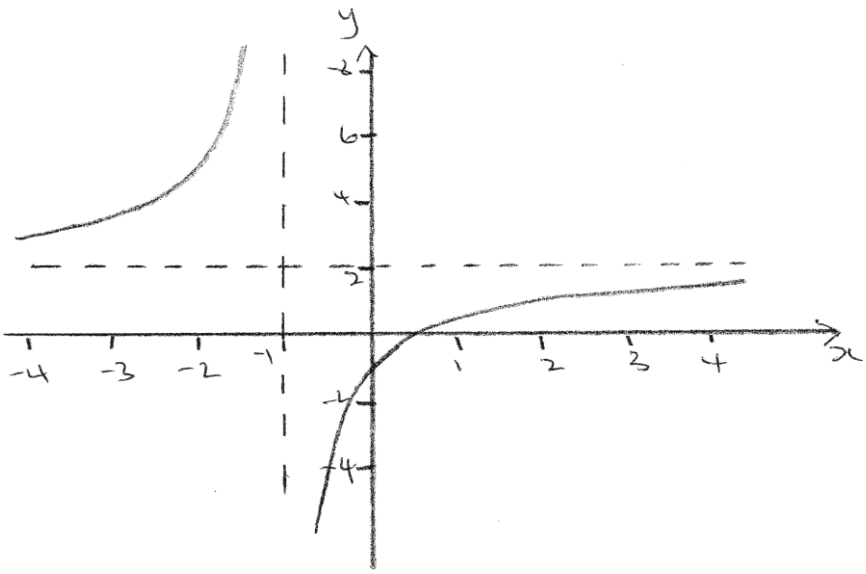
collecting terms ***A1***

*e.g.* $x+1 = 2y - xy,\ x+1 = y(2-x)$

$h^{-1}(x) = \dfrac{x+1}{2-x}$ ***A1*** ***N2***

***[4 marks]***

(b) (i)




***A1A1A1A1*** ***N4***

| |
|---|
| **Note:** Award ***A1*** for approximately correct intercepts, ***A1*** for correct shape, ***A1*** for asymptotes, ***A1*** for approximately correct domain and range. |

(ii) $x = -1,\ y = 2$ ***A1A1*** ***N2***

(iii) $\dfrac{1}{2}$ ***A1*** ***N1***

***[7 marks]***

*continued …*

*Question 9 continued*

(c) (i) $\text{area} = 2.06$ ***A2*** ***N2***

(ii) attempt to substitute into volume formula $\left(\text{do not accept } \pi\int_a^b y^2 \mathrm{d}x\right)$ ***M1***

$\text{volume} = \pi\int_{\frac{1}{2}}^{3}\left(\frac{2x-1}{x+1}\right)^2 \mathrm{d}x$ ***A2*** ***N3***

***[5 marks]***

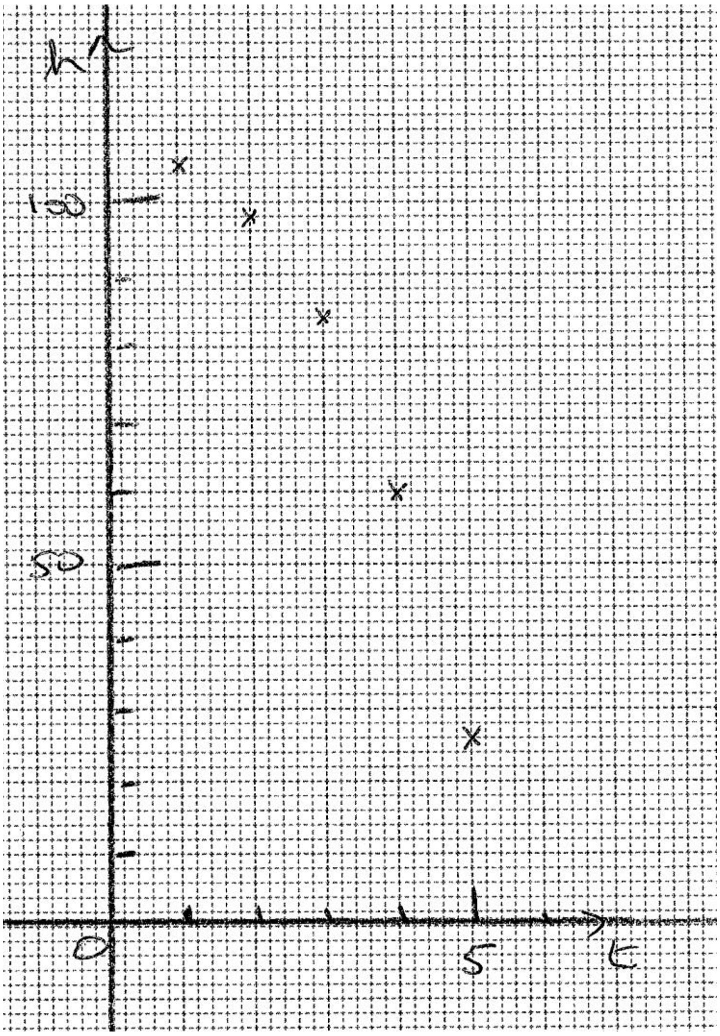
***Total [16 marks]***

| | | | | | |
|---|---|---|---|---|---|
| **10.** | (a) | (i) | 106 m | ***A1*** | ***N1*** |
| | | (ii) | substitute $t = 4.5$ | ***M1*** | |
| | | | $h = 44.9$ m | ***A1*** | ***N2*** |
| | | (iii) | set up suitable equation | ***M1*** | |
| | | | *e.g.* $f(t) = 30$ | | |
| | | | $t = 4.91$ | ***A1*** | ***N1*** |
| | | | | | ***[5 marks]*** |
| | (b) | recognising that height is 0 | | ***A1*** | |
| | | set up suitable equation | | ***M1*** | |
| | | *e.g.* $g(t) = 0$ | | | |
| | | $t = 5.39\,\text{secs}$ | | ***A1*** | ***N2*** |
| | | | | | ***[3 marks]*** |

*continued …*

*Question 10 continued*

(c) (i)




***A1A2*** ***N3***

| | |
|---|---|
| **Note:** Award ***A1*** for correct scales on axes, ***A2*** for 5 correct points, ***A1*** for 3 or 4 correct points. | |

(ii) Jane's function, with **2** valid reasons ***A1R1R1*** ***N3***

*e.g.* Jane's passes very close to all the points, Kevin's has the rock clearly going up initially – not possible if rock falls

| | |
|---|---|
| **Note:** Although Jane's also goes up initially, it only goes up very slightly, and so is the better model. | |

***[6 marks]***

***Total [14 marks]***