SAT

If x + 2x is 5 more than y + 2y, then x – y =

How do we solve this?

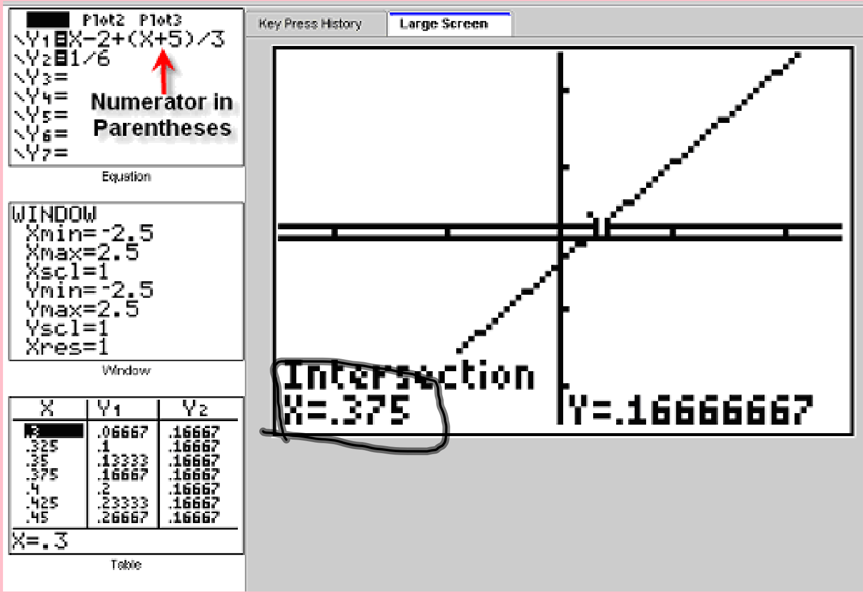
$$x - 2 + \frac{x+5}{3} = \frac{1}{6}$$

Here's a reminder of how you've solved equations with fractions before. The key idea is to find the least common denominator (LCD), and multiply the whole equation by that LCD.

$6x - 12 + 2(x+5) = 1$

$x - 2 + \frac{x+5}{3} = \frac{1}{6}$	original equation
$6x - 12 + \frac{6(x+5)}{3} = \frac{6}{6}$	multiply by 6, the LCD
$6x - 12 + 2(x+5) = 1$	reduce fractions
$6x - 12 + 2x + 10 = 1$	distribute the 2
$8x - 2 = 1$	collect like terms
$8x = 3$	Add 2
$x = \frac{3}{8}$	Divide by 3

You can also put the two sides of the equation into Y_1 and Y_2 in the calculator and use 2^{nd} - CALC - 5: intersect to find the solution. Remember to put parentheses around any numerator or denominator... Notice x = 0.375, which is the same as $\frac{3}{8}$.



Example 1: Solving by Clearing Fractions

(What is the LCD?)

The same process happens with rational functions: functions with variables in the numerator <u>and</u> the denominator. You still have to multiply by the LCD, but it can get a bit tricky...

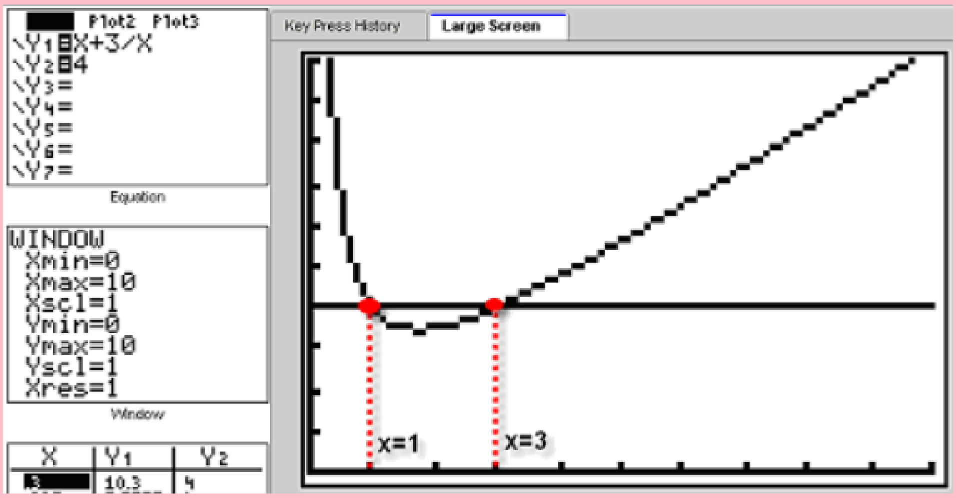
$$x\left(x + \frac{3}{x} = 4\right) \rightarrow x^2 + 3 = 4x$$

$x + \frac{3}{x} = 4$	original equation
$x^2 + \frac{3x}{x} = 4x$	multiply by x, the LCD
$x^2 + 3 = 4x$	reduce the fraction
$x^2 - 4x + 3 = 0$	subtract 4x
$(x-1)(x-3) = 0$	factor
$x - 1 = 0$ or $x - 3 = 0$	zero factor property
$x = 1$ or $x = 3$	solve each equation

Verify the solutions in your graphing calculator using Y_1 and Y_2 with 2^{nd} - CALC - 5: intersect:

$$Y_1 = x + \frac{3}{x}$$

$$Y_2 = 4$$



Now try solving this one in your notebook: $x + 5 = \frac{14}{x}$

You should get x = -7 or x = 2.

Verify both intersections on the calculator using Y_1 and Y_2.

Example 2: Solving a Rational Equation

(What is the LCD?)

$$\left(x + \frac{1}{x-4} = 0\right)(x-4)$$

Algebraically:

$$x(x-4) + 1 = 0$$

$x + \dfrac{1}{x-4} = 0$	original equation
$x(x-4) + \dfrac{1(x-4)}{x-4} = 0(x-4)$	multiply by (x - 4), the LCD
$x^2 - 4x + 1 = 0$	reduce the fraction and distribute the x

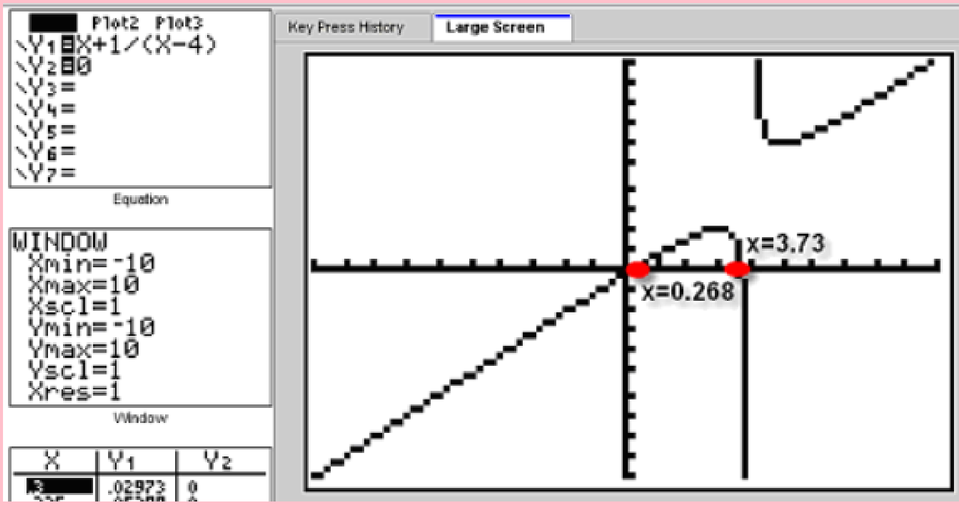
This quadratic function does not factor easily, so solve it using the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

Verify the solutions in your graphing calculator using Y_1 and Y_2 with 2^{nd} - CALC - 5: intersect:

$$Y_1 = x + \frac{1}{x-4}$$

$$Y_2 = 0$$



Now try solving this one in your notebook: $x + \frac{4x}{x-3} = \frac{12}{x-3}$

You should get x = -4 or x = 3. However, x = 3 makes the denominator equal to zero, so we throw that answer out as "extraneous".

Verify both intersections on the calculator using Y_1 and Y_2. Remember to put the numerator and denominator in parentheses!

Example 4: Eliminating Extraneous Solutions

(What is the LCD?)

$$\frac{(x-3)}{x}+\frac{3}{(x+2)}+\frac{6}{(x^2+2x)}=0$$

Algebraically:

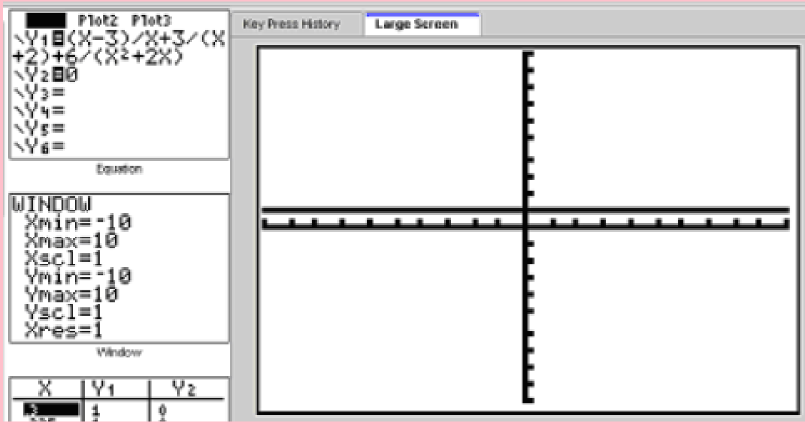
$\frac{x-3}{x}+\frac{3}{x+2}+\frac{6}{x^2+2x}=0$	original equation
$\frac{(x-3)x(x+2)}{x}+\frac{3x(x+2)}{x+2}+\frac{6x(x+2)}{x(x+2)}=0$	multiply by $x(x+2)$, the LCD
$(x-3)(x+2)+3x+6=0$	reduce the fractions
$x^2-x-6+3x+6=0$	FOIL
$x^2+2x=0$	Simplify
$x(x+2)=0$	Factor
$x=0$ or $x+2=0$	zero factor property
$x=0$ or $x=-2$	solve each equation

Look back at the original equation. $x = 0$ and $x = -2$ both make the denominator zero. So both solutions are extraneous.

Verify that there is no solution in your graphing calculator using Y_1 and Y_2 with 2^{nd} - CALC - 5: intersect:

$$Y_1 = \frac{x-3}{x} + \frac{3}{x+2} + \frac{6}{x^2+2x}$$

$$Y_2 = 0$$



Now try solving this one in your notebook $\frac{3}{x-1} + \frac{2}{x} = 8$

If you multiply and simplify correctly, you should get $8x^2 - 13x + 2 = 0$. .
Use the quadratic equation or your calculator to solve and get
$x = 0.172$ or $x = 1.45$.
Neither of these answers is "extraneous".

Homework Assignment:
page 254 (7-17odd, 23-29 odd)