

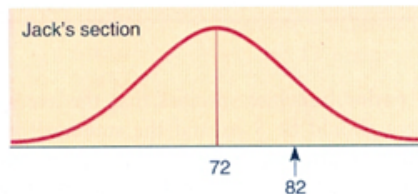
## S.6 Z-Scores and The Standard Normal Distribution

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When we're looking at very different sets of data, sometimes it's hard to make a comparison. Who did better in their section: Jack or Tina? One way to answer that question is to find how many standard deviations each student is away from their section mean.

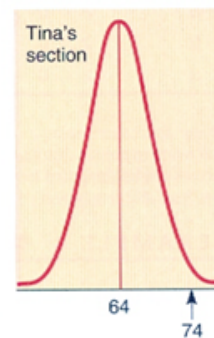
Distributions of Midterm Scores

$$\frac{82-72}{8} = \frac{10}{8} = 1.25$$



Jack:  $\mu = 72$   
 $\sigma = 8$

How many standard deviations is Jack away from his mean?



Tina:  $\mu = 64$   
 $\sigma = 4$

$$\frac{74-64}{4} = \frac{10}{4} = 2.5$$

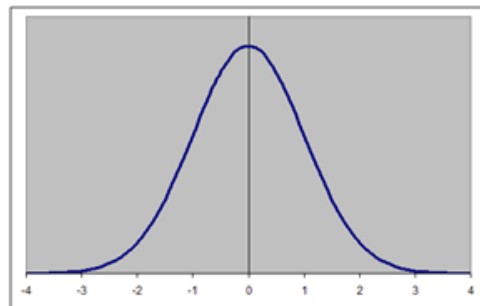
How many standard deviations is Tina away from her mean?

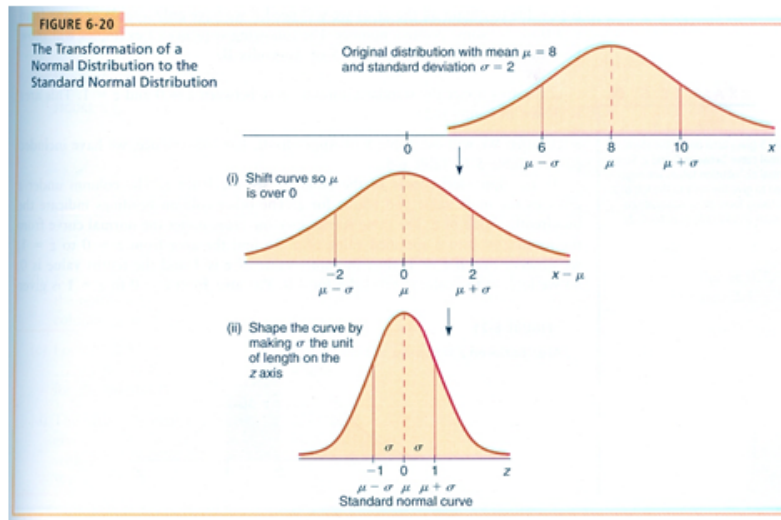
The Z-score is a measure of how many standard deviations a particular data point is away from the mean.

There are literally an infinite number of normal distributions with different combinations of means and standard deviations. Wouldn't it be nice if there was a "standard" normal distribution? Then we could have a common means of comparison between all normal distributions. Z-scores enable us to do this.

The standard normal distribution has a mean of 0 and a standard deviation of 1. All other normal distributions can be transformed into the standard normal distribution for comparison purposes.

$$Z \sim N(0,1)$$



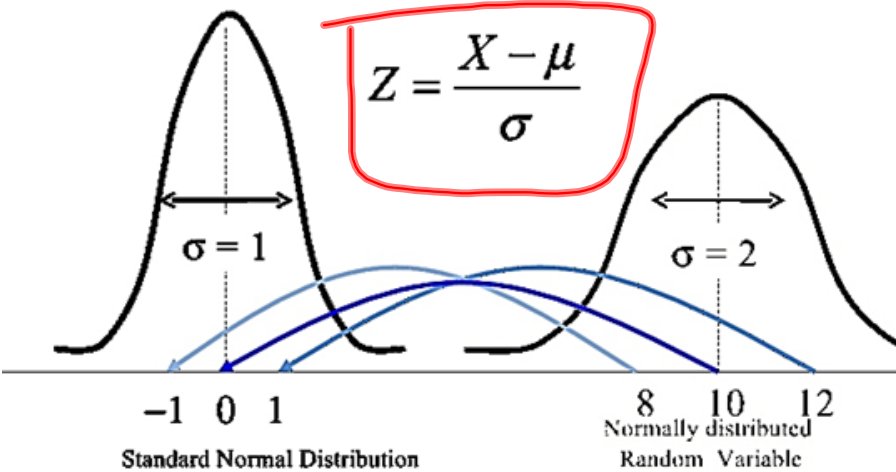


To convert a normal distribution into a standard normal distribution, we:

- 1) Shift the distribution left or right until its mean is zero
- 2) Stretch or shrink the distribution until its standard deviation is one.

This results in the following Z-score formula:

$$z = \frac{x - \mu}{\sigma}$$

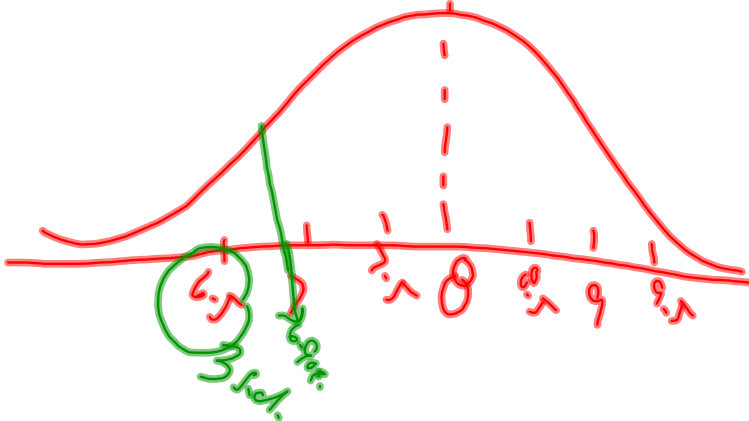


Example 1:

A pizza restaurant franchise specifies that the average (mean) amount of cheese on a large pizza should be 8 ounces and the standard deviation only 0.5 ounces. If the amount of cheese is more than 3 standard deviations below the mean, the restaurant could lose its franchise.

An inspector picks out a large pizza at random in one of the pizza restaurants and finds that it is made with 6.9 ounces of cheese. Is the restaurant in danger of losing its franchise? **no**

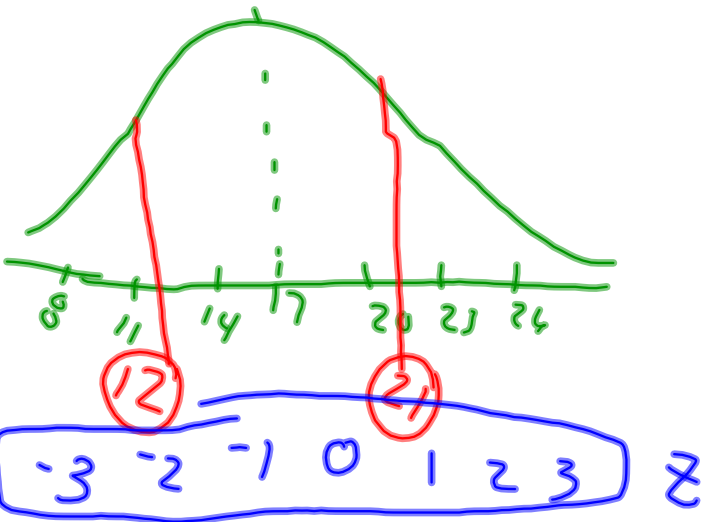
What is the minimum amount of cheese that can be placed on a large pizza according to the franchise?

Using the formula	Using the diagram
$z = \frac{x - \mu}{\sigma}$ $-3 = \frac{x - 8}{0.5}$ $-1.5 = x - 8$ $6.5 = x$	

Example 2:

A student has computed that it takes an average (mean) of 17 minutes to walk home, with a standard deviation of 3 minutes.

- One day it took the student 21 minutes to get home. How many standard deviations from the mean is that?
- Another day it took only 12 minutes for the student to get to class. How many standard deviations from the mean is that?
- Another day it took only 17 minutes for the student to get to class. What is the z-score? Why?

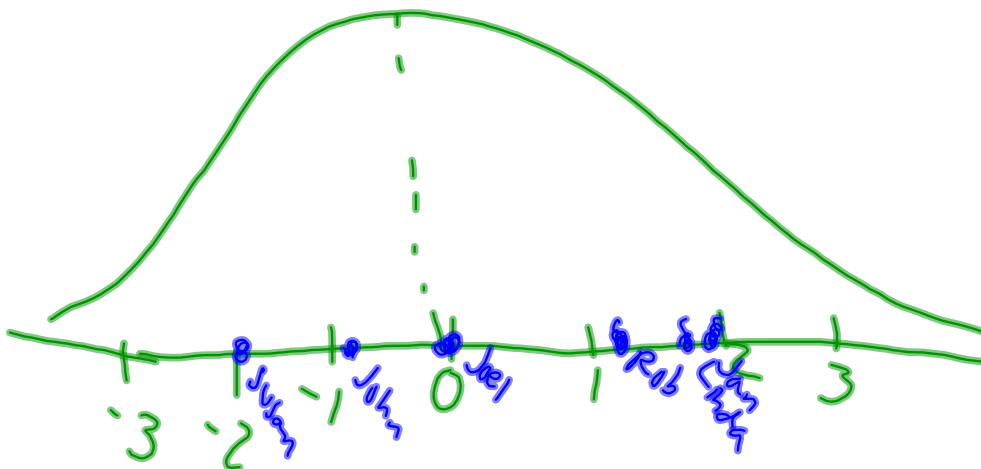
Using the formula	Using the diagram
<p>a) <math>z = \frac{21-17}{3}</math>  <math>z = 1.33</math></p> <p>b) <math>z = \frac{12-17}{3}</math>  <math>z = -1.67</math></p> <p>c) <math>z = \frac{17-17}{3} = 0</math></p>	

Example 3:

The college physical education department offered an Advanced First Aid course last semester. The scores on the comprehensive final exam were normally distributed, and the z-scores for some of the students are shown below:

Robert:	1.10	Jan:	1.70	Susan:	-2.00
Joel:	0.00	John:	-0.80	Linda:	1.60

- a) Draw a standard normal distribution. Place each student in their proper place along the x-axis according to their z-score.



- b) Which of these students scored above the mean?

Robert, Linda, Jan

- c) Which of these students scored at the mean?

Joel

- d) Which of these students scored below the mean?

Susan, John

- e) The mean score on the final exam was 150 and the standard deviation was 20. Calculate the final exam score for each student. Show your work!

$$1.1 = \frac{x - 150}{20}$$

$$\text{Robert} = 172$$

$$\text{Joel} = 150 \quad \text{John} = 134$$

$$\text{Jan} = 184 \quad \text{Susan} = 110$$

$$\text{Linda} = 102$$



A very typical kind of problem that you will encounter in IB is, given a particular situation, find the mean or the standard deviation. How can you use the calculator to find the additional information you need?

Example 4:

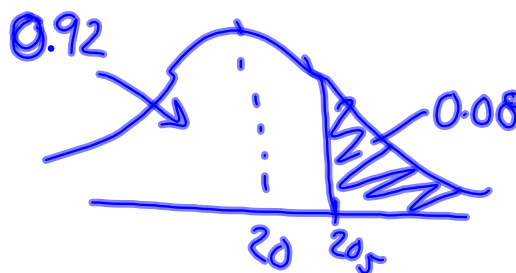
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A machine produces components whose lengths are normally distributed with a mean of 20 cm. Given that 8% of the components produced by the machine have a length greater than 20.5 cm, find the standard deviation.

$$\text{invNorm}(0.92, 0, 1)$$

$$z = 1.41$$

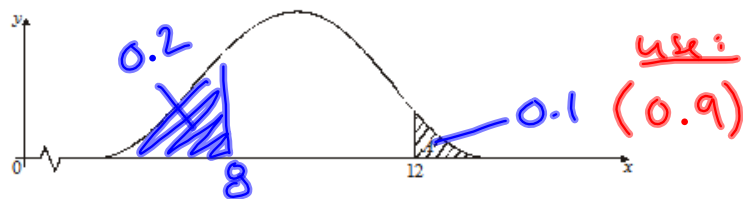
$$z = \frac{x - \mu}{\sigma} \quad 1.41 = \frac{20.5 - 20}{\sigma}$$



$$\sigma = 0.355$$

## Example 5:

The graph shows a normal curve for the random variable  $X$ , with mean  $\mu$  and standard deviation  $\sigma$ .



It is known that  $P(X \geq 12) = 0.1$ . It is also known that  $P(X \leq 8) = 0.2$ .

Find the value of  $\mu$  and  $\sigma$ , showing your method in full.

$$z = 1.28$$

$$1.28 = \frac{12 - \mu}{\sigma}$$

$$z = -0.842$$

$$-0.842 = \frac{8 - \mu}{\sigma}$$

Prodly Sm It:  $1.28\sigma + \mu = 12$

$$-0.84\sigma + \mu = 8$$

$$\sigma = 1.89, \mu = 9.59$$