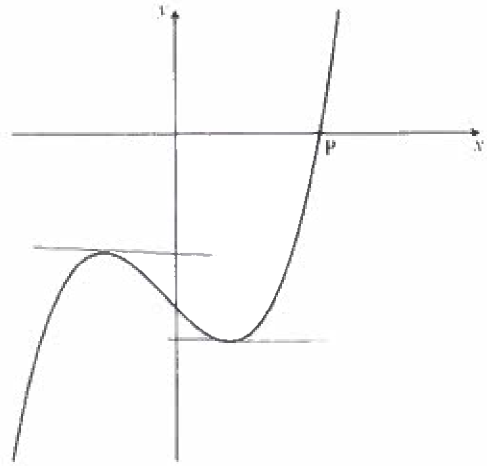
Key

REVIEW Test on Differentiation Rules [47 marks – 50 minutes]

1. Let $f(x) = x^3 - 2x - 4$. The following diagram shows part of the curve of f.



The curve crosses the x-axis at the point $P(2, 0)$.

a) Draw tangent line(s) on the diagram where $f'(x) = 0$. [1]

b) Find the gradient of the curve at P. $\rightarrow x = 2$ [2]

$f'(x) = 3x^2 - 2$

$f'(2) = 3(2)^2 - 2$

$= 12 - 2 = 10$

c) Find the equation of the normal to the curve at P, giving your equation in the form $y = ax + b$. [3]

$m = \frac{-1}{10}$

$y - 0 = -\frac{1}{10}(x - 2)$

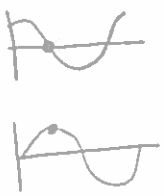
$y = -\frac{1}{10}x + \frac{1}{5}$

2. Consider $f(x) = x^2 \sin x$.

a) Find $f'(x)$. [4]

$f'(x) = x^2 \cos x + 2x \sin x$

b) Use your answer from a) to find the exact gradient of the curve of f at $x = \frac{\pi}{2}$. [3]



$f'(\frac{\pi}{2}) = (\frac{\pi}{2})^2 \cos(\frac{\pi}{2}) + 2(\frac{\pi}{2}) \sin(\frac{\pi}{2})$

$= 0 + \pi(1)$

$= \pi$

3. Let $f(x) = \frac{x}{-2x^2+5x-2}$.

Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. [6]

$$f'(x) = \frac{(-2x^2+5x-2)(1) - x(-4x+5)}{(-2x^2+5x-2)^2}$$

$$= \frac{-2x^2+5x-2+4x^2-5x}{(-2x^2+5x-2)^2}$$

$$= \frac{2x^2-2}{(-2x^2+5x-2)^2} \checkmark$$

4. Let $f(x) = e^{6x}$.

a) Write down $f'(x)$. [1]

$f'(x) = 6e^{6x}$

The tangent to the graph of f at the point $P(0, b)$ has gradient m.

bi) Show that $m = 6$. [2]

$f'(0) = 6e^{6(0)} = 6(1) = 6$

bii) Find b. [2]

$f(0) = e^{6(0)} = e^{0} = 1$

c) Hence, write down the equation of this tangent. [1]

$m = 6 \quad (0, 1)$

$y - 1 = 6(x - 0)$

$y = 6x + 1$

5. Given that $f(x) = \frac{1}{x}$, answer the following.

a) Find the first four derivatives of $f(x)$. $= x^{-1}$ [4]

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

$f^{(3)}(x) = -6x^{-4}$

$f^{(4)}(x) = 24x^{-5}$

b) Write an expression for $f^{(n)}(x)$ in terms of x and n. [3]

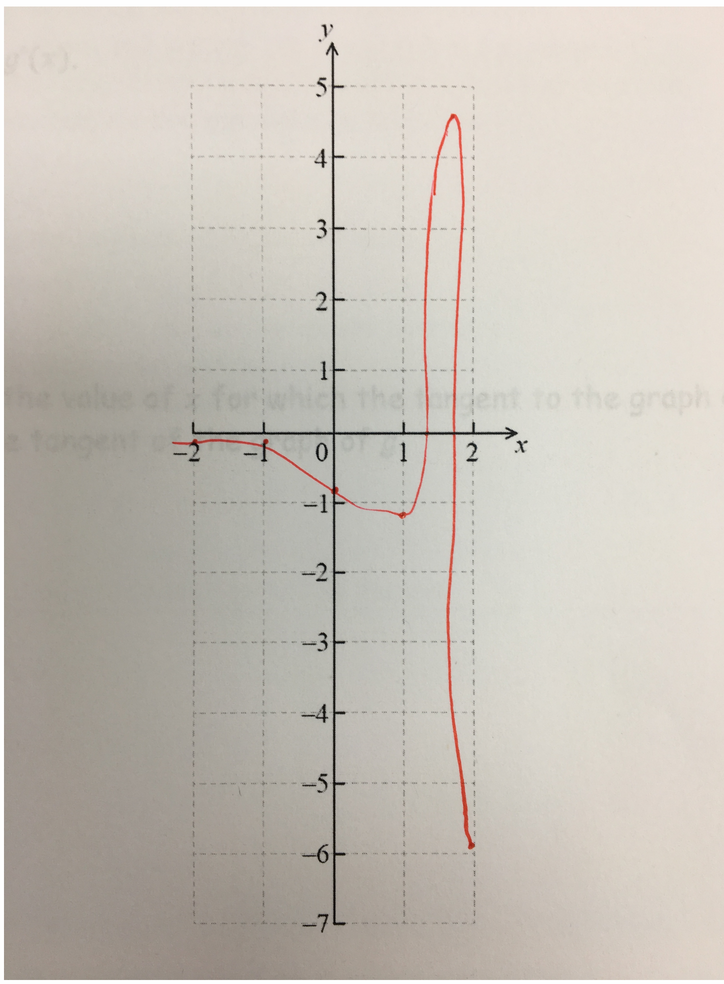
$$f^{(n)}(x) = (-1)^n n!\, x^{-n-1}$$

6. Let $f(x) = \cos(e^x)$, for $-2 \le x \le 2$.

a) Find $f'(x)$. [2]

$f'(x) = -e^x \sin(e^x)$

b) Using your graphing calculator in RADIAN mode, sketch the graph of $f'(x)$ on the grid below. [4]



7. Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \le x \le \pi$.

a) Find $f'(x)$. [3]

$f'(x) = \cos x + x - 2$

Let g be the quadratic equation $g(x) = \frac{1}{2}(x-2)^2 + 3$.

b) Find $g'(x)$. [2]

$g(x) = \frac{1}{2}(x^2 - 4x + 4) + 3$

$= \frac{1}{2}x^2 - 2x + 5$

$g'(x) = x - 2$

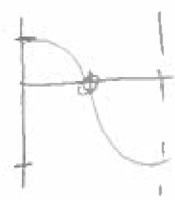
c) Find the value of x for which the tangent to the graph of f is parallel to the tangent of the graph of g. [4]

$g'(x) = f'(x)$

$x - 2 = \cos x + x - 2$

$0 = \cos x$

$x = \frac{\pi}{2}$



Domain:
$0 \le x \le \pi$