IB Math Standard Year Two

REVIEW for Test on Basic Integration **[Calculator prohibited]**

1. Find

a) $\frac{d}{dx}(3-2x)^5$

$= 5(3-2x)^4(-2)$

$= -10(3-2x)^4$

b) $\int(3-2x)^5 dx$.

$u = 3-2x$

$du = -2\,dx$

$-\frac{1}{2}du = dx$

$= -\frac{1}{2}\int u^5\,du$

$= -\frac{1}{2}\cdot\frac{u^6}{6} + C$

$= -\frac{1}{12}u^6 + C$

$= -\frac{1}{12}(3-2x)^6 + C$

2. Let $f(x) = \frac{10}{x^4}$. Find $f(x) = 10x^{-4}$

a) $f'(x)$

$f'(x) = -40x^{-5}$

$= \frac{-40}{x^5}$

b) $\int f(x)dx$

$= \frac{10x^{-3}}{-3}$

$= -\frac{10}{3x^3}$

/

3. Find

a) $\int \cos 5x \sin 5x\,dx$

$u = \sin 5x$
$du = 5\cos 5x\,dx$
$\frac{1}{5}du = \cos 5x\,dx$

$= \frac{1}{5}\int u\,du$
$= \frac{1}{5}\cdot\frac{u^2}{2} + C$
$= \frac{1}{10}u^2 + C$
$= \frac{1}{10}\sin^2 5x + C$

b) $\int \cos^4 x \sin x\,dx$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$\int (\cos x)^4 \sin x\,dx$
$= -\int u^4\,du$
$= -\frac{u^5}{5} + C$
$= -\frac{1}{5}(\cos^5 x) + C$

4. Find

a) $\int \frac{x}{3+5x^2}dx$

$u = 3+5x^2$
$du = 10x\,dx$
$\frac{1}{10}du = x\,dx$

$= \frac{1}{10}\int \frac{1}{u}\,du$
$= \frac{1}{10}\ln|u| + C$
$= \frac{1}{10}\ln(3+5x^2) + C$

b) $\int (x^2+2x-3)^2(x+1)dx$

$u = x^2+2x-3$
$du = 2x+2\,dx$
$\frac{du}{2} = \frac{2(x+1)}{2}dx$
$\frac{1}{2}du = (x+1)\,dx$

$= \frac{1}{2}\int u^2\,du$
$= \frac{1}{2}\cdot\frac{u^3}{3} + C$
$= \frac{1}{6}u^3 + C$
$= \frac{1}{6}(x^2+2x-3)^3 + C$

5. Find

a) $\int \frac{5-x^2}{x}\,dx$

$= \int \frac{5}{x}\,dx - \int x\,dx$

$= 5\int \frac{1}{x}\,dx - \int x\,dx$

$= 5\ln|x| - \frac{x^2}{2} + C$

b) $\int 4e^{3x}\,dx$

$= \frac{1}{3}\int 4e^{u}\,dx$

$= \frac{4}{3}\cdot e^{u} + C$

$= \frac{4}{3}e^{3x} + C$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

6. The acceleration, a ms^{-2}, of a particle at time t seconds is given by $a = 2t + \cos t$.

a) Find the acceleration of the particle at $t = 0$. [2]

$a(0) = 2(0) + \cos 0 = 1$

b) Find the velocity, v, at time t, given that the initial velocity of the particle is 2 ms^{-1}. [4]

$v(0) = 2$ m/s

$v(t) = \int 2t + \cos t\;dt$

$= \frac{\cancel{2}t^2}{\cancel{2}} + \sin t + C$

$v(t) = t^2 + \sin t + C$

$2 = 0^2 + \sin 0 + C$

$2 = C$

$v(t) = t^2 + \sin t + 2$

7. A particle moves along a straight line so that its velocity, v m s^{-1} at time t seconds is given by $v = 6e^{3t} + 4$. When t = 0, the displacement, s, of the particle is 7 meters. Find an expression for s in terms of t.

$s(0) = 7$ m

$$s(t) = \int 6e^{3t} + 4 \, dt$$

$u = 3t$

$du = 3dt$

$\frac{1}{3}du = dt$

$$= \frac{6}{3}e^{3t} + 4t + C$$

$$s(t) = 2e^{3t} + 4t + C$$

$$7 = 2e^{0} + 4(0) + C$$

$$7 = 2 + C$$

$$5 = C$$

$$s(t) = 2e^{3t} + 4t + 5$$

8. Find

a) $\int \frac{9r^2}{\sqrt{1-r^3}}dr$

$u = 1-r^3$
$du = -3r^2 dr$
$-\frac{1}{3}du = r^2 dr$

$= -\frac{9}{3}\int \frac{1}{u^{1/2}}\,du$
$= -3\int u^{-1/2}\,du$
$= -3\frac{u^{1/2}}{1/2} + C$
$= -6u^{1/2} + C$
$= -6(1-r^3)^{1/2} + C$

b) $\int \tan x\,dx$

Remember that $\tan x = \frac{\sin x}{\cos x}$...

$= \int \frac{\sin x}{\cos x}dx$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$= -\int \frac{1}{u}du$

$= -\ln|u| + c$

$= -\ln|\cos x| + C$

9. Find $\int\left(e^{2x+1} + \frac{3}{2x+1}\right)dx$.

$u = 2x+1$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$u = 2x+1$
$du = 2\,dx$
$\frac{3}{2}du = 3dx$

$\frac{1}{2}\int e^u\,du + \frac{3}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}e^{2x+1} + \frac{3}{2}\ln|2x+1| + C$

10. The acceleration of a particle moving back and forth along a line is $a = \pi^2 \cos \pi t$.
Find $v(t)$ if $v(0) = 8$ m/s.

$$v(t) = \int (\pi^2 \cos \pi t)\,dt$$

$u = \pi t$

$du = \pi\,dt$

$\frac{1}{\pi} du = dt$

$$= \frac{1}{\pi} \int \pi^2 \cos u \, du$$

$$= \frac{\pi^{\cancel{2}}}{\cancel{\pi}} \sin u + C$$

$$= \pi \sin \pi t + C$$

11. The marginal cost function for a certain manufacturer is
$\frac{dC}{dx} = 25 + 20x - 0.03x^2$, and it costs \$2500 to produce 10 units. Find the production cost C(x) for 100 units.

$C(10) = 2500$

$$C(x) = \int (25 + 20x - 0.03x^2)\,dx$$

$$= 25x + \frac{20x^2}{2} - \frac{0.03x^3}{3} + C$$

$$C(x) = 25x + 10x^2 - 0.01x^3 + C$$

$$2500 = 25(10) + 10(10)^2 - 0.01(10)^3 + C$$

$$2500 = 1240 + C$$

$$1260 = C$$

$$C(x) = 25x + 10x^2 - 0.01x^3 + 1260$$

$$C(100) = 25(100) + 10(100)^2 - 0.01(100)^3 + 1260$$

$$C(100) = \$93{,}760$$

$d = 480$ ft

12. A ball is thrown vertically downward from the top of a 480-foot building with an initial velocity of –64 feet per second. If acceleration due to gravity is –32 feet/sec/sec, with what velocity does the object hit the ground?

$v(0) = -64$ $a(t) = -32$

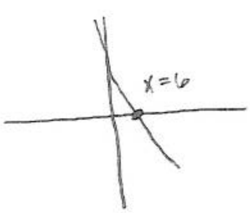
a) Find v(t). [4]

$$v(t) = \int -32\,dt$$
$$v(t) = -32t + C$$
$$-64 = -32(0) + C$$
$$-64 = C$$
$$\boxed{v(t) = -32t - 64}$$

b) Find s(t). [4]

$$s(t) = \int (-32t - 64)\,dt$$
$$= -\frac{32t^2}{2} - 64t + C$$
$$s(t) = -16t^2 - 64t + C$$
$$480 = -16(0)^2 - 64(0) + C$$
$$480 = C$$
$$\boxed{s(t) = -16t^2 - 64t + 480}$$

c) Use your graphing calculator to find when the object hits the ground. [2]



$$-16t^2 - 64t + 480 = 0$$
$$t =$$

d) Find the velocity of the object when it hits the ground. [2]

$$v(6) = -32(6) - 64$$
$$= -2[illegible]6 \text{ ft./sec.}$$