SAT

If $x^{-2} = 16$, what is the value of x^{2}?

R-2 Radicals and Rational Exponents

Why do negative exponents mean "move it"?

Consider $\frac{a^2}{a^5}$. Simplify this in 2 different ways:

1. By writing out all the variables in the top and bottom and cancelling

$$\frac{a^2}{a^5} = \frac{a \cdot a}{a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a^3}$$

2. By using the rules of exponents

$$\frac{a^2}{a^5} = a^{2-5} = a^{-3} = \frac{1}{a^3}$$

What do you notice?

Why do fraction exponents mean "roots"?

$\sqrt{3} \cdot \sqrt{3} =$ 3

$\sqrt{a} \cdot \sqrt{a} =$ a

$\sqrt{3} = 3^{1/2}$

$3^{1/2}(3^{1/2}) = 3^{1}$

What do you notice?

Radicals and Rational Exponents: Practice Exercises

Summary rules:

$b^x \cdot b^y =$ b^{x+y}

$\frac{b^x}{b^y} =$ b^{x-y}

$(b^x)^y =$ b^{xy}

$b^0 =$ 1

If $b^x = b^y$ then $x = y$

If $a^x = b^x$ then $a = b$

$(ab)^x =$ $a^x b^x$

$\left(\frac{a}{b}\right)^x =$ $\frac{a^x}{b^x}$

$b^{\frac{x}{y}} =$ $\sqrt[y]{b^x}$

$b^{-x} =$ $\frac{1}{b^x}$

$\sqrt{a} = a^{1/2}$

Simplify each expression without negative exponents. Do not use a calculator.

1. $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

3. $\left(\frac{3}{xy}\right)^{-2} = \left(\frac{xy}{3}\right)^2 = \frac{x^2y^2}{9}$

4. $\frac{(-2)^8}{(-2)^3} = (-2)^5$

6. $\frac{12^3}{6^3} = 2^3 = 8$

7. $\left(2^{-1} + 4^{-1}\right)^{-1} = \left(\frac{1}{2} + \frac{1}{4}\right)^{-1} = \left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

9. $\frac{u^2v^{-2}}{u^{-1}v^3} = u^3v^{-5} = \frac{u^3}{v^5}$

10. $\left(\frac{4a^3b}{a^2b^3}\right)\left(\frac{3b^2}{2a^2b^4}\right)$

$\frac{4a}{b^2} \cdot \frac{3}{2a^2b^2} = \frac{12a}{2a^2b^4}$

$\frac{6}{ab^4}$

12. $\frac{5b^3 + 10b^6}{5b^{-2}}$

$\frac{5b^3}{5b^{-2}} + \frac{10b^6}{5b^{-2}}$

Solve each equation for x.

13. $3^{2x} = 3^{12}$

$2x = 12$

$x = 6$

14. $9^{x} = 3^{5}$

$(3^2)^x = 3^5$

$3^{2x} = 3^5$

$2x = 5$

$x = \frac{5}{2}$

9^(5/2)

15. $x^{3/2} = 4$

$\sqrt{x}^3 = 4^2$

$\sqrt[3]{x^3} = \sqrt[3]{16}$

$x = \sqrt[3]{16}$

Radicals and Rational Exponents: Practice Exercises

Fractional exponents are easy to understand if you remember that $x^{a/b} = \sqrt[b]{x^a}$:

- the number in the numerator is the power inside the radical
- the number in the denominator is the root outside the radical
- all other exponent rules still apply...
- and negative exponents still mean "move it"...

Examples:

$x^{1/2} = \sqrt{x}$ $\qquad$ $x^{4/3} = \sqrt[3]{x^4}$ $\qquad$ $x^{-2/5} = \dfrac{1}{\sqrt[5]{x^2}}$ $\qquad$ $x^{2/3} \cdot x^{3/4} = x^{(2/3)+(3/4)} = x^{17/12}$

Rewrite each expression using radicals and no negative exponents.

1. $x^{2/3}$ $= \sqrt[3]{x^2}$

3. $5^{1/2} \cdot x^{-1/2}$ $\frac{\sqrt{5}}{\sqrt{x}}$

4. $3x^{5/3}$ $3\sqrt[3]{x^5}$

6. $a^{3/4}b^{1/4}$ $= \sqrt[4]{a^3}\sqrt[4]{b}$ $= \sqrt[4]{a^3 b}$

Rewrite each expression in its simplest exponential form.

10. $\sqrt{x^5} = x^{\frac{5}{2}}$

12. $\left(\sqrt[6]{2a}\right)^5 = \left((2a)^{\frac{1}{6}}\right)^5$

$= (2a)^{5/6}$

14. $\sqrt[3]{27x^{-6}y^2}$

$= 3(x^{-6})^{\frac{1}{3}}(y^2)^{\frac{1}{3}}$

$= 3x^{-2}y^{2/3}$

$= \frac{3y^{2/3}}{x^2}$

18. $\sqrt[3]{\frac{x^2}{y}} = \frac{(x^2)^{1/3}}{y^{1/3}}$

$= \frac{x^{2/3}}{y^{1/3}}$

Homework Assignment:
WS R-2