1. (a) $P(X=2)=\frac{4}{14}\left(=\frac{2}{7}\right)$ A1 N1 1

(b) $P(X=1)=\frac{1}{14}$ (A1)

$P(X=k)=\frac{k^2}{14}$ (A1)

setting the sum of probabilities = 1 M1

e.g. $\frac{1}{14}+\frac{4}{14}+\frac{k^2}{14}=1$, $5+k^2=14$

$k^2=9\left(\text{accept } \frac{k^2}{14}=\frac{9}{14}\right)$ A1

$k=3$ AG N0 4

(c) correct substitution into $E(X)=\sum xP(X=x)$ A1

e.g. $1\left(\frac{1}{14}\right)+2\left(\frac{4}{14}\right)+3\left(\frac{9}{14}\right)$

$E(X)=\frac{36}{14}\left(=\frac{18}{7}\right)$ A1 N1 2

[7]

2. (a) (i) $s=1$ A1 N1

(ii) evidence of appropriate approach (M1)
e.g. 21–16, $12+8-q=15$
$q=5$ A1 N2

(iii) $p=7$, $r=3$ A1A1 N2 5

(b) (i) P(art|music) = $\frac{5}{8}$ A2 N2

(ii) **METHOD 1**

$P(\text{art}) = \frac{12}{16}\left(=\frac{3}{4}\right)$ A1

evidence of correct reasoning R1

e.g. $\frac{3}{4} \neq \frac{5}{8}$

the events are not independent AG N0

METHOD 2

P(art) × P(music) = $\frac{96}{256}\left(=\frac{3}{8}\right)$ A1

evidence of correct reasoning R1

e.g. $\frac{12}{16} \times \frac{8}{16} \neq \frac{5}{16}$

the events are not independent AG N0 4

(c) P(first takes only music) = $\frac{3}{16}$ = (seen anywhere) A1

P(second takes only art)= $\frac{7}{15}$ (seen anywhere) A1

evidence of valid approach (M1)

e.g. $\frac{3}{16} \times \frac{7}{15}$

P(music and art)= $\frac{21}{240}\left(=\frac{7}{80}\right)$ A1 N2 4

[13]

3. (a) (i) $n = 0.1$ A1 N1

(ii) $m = 0.2$, $p = 0.3$, $q = 0.4$ A1A1A1 N3 4

(b) appropriate approach

e.g. $P(B') = 1 - P(B)$, $m + q$, $1-(n + p)$ (M1)

$P(B') = 0.6$ A1 N2 2

[6]

4. (a) (i) $p = 0.2$ A1 N1

(ii) $q = 0.4$ A1 N1

(iii) $r = 0.1$ A1 N1

(b) $\mathrm{P}(A \mid B') = \frac{2}{3}$ A2 N2

Note: *Award A1 for an unfinished answer such as* $\frac{0.2}{0.3}$.

(c) valid reason R1

e.g. $\frac{2}{3} \neq 0.5,\ 0.35 \neq 0.3$

thus, *A* and *B* are not independent AG N0

[6]

5. (a) (i) $\frac{7}{24}$ A1 N1

(ii) evidence of **multiplying** along the branches (M1)

e.g. $\frac{2}{3} \times \frac{5}{8}, \frac{1}{3} \times \frac{7}{8}$

adding probabilities of two mutually exclusive paths (M1)

e.g. $\left(\frac{1}{3} \times \frac{7}{8}\right) + \left(\frac{2}{3} \times \frac{3}{8}\right), \left(\frac{1}{3} \times \frac{1}{8}\right) + \left(\frac{2}{3} \times \frac{5}{8}\right)$

$\mathrm{P}(F) = \frac{13}{24}$ A1 N2

(b) (i) $\frac{1}{3} \times \frac{1}{8}$ (A1)

$\frac{1}{24}$ A1

(ii) recognizing this is $P(E|F)$ (M1)

e.g. $\frac{7}{24} \div \frac{13}{24}$

$\frac{168}{312}\left(=\frac{7}{13}\right)$ A2 N3

(c)

***X* (cost in euros)**	0	3	6
P (*X*)	$\frac{1}{9}$	$\mathbf{\frac{4}{9}}$	$\mathbf{\frac{4}{9}}$

A2A1 N3

(d) correct substitution into E(*X*) formula (M1)

e.g. $0 \times \frac{1}{9} + 3 \times \frac{4}{9} + 6 \times \frac{4}{9}, \frac{12}{9} + \frac{24}{9}$

E(*X*) = 4 (euros) A1 N2

[14]

6. (a) $p = \frac{4}{5}$ A1 N1

(b) multiplying along the branches (M1)

e.g. $\frac{1}{5} \times \frac{1}{4}, \frac{12}{40}$

adding products of probabilities of two mutually exclusive paths (M1)

e.g. $\frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8}, \frac{1}{20} + \frac{12}{40}$

$P(B) = \frac{14}{40}\left(=\frac{7}{20}\right)$ A1 N2

(c) appropriate approach which must include A' (may be seen on diagram) (M1)

e.g. $\frac{P(A' \cap B)}{P(B)} \left(\text{do not accept} \frac{P(A \cap B)}{P(B)}\right)$

$P(A' | B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}}$ (A1)

$P(A' | B) = \frac{12}{14} \left(= \frac{6}{7}\right)$ A1 N2

[7]

7. (a) $P(A) = \frac{1}{11}$ A1 N1

(b) $P(B | A) = \frac{2}{10}$ A2 N2

(c) recognising that $P(A \cap B) = P(A) \times P(B | A)$ (M1)

correct values (A1)

e.g. $P(A \cap B) = \frac{1}{11} \times \frac{2}{10}$

$P(A \cap B) = \frac{2}{110}$

A1 N3

[6]

8. (a)

3, 9	**4, 9**	**5, 9**
3, 10	**4, 10**	**5, 10**
3, 10	**4, 10**	**5, 10**

A2 N2

(b) 12, 13, 14, 15 (accept 12, 13, 13, 13, 14, 14, 14, 15, 15) A2 N2

(c) $P(12) = \frac{1}{9}$, $P(13) = \frac{3}{9}$, $P(14) = \frac{3}{9}$, $P(15) = \frac{2}{9}$ A2 N2

(d) correct substitution into formula for E(X) A1

e.g. $\mathrm{E}(S) = 12 \times \frac{1}{9} + 13 \times \frac{3}{9} + 14 \times \frac{3}{9} + 15 \times \frac{2}{9}$

$\mathrm{E}(S) = \frac{123}{9}$ A2 N2

(e) **METHOD 1**

correct expression for expected gain E(A) for 1 game (A1)

e.g. $\frac{4}{9} \times 50 - \frac{5}{9} \times 30$

$\mathrm{E}(A) = \frac{50}{9}$

amount at end = expected gain for 1 game × 36 (M1)

= 200 (dollars) A1 N2

METHOD 2

attempt to find expected number of wins and losses (M1)

e.g. $\frac{4}{5} \times 36, \frac{5}{9} \times 36$

attempt to find expected gain E(G) (M1)

e.g. 16 × 50 – 30 × 20

E(G) = 200 (dollars) A1 N2

[12]

9. (a) appropriate approach (M1)

e.g. tree diagram or a table

P(win) = P($H \cap W$) + P($A \cap W$)) (M1)

= (0.65)(0.83) + (0.35)(0.26) A1

= 0.6305 (or 0.631) A1 N2

(b) evidence of using complement (M1)
e.g. $1 - p$, 0.3695

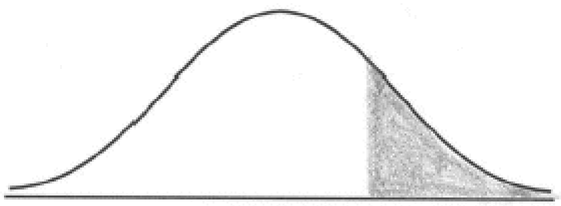
choosing a formula for conditional probability (M1)
e.g. $P(H \mid W') = \frac{P(W' \cap H)}{P(W')}$

correct substitution
e.g. $\frac{(0.65)(0.17)}{0.3695} \left(= \frac{0.1105}{0.3695} \right)$ A1

P(home) = 0.299 A1 N3

[8]

10. (a)



A1A1 N2

Note: *Award A1 for vertical line to right of mean, A1 for shading to right of **their** vertical line.*

(b) evidence of recognizing symmetry (M1)
e.g. 105 is one standard deviation above the mean so d is one standard deviation below the mean, shading the corresponding part,
$105 - 100 = 100 - d$

$d = 95$ A1 N2

(c) evidence of using complement (M1)
e.g. $1 - 0.32$, $1 - p$

$P(d < X < 105) = 0.68$ A1 N2

[6]

11. (a) (i) evidence of substituting into $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ (M1)
e.g. $75 + 55 - 100$, Venn diagram

30 A1 N2

(ii) 45 A1 N1

(b) (i) **METHOD 1**

evidence of using complement, Venn diagram (M1)

e.g. $1 - p$, $100 - 30$

$\frac{70}{100}\left(=\frac{7}{10}\right)$ A1 N2

METHOD 2

attempt to find P(only one sport), Venn diagram (M1)

e.g. $\frac{25}{100}+\frac{45}{100}$

$\frac{70}{100}\left(=\frac{7}{10}\right)$ A1 N2

(ii) $\frac{45}{70}\left(=\frac{9}{14}\right)$ A2 N2

(c) valid reason in words or symbols (R1)

e. g. $P(A \cap B) = 0$ if mutually exclusive, $P(A \cap B)$ if not mutually exclusive

correct statement in words or symbols A1 N2

e.g. $P(A \cap B) = 0.3$, $P(A \cup B) \neq P(A) + P(B)$, $P(A) + P(B) > 1$, some students play both sports, sets intersect

(d) valid reason for independence (R1)

e.g. $P(A \cap B) = P(A) \times P(B)$, $P(B | A) = P(B)$

correct substitution A1A1 N3

e.g. $\frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}, \frac{30}{55} \neq \frac{75}{100}$

[12]

12. (a) (i) $P(B) = \frac{3}{4}$ A1 N1

(ii) $P(R) = \frac{1}{4}$ A1 N1

(b) $p = \frac{3}{4}$ A1 N1

$s = \frac{1}{4}$, $t = \frac{3}{4}$ A1 N1

(c) (i) $\mathrm{P}(X = 3)$

$= \mathrm{P}$ (getting 1 and 2) $= \frac{1}{4} \times \frac{3}{4}$ A1

$= \frac{3}{16}$ AG N0

(ii) $\mathrm{P}(X = 2) = \frac{1}{4} \times \frac{1}{4} + \frac{3}{4} \left(\text{or} 1 - \frac{3}{16}\right)$ (A1)

$= \frac{13}{16}$ A1 N2

(d) (i)

X	2	3
$\mathrm{P}(X = x)$	$\frac{13}{16}$	$\frac{3}{16}$

A2 N2

(ii) evidence of using $\mathrm{E}(X) = \sum x\mathrm{P}(X = x)$ (M1)

$\mathrm{E}(X) = 2\left(\frac{13}{16}\right) + 3\left(\frac{3}{16}\right)$ (A1)

$= \frac{35}{16} \left(= 2\frac{3}{16}\right)$ A1 N2

(e) win \$10 $\Rightarrow$ scores 3 one time, 2 other time (M1)

$P(3) \times P(2) = \frac{13}{16} \times \frac{3}{16}$ (seen anywhere) A1

evidence of recognizing there are different ways of winning \$10 (M1)

e.g. $P(3) \times P(2) + P(2) \times P(3)$, $2\left(\frac{13}{16} \times \frac{3}{16}\right)$,

$\frac{36}{256} + \frac{3}{256} + \frac{36}{256} + \frac{3}{256}$

$P(\text{win } \$10) = \frac{78}{256} \left(= \frac{39}{128}\right)$ A1 N3

[16]

13. (a) (i) correct calculation (A1)

e.g. $\frac{9}{20} + \frac{5}{20} - \frac{2}{20}$, $\frac{4+2+3+3}{20}$

P(male or tennis) = $\frac{12}{20} \left(= \frac{3}{5}\right)$ A1 N2

(ii) correct calculation (A1)

e.g. $\frac{6}{20} \div \frac{11}{20}$, $\frac{3+3}{11}$

P(not football | female) = $\frac{6}{11}$ A1 N2

(b) P(first not football) = $\frac{11}{20}$, P(second not football) = $\frac{10}{19}$ A1

P(neither football) = $\frac{11}{20} \times \frac{10}{19}$ A1

P(neither football) = $\frac{110}{380} \left(= \frac{11}{38}\right)$ A1 N1

[7]

14. (a) evidence of using $\sum p_i = 1$ (M1)
correct substitution A1
e.g. $10k^2 + 3k + 0.6 = 1$, $10k^2 + 3k - 0.4 = 0$
$k = 0.1$ A2 N2

(b) evidence of using $E(X) = \sum p_i x_i$ (M1)

correct substitution (A1)

e.g. $-1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3$

$\mathrm{E}(X) = 1.5$ A1 N2

[7]

15. (a) evidence of binomial distribution (seen anywhere) (M1)

e.g. $X \sim \mathrm{B}\left(3, \frac{1}{4}\right)$

mean = $\frac{3}{4}$ $(= 0.75)$ A1 N2

(b) $\mathrm{P}(X = 2) = \binom{3}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$ (A1)

$\mathrm{P}(X = 2) = 0.141$ $\left(= \frac{9}{64}\right)$ A1 N2

(c) evidence of appropriate approach M1

e.g. complement, $1 - \mathrm{P}(X = 0)$, adding probabilities

$\mathrm{P}(X = 0) = (0.75)^3$ $\left(= 0.422, \frac{27}{64}\right)$ (A1)

$\mathrm{P}(X \geq 1) = 0.578$ $\left(= \frac{37}{64}\right)$ A1 N2

[7]

16. (a) $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ $(= 0.6x)$ A1 N1

(b) (i) evidence of using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ (M1)

correct substitution A1

e.g. $0.80 = 0.6 + x - 0.6x$, $0.2 = 0.4x$

$x = 0.5$ A1 N2

(ii) $\mathrm{P}(A \cap B) = 0.3$ A1 N1

(c) valid reason, with reference to $P(A \cap B)$ R1 N1

e.g. $P(A \cap B) \neq 0$

[6]

17. (a) (i) number of ways of getting $X = 6$ is 5 (A1)

$P(X = 6) = \frac{5}{36}$ A1 N2

(ii) number of ways of getting $X > 6$ is 21 (A1)

$P(X > 6) = \frac{21}{36}\left(=\frac{7}{12}\right)$ A1 N2

(iii) $P(X = 7 | X > 5) = \frac{6}{26}\left(=\frac{3}{13}\right)$ A2 N2

(b) evidence of substituting into E(X) formula (M1)

finding $P(X < 6) = \frac{10}{36}$ (seen anywhere) (A2)

evidence of using $E(W) = 0$ (M1)

correct substitution A2

e.g. $3\left(\frac{5}{36}\right)+1\left(\frac{21}{36}\right)-k\left(\frac{10}{36}\right)=0$, $15 + 21 - 10k = 0$

$k = \frac{36}{10}$ $(= 3.6)$ A1 N4

[13]

18. **METHOD 1**

(a) $\sigma = 10$ (A1)

$1.12 \times 10 = 11.2$ A1

$11.2 + 100$ (M1)

$x = 111.2$ A1 N2

(b) $100 - 11.2$ (M1)

$= 88.8$ A1 N2

[6]

METHOD 2

(a) $\sigma = 10$ (A1)

Evidence of using standardisation formula (M1)

$\frac{x-100}{10} = 1.12$ A1

$x = 111.2$ A1 N2

(b) $\frac{100-x}{10} = 1.12$ A1

$x = 88.8$ A1 N2

[6]

19. (a) For summing to 1 (M1)

e.g. $\frac{1}{5}+\frac{2}{5}+\frac{1}{10}+x=1$

$x = \frac{3}{10}$ A1 N2

(b) For evidence of using $E(X) = \sum x\, f(x)$ (M1)

Correct calculation A1

e.g. $\frac{1}{5}\times 1 + 2\times\frac{2}{5} + 3\times\frac{1}{10} + 4\times\frac{3}{10}$

$E(X) = \frac{25}{10} (= 2.5)$ A1 N2

(c) $\frac{1}{10}\times\frac{1}{10}$ (M1)

$\frac{1}{100}$ A1 N2

[7]

20. (a) Evidence of using the complement *e.g.* 1 – 0.06 (M1)

$p = 0.94$ A1 N2

(b) For evidence of using symmetry (M1)

Distance from the mean is 7 (A1)

e.g. diagram, D = mean – 7

$D = 10$ A1 N2

(c) P(17 < H < 24) = 0.5 – 0.06 (M1)
= 0.44 A1

E(trees) = 200 × 0.44 (M1)
= 88 A1 N2

[9]

21. (a) (i) Attempt to find P(3H) = $\left(\frac{1}{3}\right)^3$ (M1)

$= \frac{1}{27}$ A1 N2

(ii) Attempt to find P(2H, 1T) (M1)

$= 3\left(\frac{1}{3}\right)^2 \frac{2}{3}$ A1

$= \frac{2}{9}$ A1 N2

(b) (i) Evidence of using np $\left(\frac{1}{3} \times 12\right)$ (M1)

expected number of heads = 4 A1 N2

(ii) 4 heads, so 8 tails (A1)
E(winnings) = 4 × 10 – 8 × 6 (= 40 – 48) (M1)
= –$ 8 A1 N1

[10]

22. (a) $\frac{3}{4}$ A1 N1

(b) P($A \cup B$) = P(A) + P(B) – P($A \cap B$) (M1)

P($A \cap B$) = P(A) + P(B) – P($A \cup B$)

$= \frac{2}{5} + \frac{3}{4} - \frac{7}{8}$ A1

$= \frac{11}{40}$ (0.275) A1 N2

(c) $P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\left(= \frac{\frac{11}{40}}{\frac{3}{4}} \right)$ A1

$= \frac{11}{30}$ (0.367) A1 N1

[6]

23. (a) $\frac{46}{97}$ $(=0.474)$ A1A1 N2

(b) $\frac{13}{51}$ $(=0.255)$ A1A1 N2

(c) $\frac{59}{97}$ $(=0.608)$ A2 N2

[6]

24. (a) $\frac{19}{120}$ $(=0.158)$ A1 N1

(b) 35 – (8 + 5 + 7)(= 15) (M1)

Probability = $\frac{15}{120}\left(=\frac{3}{24}=\frac{1}{8}=0.125\right)$ A1 N2

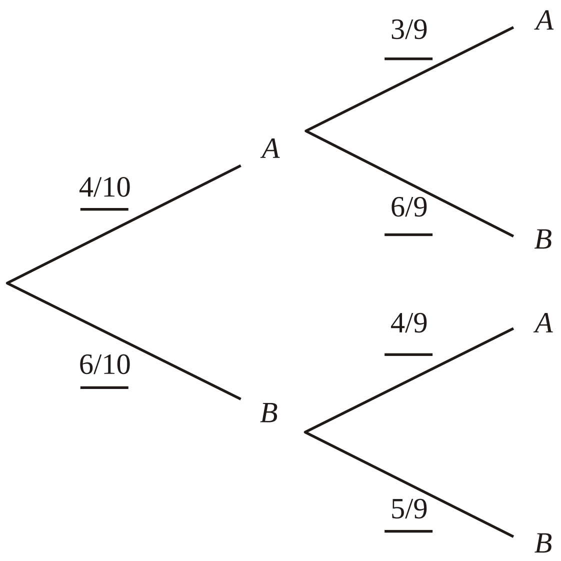
(c) Number studying = 76 (A1)

Number not studying = 120 – number studying = 44 (M1)

Probability = $\frac{44}{120}\left(=\frac{11}{30}=0.367\right)$ A1 N3

[6]

25. (a)



A1A1A1 N3

(b) $\left(\frac{4}{10}\times\frac{6}{9}\right)+\left(\frac{6}{10}\times\frac{4}{9}\right)$ M1M1

$=\frac{48}{90}\ \left(\frac{8}{15}, 0.533\right)$ A1 N1

[6]

-

26. (a) For summing to 1 (M1) 6

eg $0.1 + a + 0.3 + b = 1$

$a + b = 0.6$ A1

(b) evidence of correctly using (M1)

eg $0 \times 0.1 + 1 \times a + 2 \times 0.3 + 3 \times b$, $0.1 + a + 0.6 + 3b = 1.5$

Correct equation $0 + a + 0.6 + 3b = 1.5$ $(a + 3b = 0.9)$

Solving simultaneously gives

$a = 0.45$ $b = 0.15$ A1A1

[6]

$50 \le t < 60$

(2)
(Total 6 marks)

27. (a) Independent $\Rightarrow$ $P(A \cap B) = P(A) \times P(B)$ $(= 0.3 \times 0.8)$ (M1)

$= 0.24$ A1 N2

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $(= 0.3 + 0.8 - 0.24)$ M1

$= 0.86$ A1 N1

(c) No, **with** valid reason A2 N2

eg $P(A \cap B) \neq 0$ or $P(A \cup B) \neq P(A) + P(B)$ or correct numerical equivalent

[6]

28. (a) For using $\sum p = 1$ $(0.4 + p + 0.2 + 0.07 + 0.02 = 1)$ (M1)

$p = 0.31$ A1 N2

(b) For using $E(X) = \sum x P(X = x)$ (M1)

$E(X) = 1(0.4) + 2(0.31) + 3(0.2) + 4(0.07) + 5(0.02)$ A1

$= 2$ A2 N2

[6]

29. (a) $P(P \mid C) = \frac{20}{20+40}$ A1

$= \frac{1}{3}$ A1 N1

(b) $P(P \mid C') = \frac{30}{30+60}$ A1

$= \frac{1}{3}$ A1 N1

(c) Investigating conditions, or some relevant calculations (M1)

P is independent of C, **with** valid reason A1 N2

eg $P(P \mid C) = P(P \mid C')$, $P(P \mid C) = P(P)$,

$\frac{20}{150} = \frac{50}{150} \times \frac{60}{150}$ (*ie* $P(P \cap C) = P(P) \times P(C)$)

[6]

30. (a) Adding probabilities (M1)
Evidence of knowing that sum = 1 for probability distribution R1
eg Sum greater than 1, sum = 1.3, sum does not equal 1 N2

(b) Equating sum to 1 $(3k + 0.7 = 1)$ M1

$k = 0.1$ A1 N1

(c) (i) $P(X=0)=\frac{0+1}{20}$ (M1)

$=\frac{1}{20}$ A1 N2

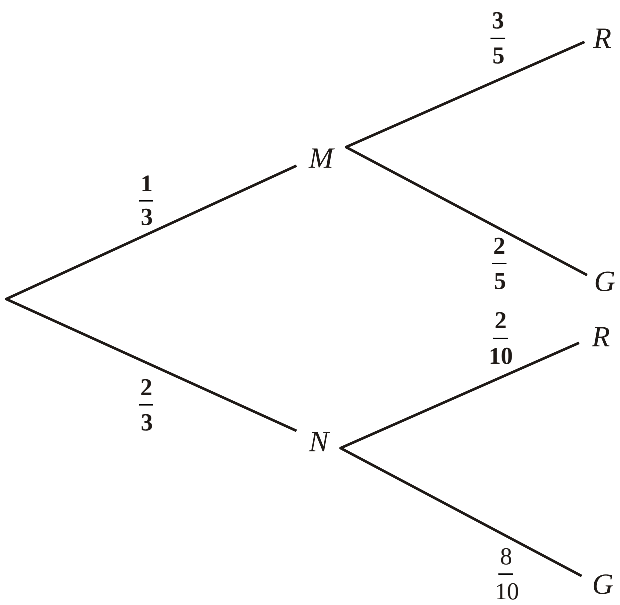
(ii) Evidence of using $P(X > 0) = 1 - P(X = 0)$

$\left(\text{or } \frac{4}{20}+\frac{5}{20}+\frac{10}{20}\right)$ (M1)

$=\frac{19}{20}$ A1 N2

[8]

31. (a)



A1A1A1 N3

(b) (i) P(M and G) = $\frac{1}{3}\times\frac{2}{5}\left(=\frac{2}{15}=0.133\right)$ A1 N1

(ii) P(G) = $\frac{1}{3}\times\frac{2}{5}+\frac{2}{3}\times\frac{8}{10}$ (A1)(A1)

$=\frac{10}{15}\left(=\frac{2}{3}=0.667\right)$ A1 N3

(iii) P($M|G$) = $\frac{\mathrm{P}(M\cap G)}{\mathrm{P}(G)}=\frac{\frac{2}{15}}{\frac{2}{3}}$ (A1)(A1)

$=\frac{1}{5}$ or 0.2 A1 N3

(c) P(R) = $1-\frac{2}{3}=\frac{1}{3}$ (A1)

Evidence of using a correct formula M1

E(win) = $2\times\frac{1}{3}+5\times\frac{2}{3}\left(\text{or } 2\times\frac{1}{3}\times\frac{3}{5}+2\times\frac{2}{3}\times\frac{2}{10}+5\times\frac{1}{3}\times\frac{2}{5}+5\times\frac{2}{3}\times\frac{8}{10}\right)$ A1

= \$4 $\left(\text{accept } \frac{12}{3}, \frac{60}{15}\right)$ A1 N2

[14]

32. (a) For attempting to use the formula (P($E\cap F$) = P(E)P(F)) (M1)

Correct substitution or rearranging the formula A1

eg $\frac{1}{3}=\frac{2}{3}$ P(F), P(F) = $\frac{\mathrm{P}(E\cap F)}{\mathrm{P}(E)}$, P($F$) = $\frac{\frac{1}{3}}{\frac{2}{3}}$

P(F) = $\frac{1}{2}$ A1 N2

(b) For attempting to use the formula ($P(E \cup F) = P(E) + P(F) - (P(E \cap F))$ (M1)

$P(E \cup F) = \frac{2}{3} + \frac{1}{2} - \frac{1}{3}$ A1

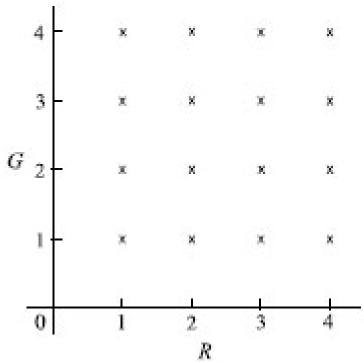
$= \frac{5}{6} (= 0.833)$ A1 N2

[6]

33. (a) (i) Attempt to set up sample space, (M1)

Any **correct** representation with 16 pairs A2 N3

eg	1,1	2,1	3,1	4,1
	1,2	2,2	3,2	4,2
	1,3	2,3	3,3	4,3
	1,4	2,4	3,4	4,4



(ii) Probability of two 4s is $\frac{1}{16}$ (= 0.0625) A1 N1

(b)

x	0	1	2
$P(X = x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

A1A1A1 N3

(c) Evidence of selecting appropriate formula for E(X) (M1)

eg $\mathrm{E}(X) = \sum_{0}^{2} x\,\mathrm{P}(X = x)$, $\mathrm{E}(X) = np$

Correct substitution

eg $\mathrm{E}(X) = 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16}$, $\mathrm{E}(X) = 2 \times \frac{1}{4}$

$\mathrm{E}(X) = \frac{8}{16}\left(= \frac{1}{2}\right)$ A1 N2

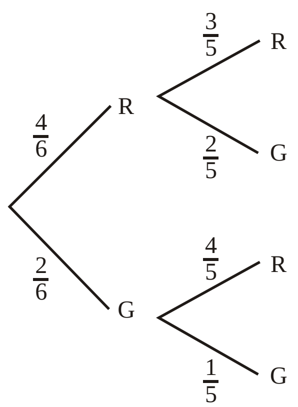
[10]

34. (a) Using $\mathrm{E}(X) = \sum_{0}^{2} x\,\mathrm{P}(X = x)$ (M1)

Substituting correctly $E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}$ A1

$= \frac{8}{10}$ (0.8) A1 3

(b) (i)



A1A1A1 3

***Note**: Award (A1) for each complementary pair of probabilities,*

ie $\frac{4}{6}$ *and* $\frac{2}{6}$, $\frac{3}{5}$ *and* $\frac{2}{5}$, $\frac{4}{5}$ *and* $\frac{1}{5}$.

(ii) $\text{P}(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$ A1

$\text{P}(Y = 1) = \text{P}(RG) + \text{P}(GR) \quad \left(= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5}\right)$ M1

$= \frac{16}{30}$ A1

$\text{P}(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$ (A1)

For forming a distribution M1 5

y	0	1	2
$\text{P}(Y = y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

(c) $\text{P}(\text{Bag A}) = \frac{2}{6} \left(= \frac{1}{3}\right)$ (A1)

$\text{P}(\text{BagA B}) = \frac{4}{6} \left(= \frac{2}{3}\right)$ (A1)

For summing $\text{P}(A \cap RR)$ and $\text{P}(B \cap RR)$ (M1)

Substituting correctly $\text{P}(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30}$ A1

$= \frac{27}{90} \left(\frac{3}{10}, 0.3\right)$ A1 5

(d) For recognising that $\text{P}(1 \text{ or } 6|RR) = \text{P}(A|RR) = \frac{\text{P}(A \cap RR)}{\text{P}(RR)}$ (M1)

$= \frac{1}{30} \div \frac{27}{90}$ A1

$= \frac{3}{27} \quad \left(\frac{1}{9}, 0.111\right)$ A1 3

[19]

35. Total number of possible outcomes = 36 (may be seen anywhere) (A1)

(a) $\text{P}(E) = \text{P}(1,1) + \text{P}(2,2) + \text{P}(3,3) + \text{P}(4,4) + \text{P}(5,5) + \text{P}(6,6)$

$= \frac{6}{36}$ (A1) (C2)

(b) $\text{P}(F) = \text{P}(6,4) + \text{P}(5,5) + \text{P}(4,6)$

$= \frac{3}{36}$ (A1) (C1)

(c) $\mathrm{P}(E \cup F) = \mathrm{P}(E) + \mathrm{P}(F) - \mathrm{P}(E \cap F)$

$\mathrm{P}(E \cap F) = \dfrac{1}{36}$ (A1)

$\mathrm{P}(E \cup F) = \dfrac{6}{36} + \dfrac{3}{36} - \dfrac{1}{36} \quad \left(= \dfrac{8}{36} = \dfrac{2}{9}, 0.222 \right)$ (M1)(A1) (C3)

[6]

36. (a) (i) $\mathrm{P}(A) = \dfrac{80}{210} = \left(\dfrac{8}{21} = 0.381 \right)$ (A1) (N1)

(ii) $\mathrm{P}(\text{year 2 art}) = \dfrac{35}{210} = \left(\dfrac{1}{6} = 0.167 \right)$ (A1) (N1)

(iii) No (the events are not independent, or, they are dependent) (A1) (N1)

EITHER

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ (to be independent) (M1)

$\mathrm{P}(B) = \dfrac{100}{210} \left(= \dfrac{10}{21} = 0.476 \right)$ (A1)

$\dfrac{1}{6} \neq \dfrac{8}{21} \times \dfrac{10}{21}$ (A1)

OR

$\mathrm{P}(A) = \mathrm{P}(A \mid B)$ (to be independent) (M1)

$\mathrm{P}(A \mid B) = \dfrac{35}{100}$ (A1)

$\dfrac{8}{21} \neq \dfrac{35}{100}$ (A1)

OR

$P(B) = P(B|A)$ (to be independent) (M1)

$P(B) = \frac{100}{210}\left(= \frac{10}{21} = 0.476\right), P(B|A) = \frac{35}{80}$ (A1)

$\frac{35}{80} \neq \frac{100}{210}$ (A1) 6

***Note:** Award the first (M1) only for a **mathematical** interpretation of independence.*

(b) $n(\text{history}) = 85$ (A1)

$P(\text{year 1}|\text{history}) = \frac{50}{85} = \left(\frac{10}{17} = 0.588\right)$ (A1)(N2) 2

(c) $\left(\frac{110}{210} \times \frac{100}{209}\right) + \left(\frac{100}{210} \times \frac{110}{209}\right) \left(= 2 \times \frac{110}{210} \times \frac{100}{209}\right)$ (M1)(A1)(A1)

$= \frac{200}{399}(= 0.501)$

(A1) (N2) 4

[12]

37. Correct probabilities $\left(\frac{13}{24}\right), \left(\frac{12}{23}\right), \left(\frac{11}{22}\right), \left(\frac{10}{21}\right)$ (A1)(A1)(A1)(A1)

Multiplying $\left(\frac{13}{24} \times \frac{12}{23} \times \frac{11}{22} \times \frac{10}{21}\right)$ (M1)

P(4 girls) = $\frac{17160}{255024}\left(= \frac{65}{966} = 0.0673\right)$ (A1) (C6)

[6]

38. For using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

Let $P(A) = x$ then $P(B) = 3x$

$P(A \cap B) = P(A) \times 3P(A)$ $(= 3x^2)$ (A1)

$0.68 = x + 3x - 3x^2$ (A1)

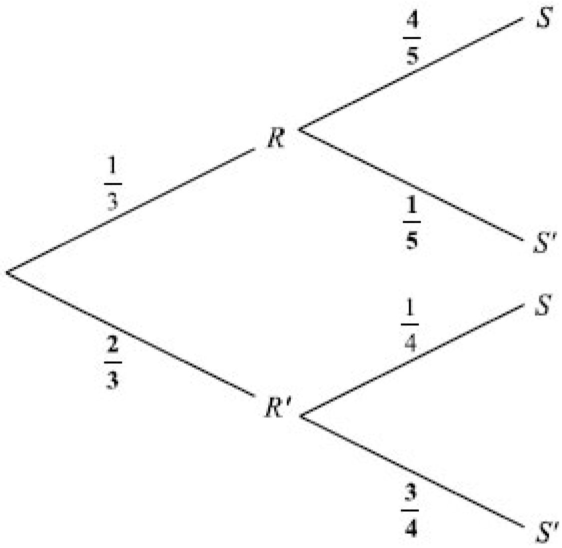
$3x^2 - 4x + 0.68 = 0$

$x = 0.2$ $(x = 1.133$, not possible) (A2)

$P(B) = 3x = 0.6$ (A1) (C6)

[6]

39. (a)



(A1)(A1)(A1)

(b) (i) $P(R \cap S) = \frac{1}{3} \times \frac{4}{5} \left(= \frac{4}{15} = 0.267 \right)$ (A1) (N1)

(ii) $P(S) = \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4}$ (A1)(A1)

$= \frac{13}{30}$ $(= 0.433)$ (A1) (N3)

(iii) $P(R \mid S) = \dfrac{\frac{4}{15}}{\frac{13}{30}}$ (A1)(A1)

$= \frac{8}{13}$ $(= 0.615)$ (A1) (N3)

[10]

40. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

$\mathrm{P}(A \cap B) = \frac{1}{2} + \frac{3}{4} - \frac{7}{8}$

$= \frac{3}{8}$ (A1) (C2)

(b) $\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} \left(= \frac{\frac{3}{8}}{\frac{3}{4}} \right)$ (M1)

$= \frac{1}{2}$ (A1) (C2)

(c) Yes, the events are independent (A1) (C1)

EITHER

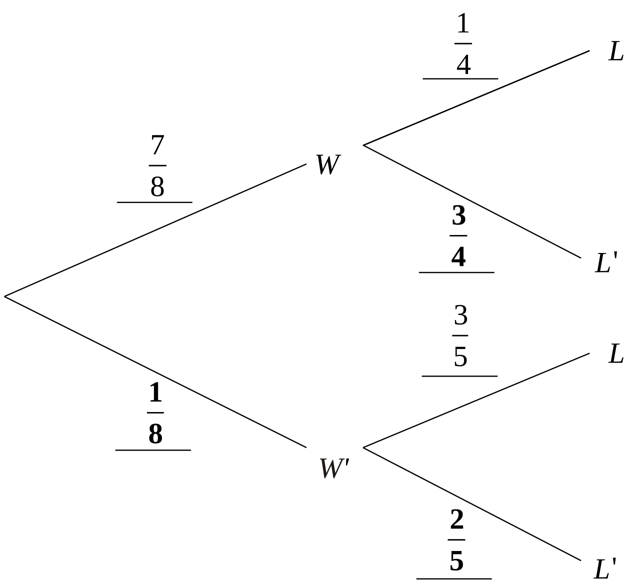
$\mathrm{P}(A|B) = \mathrm{P}(A)$ (R1) (C1)

OR

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ (R1) (C1)

[6]

41. (a)



(A1)(A1)(A1)(A1) 4

Note: *Award (A1) for the given probabilities* $\left(\frac{7}{8}, \frac{1}{4}, \frac{3}{5}\right)$ *in the correct positions, and (A1) for each* ***bold*** *value.*

(b) Probability that Dumisani will be late is $\frac{7}{8}\times\frac{1}{4}+\frac{1}{8}\times\frac{3}{5}$ (A1)(A1)

$= \frac{47}{160}$ (0.294)

(A1) (N2) 3

(c) $P(W|L) = \frac{P(W \cap L)}{P(L)}$

$P(W \cap L) = \frac{7}{8}\times\frac{1}{4}$ (A1)

$P(L) = \frac{47}{160}$ (A1)

$P(W|L) = \frac{\frac{7}{32}}{\frac{47}{160}}$ (M1)

$= \frac{35}{47}$ (= 0.745) (A1) (N3) 4

[11]

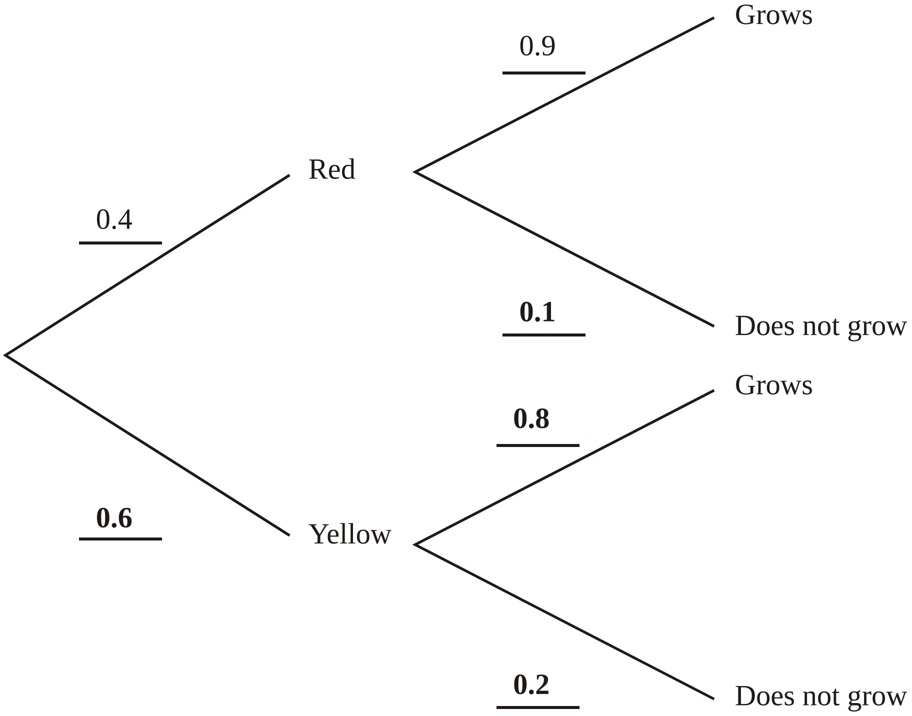
42. (a) $\frac{120}{360}\left(=\frac{1}{3}=0.333\right)$ (A1)(A1) (C2)

(b) $\frac{90+120}{360}\left(=\frac{210}{360}=\frac{7}{12}=0.583\right)$ (A2) (C2)

(c) $\frac{90}{210}\left(=\frac{3}{7}=0.429\right)$ $\left(\text{Accept } \frac{\frac{1}{4}}{\frac{7}{12}}\right)$ (A1)(A1) (C2)

[6]

43. (a)



(A3) (N3) 3

(b) (i) 0.4×0.9 (A1)

$= 0.36$ (A1) (N2)

(ii) $0.36 + 0.6 \times 0.8 \quad (= 0.36 + 0.48)$ (A1)

$= 0.84$ (A1) (N1)

(iii) $\frac{\text{P}(\text{red} \cap \text{grows})}{\text{P}(\text{grows})}$ (may be implied) (M1)

$= \frac{0.36}{0.84}$ (A1)

$= 0.429 \left(\frac{3}{7}\right)$

(A1) (N2) 7

[10]

44. (a) Independent (I) (C2)

(b) Mutually exclusive (M) (C2)

(c) Neither (N) (C2)

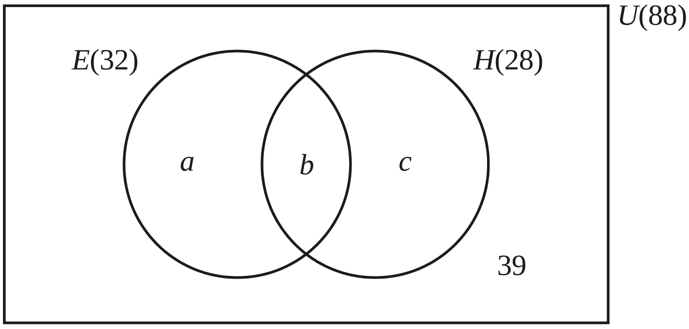
Note: *Award part marks if the candidate shows understanding of I and/or M*

eg I $P(A \cap B) = P(A)P(B)$ *(M1)*

M $P(A \cup B) = P(A) + P(B)$ *(M1)*

[6]

45. (a)



$n(E \cup H) = a + b + c = 88 - 39 = 49$ (M1)

$n(E \cup H) = 32 + 28 - b = 49$

$60 - 49 = b = 11$ (A1)

$a = 32 - 11 = 21$ (A1)

$c = 28 - 11 = 17$ (A1) 4

Note: *Award (A3) for correct answers with no working.*

(b) (i) $P(E \cap H) = \frac{11}{88} = \frac{1}{8}$ (A1)

(ii) $P(H' | E) = \frac{P(H' \cap E)}{P(E)} = \frac{\frac{21}{88}}{\frac{32}{88}}$ (M1)

$= \frac{21}{32}$ (= 0.656) (A1)

OR

Required probability = $\frac{21}{32}$ (A1)(A1) 3

(c) (i) P(none in economics) = $\frac{56 \times 55 \times 54}{88 \times 87 \times 86}$ (M1)(A1)

$= 0.253$ (A1)

Notes: *Award (M0)(A0)(A1)(ft) for* $\left(\frac{56}{88}\right)^3 = 0.258$.

Award no marks for $\frac{56 \times 55 \times 54}{88 \times 88 \times 88}$.

(ii) P(at least one) = 1 – 0.253 (M1)

$= 0.747$ (A1)

OR

$3\left(\frac{32}{88} \times \frac{56}{87} \times \frac{55}{86}\right) + 3\left(\frac{32}{88} \times \frac{31}{87} \times \frac{56}{86}\right) + \frac{32}{88} \times \frac{31}{87} \times \frac{30}{86}$ (M1)

$= 0.747$ (A1) 5

[12]

46. P(RR) = $\frac{7}{12} \times \frac{6}{11}\left(= \frac{7}{22}\right)$ (M1)(A1)

P(YY) = $\frac{5}{12} \times \frac{4}{11}\left(= \frac{5}{33}\right)$ (M1)(A1)

P (same colour) = P(RR) + P(YY) (M1)

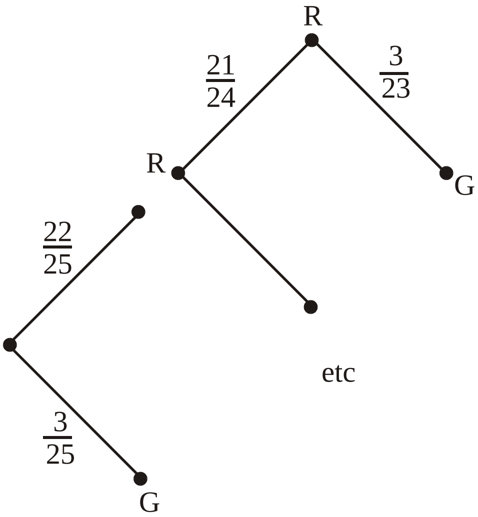
$= \frac{31}{66}$ (= 0.470, 3 sf) (A1) (C6)

Note: *Award C2 for* $\left(\frac{7}{12}\right)^2 + \left(\frac{5}{12}\right)^2 = \frac{74}{144}$.

[6]

47. (a) P = $\frac{22}{23}$ (= 0.957 (3 sf)) (A2) (C2)

(b)



(M1)

OR

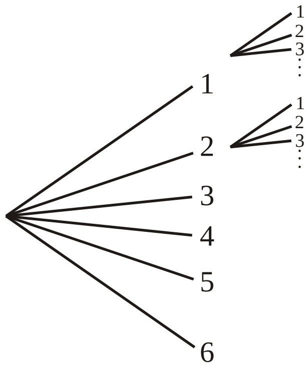
P = P (RRG) + P (RGR) + P (GRR) (M1)

$$\frac{22}{25}\times\frac{21}{24}\times\frac{3}{23}+\frac{22}{25}\times\frac{3}{24}\times\frac{21}{23}+\frac{3}{25}\times\frac{22}{24}\times\frac{21}{23}$$ (M1)(A1)

$$=\frac{693}{2300}$$ (= 0.301 (3 sf)) (A1) (C4)

[6]

48. Sample space ={(1, 1), (1, 2) ... (6, 5), (6, 6)}
(This may be indicated in other ways, for example, a grid or a tree diagram, partly or fully completed)



(a) $P(S < 8) = \frac{6+5+4+3+2+1}{36}$ (M1)

$= \frac{7}{12}$ (A1)

OR

$P(S < 8) = \frac{7}{12}$ (A2)

(b) P (at least one 3) = $\frac{1+1+6+1+1+1}{36}$ (M1)

$= \frac{11}{36}$ (A1)

OR

P (at least one 3) = $\frac{11}{36}$ (A2)

(c) $P(\text{at least one } 3 \mid S < 8) = \frac{P(\text{at least one } 3 \cap S < 8)}{P(S < 8)}$ (M1)

$= \frac{7/36}{7/12}$ (A1)

$= \frac{1}{3}$ (A1)

[7]

49. (a) P $(A \cup B)$ = P (A) + P (B) – P $(A \cap B) \Rightarrow$ P $(A \cap B)$ = P (A) + P (B) – P $(A \cup B)$ (M1)

$= \frac{3}{11} + \frac{4}{11} - \frac{6}{11}$ (M1)

$= \frac{1}{11}$ (0.0909) (A1) (C3)

(b) For independent events, P $(A \cap B)$ = P (A) × P (B) (M1)

$= \frac{3}{11} \times \frac{4}{11}$ (A1)

$= \frac{12}{121}$ (0.0992) (A1) (C3)

[6]

50. P(different colours) = 1 – [P(GG) + P(RR) + P(WW)] (M1)

$= 1 - \left(\frac{10}{6} \times \frac{9}{25} + \frac{10}{26} \times \frac{9}{25} + \frac{6}{26} \times \frac{5}{25}\right)$ (A1)

$= 1 - \left(\frac{210}{650}\right)$ (A1)

$= \frac{44}{65}$ (= 0.677, to 3 sf) (A1) (C4)

OR

P(different colours) = P(GR) + P(RG) + P(GW) + P(WG) + P(RW) + P(WR) (A1)

$= 4\left(\frac{10}{26} \times \frac{6}{25}\right) + 2\left(\frac{10}{26} \times \frac{10}{25}\right)$ (A1)(A1)

$= \frac{44}{65}$ (= 0.677, to 3 sf) (A1) (C4)

[4]

51. (a) s = 7.41(3 sf) (G3) 3

(b)

Weight (W)	$W \le 85$	$W \le 90$	$W \le 95$	$W \le 100$	$W \le 105$	$W \le 110$	$W \le 115$
Number of packets	5	15	**30**	**56**	**69**	**76**	80

(A1) 1

(c) (i) From the graph, the median is approximately 96.8.
Answer: 97 (nearest gram). (A2)

(ii) From the graph, the upper or third quartile is approximately 101.2.
Answer: 101 (nearest gram). (A2) 4

(d) Sum = 0, since the sum of the deviations from the mean is zero. (A2)
OR

$$\sum (W_i - \overline{W}) = \sum W_i - \left(80 \frac{\sum W_i}{80}\right) = 0$$ (M1)(A1) 2

(e) Let A be the event: $W > 100$, and B the event: $85 < W \le 110$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ (M1)

$P(A \cap B) = \frac{20}{80}$ (A1)

$P(B) = \frac{71}{80}$ (A1)

$P(A|B) = 0.282$ (A1)

OR

71 packets with weight $85 < W \le 110$. (M1)
Of these, 20 packets have weight $W > 100$. (M1)

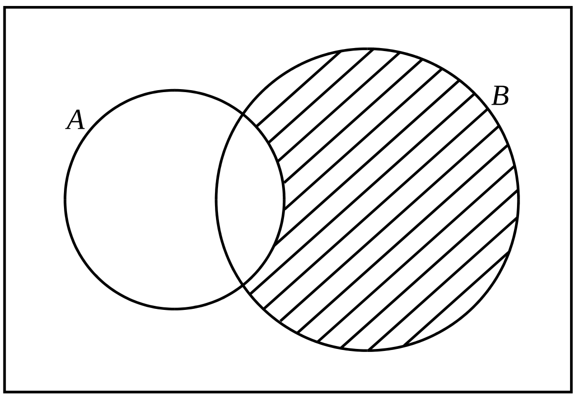
Required probability = $\frac{20}{71}$ (A1)

= 0.282 (A1) 4

Notes: *Award (A2) for a correct final answer with no reasoning.*
Award up to (M2) for correct reasoning or method.

[14]

52. (a) U



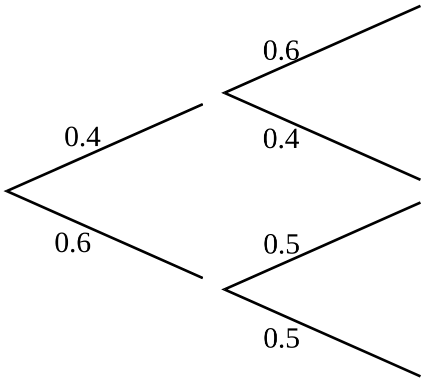
(A1) (C1)

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$65 = 30 + 50 - n(A \cap B)$
$\Rightarrow n(A \cap B) = 15$ (may be on the diagram) (M1)
$n(B \cap A') = 50 - 15 = 35$ (A1) (C2)

(c) $\text{P}(B \cap A') = \dfrac{n(B \cap A')}{n(U)} = \dfrac{35}{100} = 0.35$ (A1) (C1)

[4]

53. (a)



(A1) (C1)

(b) $\text{P}(B) = 0.4(0.6) + 0.6\,(0.5) = 0.24 + 0.30$ (M1)
$= 0.54$ (A1) (C2)

(c) $\text{P}(C \mid B) = \dfrac{\text{P}(B \cap C)}{\text{P}(B)} = \dfrac{0.24}{0.54} = \dfrac{4}{9}$ (= 0.444, 3 sf) (A1) (C1)

[4]

54. (a)

	Males	Females	Totals
Unemployed	**20**	**40**	**60**
Employed	**90**	**50**	**140**
Totals	**110**	**90**	200

Note: *Award (A1) if at least 4 entries are correct.*
Award (A2) if all 8 entries are correct.

(b) (i) P(unemployed female) = $\dfrac{40}{200} = \dfrac{1}{5}$ (A1)

(ii) P(male I employed person) = $\dfrac{90}{140} = \dfrac{9}{14}$ (A1)

[4]

55. (a)

	Boy	Girl	Total

TV	**13**	**25**	**38**
Sport	33	29	**62**
Total	46	**54**	100

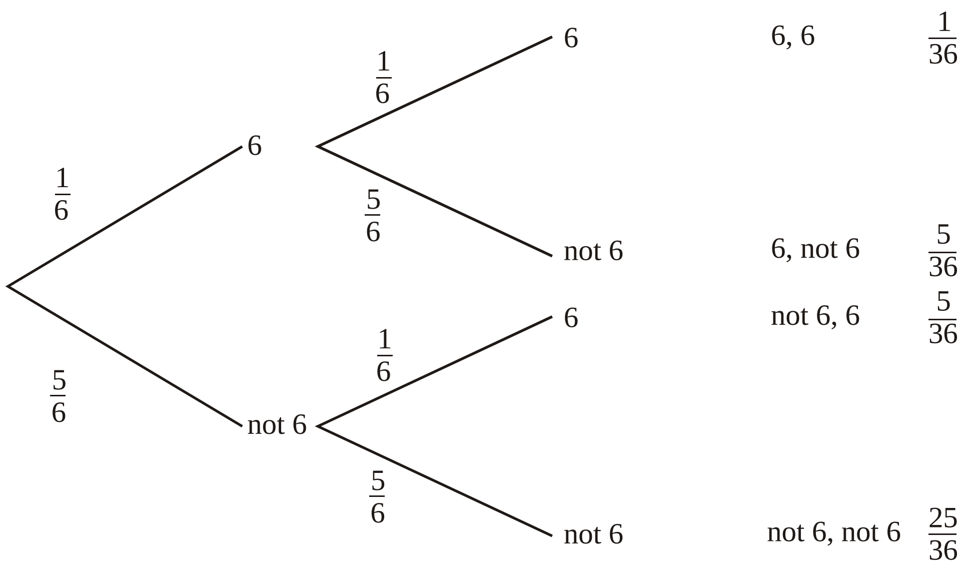
P(TV) = $\frac{38}{100}$ (A1) (C2)

(b) P(TV | Boy) = $\frac{13}{46}$ (= 0.283 to 3 sf) (A2) (C2)

Notes: *Award (A1) for numerator and (A1) for denominator. Accept equivalent answers.*

[4]

56. (a)



(M2) (C2)

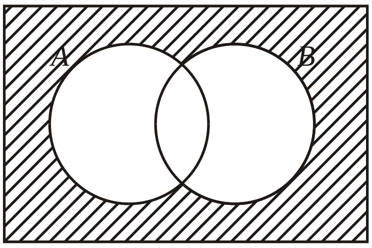
Notes: *Award (M1) for probabilities* $\frac{1}{6}, \frac{5}{6}$ *correctly entered on diagram.*
Award (M1) for correctly listing the outcomes 6, 6; 6 not 6; not 6, 6; not 6, not 6, or the corresponding probabilities.

(b) P(one or more sixes) = $\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}$ **or** $\left(1-\frac{5}{6}\times\frac{5}{6}\right)$ (M1)

$= \frac{11}{36}$ (A1) (C2)

[4]

57. (a)



(A1) (C1)

(b) (i) $n(A \cap B) = 2$ (A1) (C1)

(ii) $P(A \cap B) = \frac{2}{36}\left(\text{or } \frac{1}{18}\right)$ (allow **ft** from (b)(i)) (A1) (C1)

(c) $n(A \cap B) \neq 0$ (or equivalent) (R1) (C1)

[4]

58. $p(\text{Red}) = \frac{35}{40} = \frac{7}{8}$ $p(\text{Black}) = \frac{5}{40} = \frac{1}{8}$

(a) (i) $p(\text{one black}) = \binom{8}{1}\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^7$ (M1)(A1)

$= 0.393$ to 3 sf (A1) 3

(ii) $p(\text{at least one black}) = 1 - p(\text{none})$ (M1)

$= 1 - \binom{8}{0}\left(\frac{1}{8}\right)^0\left(\frac{7}{8}\right)^8$ (A1)

$= 1 - 0.344$

$= 0.656$ (A1) 3

(b) 400 draws: expected number of blacks = $\frac{400}{8}$ (M1)

$= 50$ (A1) 2

[8]

59. (a) $p(A \cap B) = 0.6 + 0.8 - 1$ (M1)

$= 0.4$ (A1) (C2)

(b) $p(\complement A \cup \complement B) = p(\complement(A \cap B)) = 1 - 0.4$ (M1)

$= 0.6$ (A1) (C2)

[4]