1. The probability distribution of a discrete random variable $X$ is given by

$$
\mathrm{P}(X=x)=\frac{x^{2}}{14}, x \in\{1,2, k\}, \text { where } k>0 .
$$

(a) Write down $\mathrm{P}(X=2)$.
(b) Show that $k=3$.
(c) Find $\mathrm{E}(X)$.
(2)
(Total 7 marks)
2. In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values $p, q, r$ and $s$ represent numbers of students.

(a) (i) Write down the value of $s$.
(ii) Find the value of $q$.
(iii) Write down the value of $p$ and of $r$.
(b) (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art.
(ii) Hence, show that taking music and taking art are not independent events.
(c) Two students are selected at random, one after the other. Find the probability that the first student takes only music and the second student takes only art.
3. The Venn diagram below shows events $A$ and $B$ where $\mathrm{P}(A)=0.3, \mathrm{P}(A \cup B)=0.6$ and $\mathrm{P}(A \cap B)=0.1$. The values $m, n, p$ and $q$ are probabilities.

(a) (i) Write down the value of $n$.
(ii) Find the value of $m$, of $p$, and of $q$.
(b) Find $\mathrm{P}\left(B^{\prime}\right)$.
4. Consider the events $A$ and $B$, where $\mathrm{P}(A)=0.5, \mathrm{P}(B)=0.7$ and $\mathrm{P}(A \cap B)=0.3$.

The Venn diagram below shows the events $A$ and $B$, and the probabilities $p, q$ and $r$.

(a) Write down the value of
(i) $p$;
(ii) $q$;
(iii) $r$.
(b) Find the value of $\mathrm{P}\left(A \mid B^{\prime}\right)$.
(c) Hence, or otherwise, show that the events $A$ and $B$ are not independent.
5. José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.

If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.
If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.
Let $E$ be the event "he misses his bus" and F the event "he is late for school". The information above is shown on the following tree diagram.

(a) Find
(i) $\mathrm{P}(E \cap F)$;
(ii) $\mathrm{P}(F)$.
(b) Find the probability that
(i) José misses his bus and is not late for school;
(ii) José missed his bus, given that he is late for school.

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.
(c) Copy and complete the probability distribution table.

| $X$ (cost in euros) | 0 | 3 | 6 |
| :--- | :---: | :---: | :---: |
| $\mathbf{P}(\boldsymbol{X})$ | $\frac{1}{9}$ |  |  |

(d) Find the expected cost for José for both days.
6. The diagram below shows the probabilities for events $A$ and $B$, with $\mathrm{P}\left(A^{\prime}\right)=p$.

(a) Write down the value of $p$.
(b) Find $\mathrm{P}(B)$.
(c) Find $\mathrm{P}\left(A^{\prime} \mid B\right)$.
7. The letters of the word PROBABILITY are written on 11 cards as shown below.


Two cards are drawn at random without replacement.
Let $A$ be the event the first card drawn is the letter A .
Let $B$ be the event the second card drawn is the letter B .
(a) Find $\mathrm{P}(A)$.
(b) Find $\mathrm{P}(B \mid A)$.
(c) Find $\mathrm{P}(A \cap B)$.
8. Two boxes contain numbered cards as shown below.


Two cards are drawn at random, one from each box.
(a) Copy and complete the table below to show all nine equally likely outcomes.

| 3,9 |  |  |
| :---: | :--- | :--- |
| 3,10 |  |  |
| 3,10 |  |  |

Let $S$ be the sum of the numbers on the two cards.
(b) Write down all the possible values of $S$.
(c) Find the probability of each value of $S$.
(d) Find the expected value of $S$.
(e) Anna plays a game where she wins $\$ 50$ if $S$ is even and loses $\$ 30$ if $S$ is odd. Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games.
9. In any given season, a soccer team plays $65 \%$ of their games at home.

When the team plays at home, they win $83 \%$ of their games.
When they play away from home, they win $26 \%$ of their games.
The team plays one game.
(a) Find the probability that the team wins the game.
(b) If the team does not win the game, find the probability that the game was played at home.
10. Let $X$ be normally distributed with mean 100 cm and standard deviation 5 cm .
(a) On the diagram below, shade the region representing $\mathrm{P}(X>105)$.

(b) Given that $\mathrm{P}(X<d)=\mathrm{P}(X>105)$, find the value of $d$.
(c) Given that $\mathrm{P}(X>105)=0.16$ (correct to two significant figures), find $\mathrm{P}(d<X<105)$.
11. In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.
(a) (i) Find the number of boys who play both sports.
(ii) Write down the number of boys who play only rugby.
(b) One boy is selected at random.
(i) Find the probability that he plays only one sport.
(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

Let $A$ be the event that a boy plays football and $B$ be the event that a boy plays rugby.
(c) Explain why $A$ and $B$ are not mutually exclusive.
(d) Show that $A$ and $B$ are not independent.
12. A four-sided die has three blue faces and one red face. The die is rolled.

Let $B$ be the event a blue face lands down, and $R$ be the event a red face lands down.
(a) Write down
(i) $\mathrm{P}(B)$;
(ii) $\mathrm{P}(R)$.
(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where $p, s, t$ are probabilities.


Find the value of $p$, of $s$ and of $t$.

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let $X$ be the total score obtained.
(c) (i) Show that $\mathrm{P}(X=3)=\frac{3}{16}$.
(ii) Find $\mathrm{P}(X=2)$.
(d) (i) Construct a probability distribution table for $X$.
(ii) Calculate the expected value of $X$.
(e) If the total score is 3 , Guiseppi wins $\$ 10$. If the total score is 2 , Guiseppi gets nothing. Guiseppi plays the game twice. Find the probability that he wins exactly $\$ 10$.
13. There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

|  | Football | Tennis | Hockey |
| :--- | :---: | :---: | :---: |
| Female | 5 | 3 | 3 |
| Male | 4 | 2 | 3 |

(a) One student is selected at random.
(i) Calculate the probability that the student is a male or is a tennis player.
(ii) Given that the student selected is female, calculate the probability that the student does not play football.
(b) Two students are selected at random. Calculate the probability that neither student plays football.
14. The following table shows the probability distribution of a discrete random variable $X$.

| $x$ | -1 | 0 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.2 | $10 k^{2}$ | 0.4 | $3 k$ |

(a) Find the value of $k$.
(b) Find the expected value of $X$.
15. Paula goes to work three days a week. On any day, the probability that she goes on a red bus is $\frac{1}{4}$.
(a) Write down the expected number of times that Paula goes to work on a red bus in one week.

In one week, find the probability that she goes to work on a red bus
(b) on exactly two days;
(c) on at least one day.
16. Let $A$ and $B$ be independent events, where $\mathrm{P}(A)=0.6$ and $\mathrm{P}(B)=x$.
(a) Write down an expression for $\mathrm{P}(A \cap B)$.
(b) Given that $\mathrm{P}(A \cup B)=0.8$,
(i) find $x$;
(ii) find $\mathrm{P}(A \cap B)$.
(c) Hence, explain why $A$ and $B$ are not mutually exclusive.
17. Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.


Let $X$ be the sum of the scores on the two dice.
(a) Find
(i) $\mathrm{P}(X=6)$;
(ii) $\mathrm{P}(X>6)$;
(iii) $\mathrm{P}(X=7 \mid X>5)$.
(b) Elena plays a game where she tosses two dice.

If the sum is 6 , she wins 3 points.
If the sum is greater than 6 , she wins 1 point.
If the sum is less than 6 , she loses $k$ points.
Find the value of $k$ for which Elena's expected number of points is zero.
18. A random variable $X$ is distributed normally with a mean of 100 and a variance of 100 .
(a) Find the value of $X$ that is 1.12 standard deviations above the mean.
(b) Find the value of $X$ that is 1.12 standard deviations below the mean.
19. In a game a player rolls a biased four-faced die. The probability of each possible score is shown below.

| Score | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Probability | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{1}{10}$ | $x$ |

(a) Find the value of $x$.
(b) Find $\mathrm{E}(X)$.
(c) The die is rolled twice. Find the probability of obtaining two scores of 3.
20. The heights of trees in a forest are normally distributed with mean height 17 metres. One tree is selected at random. The probability that a selected tree has a height greater than 24 metres is 0.06 .
(a) Find the probability that the tree selected has a height less than 24 metres.
(b) The probability that the tree has a height less than $D$ metres is 0.06 . Find the value of $D$.
(c) A woodcutter randomly selects 200 trees. Find the expected number of trees whose height lies between 17 metres and 24 metres.
21. The probability of obtaining heads on a biased coin is $\frac{1}{3}$.
(a) Sammy tosses the coin three times. Find the probability of getting
(i) three heads;
(ii) two heads and one tail.
(b) Amir plays a game in which he tosses the coin 12 times.
(i) Find the expected number of heads.
(ii) Amir wins \$ 10 for each head obtained, and loses $\$ 6$ for each tail.

Find his expected winnings.
22. Consider the events $A$ and $B$, where $\mathrm{P}(A)=\frac{2}{5}, \mathrm{P}\left(B^{\prime}\right)=\frac{1}{4}$ and $\mathrm{P}(A \cup B)=\frac{7}{8}$.
(a) Write down $\mathrm{P}(B)$.
(b) Find $\mathrm{P}(A \cap B)$.
(c) Find $\mathrm{P}(A \mid B)$.
23. The eye colour of 97 students is recorded in the chart below.

|  | Brown | Blue | Green |
| :---: | :---: | :---: | :---: |
| Male | 21 | 16 | 9 |
| Female | 19 | 19 | 13 |

One student is selected at random.
(a) Write down the probability that the student is a male.
(b) Write down the probability that the student has green eyes, given that the student is a female.
(c) Find the probability that the student has green eyes or is male.
(Total 6 marks)
24. The Venn diagram below shows information about 120 students in a school. Of these, 40 study Chinese ( $C$ ), 35 study Japanese ( $J$ ), and 30 study Spanish $(S)$.


A student is chosen at random from the group. Find the probability that the student
(a) studies exactly two of these languages;
(b) studies only Japanese;
(c) does not study any of these languages.
25. A bag contains four apples $(A)$ and six bananas $(B)$. A fruit is taken from the bag and eaten. Then a second fruit is taken and eaten.
(a) Complete the tree diagram below by writing probabilities in the spaces provided.

(b) Find the probability that one of each type of fruit was eaten.
26. A discrete random variable $X$ has a probability distribution as shown in the table below.

| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.1 | $a$ | 0.3 | $b$ |

(a) Find the value of $a+b$.
(b) Given that $\mathrm{E}(X)=1.5$, find the value of $a$ and of $b$.
27. Let $A$ and $B$ be independent events such that $\mathrm{P}(A)=0.3$ and $\mathrm{P}(B)=0.8$.
(a) Find $\mathrm{P}(A \cap B)$.
(b) Find $\mathrm{P}(A \cup B)$.
(c) Are $A$ and $B$ mutually exclusive? Justify your answer.
(Total 6 marks)
28. The probability distribution of the discrete random variable $X$ is given by the following table.

| $x$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.4 | $p$ | 0.2 | 0.07 | 0.02 |

(a) Find the value of $p$.
(b) Calculate the expected value of $X$.
(Total 6 marks)
29. In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.
(a) Find the probability that a student takes physics given that the student takes chemistry.
(b) Find the probability that a student takes physics given that the student does not take chemistry.
(c) State whether the events "taking chemistry" and "taking physics" are mutually exclusive, independent, or neither. Justify your answer.
30. Three students, Kim, Ching Li and Jonathan each have a pack of cards, from which they select a card at random. Each card has a $0,3,4$, or 9 printed on it.
(a) Kim states that the probability distribution for her pack of cards is as follows.

| $x$ | 0 | 3 | 4 | 9 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.3 | 0.45 | 0.2 | 0.35 |

Explain why Kim is incorrect.
(b) Ching Li correctly states that the probability distribution for her pack of cards is as follows.

| $x$ | 0 | 3 | 4 | 9 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.4 | $k$ | $2 k$ | 0.3 |

Find the value of $k$.
(c) Jonathan correctly states that the probability distribution for his pack of cards is given by $\mathrm{P}(X=x)=\frac{x+1}{20}$. One card is drawn at random from his pack.
(i) Calculate the probability that the number on the card drawn is 0 .
(ii) Calculate the probability that the number on the card drawn is greater than 0 .
31. A game is played, where a die is tossed and a marble selected from a bag.

Bag M contains 3 red marbles $(R)$ and 2 green marbles $(G)$.
Bag N contains 2 red marbles and 8 green marbles.
A fair six-sided die is tossed. If a 3 or 5 appears on the die, bag $M$ is selected ( $M$ ). If any other number appears, bag N is selected ( $N$ ).
A single marble is then drawn at random from the selected bag.
(a) Copy and complete the probability tree diagram on your answer sheet.

(b) (i) Write down the probability that bag M is selected and a green marble drawn from it.
(ii) Find the probability that a green marble is drawn from either bag.
(iii) Given that the marble is green, calculate the probability that it came from Bag M.
(c) A player wins $\$ 2$ for a red marble and $\$ 5$ for a green marble. What are his expected winnings?
32. Events $E$ and $F$ are independent, with $\mathrm{P}(E)=\frac{2}{3}$ and $\mathrm{P}(E \cap F)=\frac{1}{3}$. Calculate
(a) $\mathrm{P}(F)$;
(b) $\mathrm{P}(E \cup F)$.
(Total 6 marks)
33. Two fair four-sided dice, one red and one green, are thrown. For each die, the faces are labelled $1,2,3,4$. The score for each die is the number which lands face down.
(a) Write down
(i) the sample space;
(ii) the probability that two scores of 4 are obtained.

Let $X$ be the number of 4 s that land face down.
(b) Copy and complete the following probability distribution for $X$.

| $x$ | 0 | 1 | 2 |
| :---: | :--- | :--- | :--- |
| $\mathrm{P}(X=x)$ |  |  |  |

(c) Find $\mathrm{E}(X)$.
34. Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let $X$ denote the number of red balls chosen. The following table shows the probability distribution for $X$

| $X$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $\frac{3}{10}$ | $\frac{6}{10}$ | $\frac{1}{10}$ |

(a) Calculate $\mathrm{E}(X)$, the mean number of red balls chosen.

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.
(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.
(ii) Hence find the probability distribution for $Y$, where $Y$ is the number of red balls chosen.

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.
(c) Calculate the probability that two red balls are chosen.
(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die.
35. Two unbiased 6 -sided dice are rolled, a red one and a black one. Let $E$ and $F$ be the events
$E:$ the same number appears on both dice;
$F:$ the sum of the numbers is 10 .

Find
(a) $\mathrm{P}(E)$;
(b) $\mathrm{P}(F)$;
(c) $\mathrm{P}(E \cup F)$.


Answers:
(a) $\qquad$
(b) $\qquad$
(c) $\qquad$
36. The table below shows the subjects studied by 210 students at a college.

|  | Year 1 | Year 2 | Totals |
| :---: | :---: | :---: | :---: |
| History | 50 | 35 | 85 |
| Science | 15 | 30 | 45 |
| Art | 45 | 35 | 80 |
| Totals | 110 | 100 | 210 |

(a) A student from the college is selected at random.

Let $A$ be the event the student studies Art.
Let $B$ be the event the student is in Year 2.
(i) Find $P(A)$.
(ii) Find the probability that the student is a Year 2 Art student.
(iii) Are the events $A$ and $B$ independent? Justify your answer.
(b) Given that a History student is selected at random, calculate the probability that the student is in Year 1.
(c) Two students are selected at random from the college. Calculate the probability that one student is in Year 1, and the other in Year 2.
37. A class contains 13 girls and 11 boys. The teacher randomly selects four students. Determine the probability that all four students selected are girls.


Answers:
(Total 6 marks)
38. The events $A$ and $B$ are independent such that $\mathrm{P}(B)=3 \mathrm{P}(A)$ and $\mathrm{P}(A \cup B)=0.68$. Find $\mathrm{P}(B)$


Answers:
$\qquad$
39. The following probabilities were found for two events $R$ and $S$.

$$
\mathrm{P}(R)=\frac{1}{3}, \mathrm{P}(S \mid R)=\frac{4}{5}, \mathrm{P}\left(S \mid R^{\prime}\right)=\frac{1}{4} .
$$

(a) Copy and complete the tree diagram.

(b) Find the following probabilities.
(i) $\quad \mathrm{P}(R \cap S)$.
(ii) $\mathrm{P}(S)$.
(iii) $\mathrm{P}(R \mid S)$.
(Total 10 marks)
40. Let $A$ and $B$ be events such that $\mathrm{P}(A)=\frac{1}{2}, \mathrm{P}(B)=\frac{3}{4}$ and $\mathrm{P}(A \cup B)=\frac{7}{8}$.
(a) Calculate $\mathrm{P}(A \cap B)$.
(b) Calculate $\mathrm{P}(A \mid B)$.
(c) Are the events $A$ and $B$ independent? Give a reason for your answer.


Answers:
(a)
(b)
(c)
41. Dumisani is a student at IB World College.

The probability that he will be woken by his alarm clock is $\frac{7}{8}$.
If he is woken by his alarm clock the probability he will be late for school is $\frac{1}{4}$.
If he is not woken by his alarm clock the probability he will be late for school is $\frac{3}{5}$.
Let $W$ be the event "Dumisani is woken by his alarm clock".
Let $L$ be the event "Dumisani is late for school".
(a) Copy and complete the tree diagram below.

(b) Calculate the probability that Dumisani will be late for school.
(c) Given that Dumisani is late for school what is the probability that he was woken by his alarm clock?
42. The following diagram shows a circle divided into three sectors $\mathrm{A}, \mathrm{B}$ and C . The angles at the centre of the circle are $90^{\circ}, 120^{\circ}$ and $150^{\circ}$. Sectors A and B are shaded as shown.


The arrow is spun. It cannot land on the lines between the sectors. Let $A, B, C$ and $S$ be the events defined by

A: Arrow lands in sector A
B: Arrow lands in sector B
C: Arrow lands in sector C
$S$ : Arrow lands in a shaded region.

Find
(a) $\mathrm{P}(B)$;
(b) $\mathrm{P}(S)$;
(c) $\mathrm{P}(A \mid S)$.

Working:

Answers:
(a)
(b)
(c)
(Total 6 marks)
43. A packet of seeds contains $40 \%$ red seeds and $60 \%$ yellow seeds. The probability that a red seed grows is 0.9 , and that a yellow seed grows is 0.8 . A seed is chosen at random from the packet.
(a) Complete the probability tree diagram below.

(b) (i) Calculate the probability that the chosen seed is red and grows.
(ii) Calculate the probability that the chosen seed grows.
(iii) Given that the seed grows, calculate the probability that it is red.
44. Consider events $A, B$ such that $\mathrm{P}(A) \neq 0, \mathrm{P}(A) \neq 1, \mathrm{P}(\mathrm{B}) \neq 0$, and $\mathrm{P}(\mathrm{B}) \neq 1$.

In each of the situations (a), (b), (c) below state whether $A$ and $B$ are
mutually exclusive (M);
independent (I);
neither ( N ).
(a) $\quad \mathrm{P}(\mathrm{A} \mid \mathrm{B})=\mathrm{P}(\mathrm{A})$
(b) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})=0$
(c) $\quad \mathrm{P}(\mathrm{A} \cap \mathrm{B})=\mathrm{P}(\mathrm{A})$

## Working:

Answers:
(a)
(b)
(c)
45. In a school of 88 boys, 32 study economics (E), 28 study history (H) and 39 do not study either subject. This information is represented in the following Venn diagram.


U (88)
(a) Calculate the values $a, b, c$.
(b) A student is selected at random.
(i) Calculate the probability that he studies both economics and history.
(ii) Given that he studies economics, calculate the probability that he does not study history.
(c) A group of three students is selected at random from the school.
(i) Calculate the probability that none of these students studies economics.
(ii) Calculate the probability that at least one of these students studies economics.
46. A painter has 12 tins of paint. Seven tins are red and five tins are yellow. Two tins are chosen at random. Calculate the probability that both tins are the same colour.


Answer:
$\qquad$
47. A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.
(a) The first two apples are green. What is the probability that the third apple is red?
(b) What is the probability that exactly two of the three apples are red?

## Working:

Answers:
(a)
(b) $\qquad$
(Total 6 marks)
48. Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is $S$. Find the probability that
(a) $\quad S$ is less than 8 ;
(b) at least one die shows a 3;
(c) at least one die shows a 3, given that $S$ is less than 8 .
49. For events $A$ and $B$, the probabilities are $\mathrm{P}(A)=\frac{3}{11}, \mathrm{P}(B)=\frac{4}{11}$.

Calculate the value of $\mathrm{P}(A \cap B)$ if
(a) $\mathrm{P}(A \cup B)=\frac{6}{11}$;
(b) events $A$ and $B$ are independent.


Answers:
(a) $\qquad$
(b) $\qquad$
50. A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?


Answer:

Total 4 marks)
51. The table below represents the weights, $W$, in grams, of 80 packets of roasted peanuts.

| Weight $(W)$ | $80<W \leq 85$ | $85<W \leq 90$ | $90<W \leq 95$ | $95<W \leq 100$ | $100<W \leq 105$ | $105<W \leq 110$ | $110<W \leq 115$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> packets | 5 | 10 | 15 | 26 | 13 | 7 | 4 |

(a) Use the midpoint of each interval to find an estimate for the standard deviation of the weights.
(b) Copy and complete the following cumulative frequency table for the above data.

| Weight $(W)$ | $W \leq 85$ | $W \leq 90$ | $W \leq 95$ | $W \leq 100$ | $W \leq 105$ | $W \leq 110$ | $W \leq 115$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> packets | 5 | 15 |  |  |  |  | 80 |

(c) A cumulative frequency graph of the distribution is shown below, with a scale 2 cm for 10 packets on the vertical axis and 2 cm for 5 grams on the horizontal axis.


Use the graph to estimate
(i) the median;
(ii) the upper quartile (that is, the third quartile).

Give your answers to the nearest gram.
(d) Let $W_{1}, W_{2}, \ldots, W_{80}$ be the individual weights of the packets, and let $\bar{W}$ be their mean. What is the value of the sum

$$
\begin{equation*}
\left(W_{1}-\bar{W}\right)+\left(W_{2}-\bar{W}\right)+\left(W_{3}-\bar{W}\right)+\ldots+\left(W_{79}-\bar{W}\right)+\left(W_{80}-\bar{W}\right) ? \tag{2}
\end{equation*}
$$

(e) One of the 80 packets is selected at random. Given that its weight satisfies $85<W \leq 110$, find the probability that its weight is greater than 100 grams.
52. The following Venn diagram shows the universal set $U$ and the sets $A$ and $B$.

(a) Shade the area in the diagram which represents the set $B \cap A^{\prime}$.
$n(U)=100, n(A)=30, n(B)=50, n(A \cup B)=65$.
(b) Find $n\left(B \cap A^{\prime}\right)$.
(c) An element is selected at random from $U$. What is the probability that this element is in $B \cap A^{\prime}$ ?


Answers:
(b) $\qquad$
(c) $\qquad$
(Total 4 marks)
53. The events $B$ and $C$ are dependent, where $C$ is the event "a student takes Chemistry", and $B$ is the event "a student takes Biology". It is known that

$$
\mathrm{P}(C)=0.4, \mathrm{P}(B \mid C)=0.6, \mathrm{P}\left(B \mid C^{\prime}\right)=0.5 .
$$

(a) Complete the following tree diagram.

Chemistry
Biology

(b) Calculate the probability that a student takes Biology.
(c) Given that a student takes Biology, what is the probability that the student takes Chemistry?

Working:

Answers:
(b)
(c)
$\qquad$
$\qquad$
54. In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.
(a) Using this information, complete the table below.

|  | Males | Females | Totals |
| :--- | :---: | :---: | :---: |
| Unemployed |  |  |  |
| Employed |  |  |  |
| Totals |  |  | 200 |

(b) If a person is selected at random from this group of 200, find the probability that this person is
(i) an unemployed female;
(ii) a male, given that the person is employed.

## Working:

Answers:
(b) (i) $\qquad$
(ii) $\qquad$
(b) Calculate the probability that a student takes Biology.
(c) Given that a student takes Biology, what is the probability that the student takes Chemistry?

## Working:

Answers:
(b)
(c) $\qquad$
55. In a survey, 100 students were asked "do you prefer to watch television or play sport?" Of the 46 boys in the survey, 33 said they would choose sport, while 29 girls made this choice.

|  | Boys | Girls | Total |
| :--- | :---: | :---: | :---: |
| Television |  |  |  |
| Sport | 33 | 29 |  |
| Total | 46 |  | 100 |

By completing this table or otherwise, find the probability that
(a) a student selected at random prefers to watch television;
(b) a student prefers to watch television, given that the student is a boy.

## Working:

Answers:
(a) $\qquad$
(b)
56. Two ordinary, 6-sided dice are rolled and the total score is noted.
(a) Complete the tree diagram by entering probabilities and listing outcomes.

(b) Find the probability of getting one or more sixes.

## Working:

Answer:
(b) $\qquad$
57. The following Venn diagram shows a sample space $U$ and events $A$ and $B$.

$n(U)=36, n(A)=11, n(B)=6$ and $n(A \cup B)^{\prime}=21$.
(a) On the diagram, shade the region $(A \cup B)^{\prime}$.
(b) Find
(i) $\quad n(A \cap B)$;
(ii) $\quad P(A \cap B)$.
(c) Explain why events $A$ and $B$ are not mutually exclusive.


Answers:
(b) (i) $\qquad$
(ii) $\qquad$
(c) $\qquad$
58. A box contains 35 red discs and 5 black discs. A disc is selected at random and its colour noted. The disc is then replaced in the box.
(a) In eight such selections, what is the probability that a black disc is selected
(i) exactly once?
(ii) at least once?
(b) The process of selecting and replacing is carried out 400 times.

What is the expected number of black discs that would be drawn?
59. For the events $A$ and $B, p(A)=0.6, p(B)=0.8$ and $p(A \cup B)=1$.

Find
(a) $p(A \cap B)$;
(b) $\quad p(\mathrm{C} \mathrm{A} \cup \subset \mathrm{B})$.

Working:

Answers:
(a)
(b)
$\qquad$
$\qquad$

