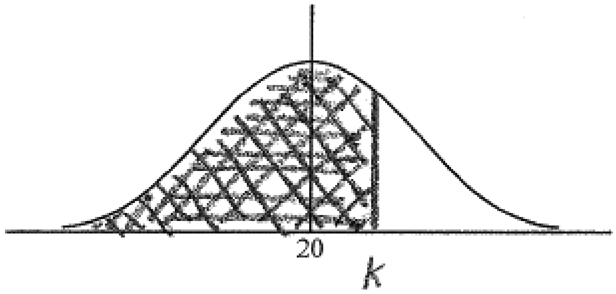
1. (a) $\sigma = 3$ (A1)

evidence of attempt to find $P(X \le 24.5)$ (M1)

e.g. $z = 1.5$, $\frac{24.5 - 20}{3}$

$P(X \le 24.5) = 0.933$ A1 N3 3

(b) (i)



A1A1 N2

Note: *Award A1 with shading that clearly extends to right of the mean, A1 for any correct label, either k, area or their value of k*

(ii) $z = 1.03(64338)$ (A1)

attempt to set up an equation (M1)

e.g. $\frac{k - 20}{3} = 1.0364, \frac{k - 20}{3} = 0.85$

$k = 23.1$ A1 N3 5

[8]

2. (a) correct substitution into formula for $E(X)$ (A1)

e.g. 0.05×240

$E(X) = 12$ A1 N2 2

(b) evidence of recognizing binomial probability (may be seen in part (a)) (M1)

e.g. $\binom{240}{15}(0.05)^{15}(0.95)^{225}$, $X \sim B(240, 0.05)$

$P(X = 15) = 0.0733$ A1 N2 2

	(c)	$P(X \leq 9) = 0.236$	(A1)		
		evidence of valid approach	(M1)		
		e.g. using complement, summing probabilities			
		$P(X \geq 10) = 0.764$	A1	N3	3

[7]

3.	(a)	symmetry of normal curve	(M1)		
		e.g. $P(X < 25) = 0.5$			
		$P(X > 27) = 0.2$	A1	N2	2
	(b)	**METHOD 1**			
		finding standardized value	(A1)		
		e.g. $\frac{27-25}{\sigma}$			
		evidence of complement	(M1)		
		e.g. $1-p$, $P(X < 27)$, 0.8			
		finding z-score	(A1)		
		e.g. $z = 0.84\ldots$			
		attempt to set up equation involving the standardized value	M1		
		e.g. $0.84 = \frac{27-25}{\sigma}, 0.84 = \frac{X-\mu}{\sigma}$			
		$\sigma = 2.38$	A1	N3	5
		METHOD 2			
		set up using normal CDF function and probability	(M1)		
		e.g. $P(25 < X < 27) = 0.3$, $P(X < 27) = 0.8$			
		correct equation	A2		
		e.g. $P(25 < X < 27) = 0.3$, $P(X > 27) = 0.2$			
		attempt to solve the equation using GDC	(M1)		
		e.g. solver, graph, trial and error (more than two trials must be shown)			
		$\sigma = 2.38$	A1	N3	5

[7]

4. (a) evidence of recognizing binomial probability (may be seen in (b) or (c)) (M1)

e.g. probability = $\binom{7}{4}(0.9)^4(0.1)^3$, $X \sim \text{B}(7, 0.9)$, complementary probabilities

probability = 0.0230 A1 N2

(b) correct expression A1A1 N2

e.g. $\binom{7}{4}p^4(1-p)^3$, $35p^4(1-p)^3$

Note: *Award A1 for binomial coefficient* $\left(\text{accept}\binom{7}{3}\right)$,

A1 for $p^4(1-p)^3$.

(c) evidence of attempting to solve **their** equation (M1)

e.g. $\binom{7}{4}p^4(1-p)^3 = 0.15$, sketch

$p = 0.356, 0.770$ A1A1 N3

[7]

5. (a) evidence of appropriate approach (M1)

e.g. 1 – 0.85, diagram showing values in a normal curve

$\text{P}(w \geq 82) = 0.15$ A1 N2

(b) (i) $z = -1.64$ A1 N1

(ii) evidence of appropriate approach (M1)

e.g. $-1.64 = \frac{x-\mu}{\sigma}, \frac{68-76.6}{\sigma}$

correct substitution A1

e.g. $-1.64 = \frac{68-76.6}{\sigma}$

$\sigma = 5.23$ A1 N1

(c) (i) $68.8 \leq \text{weight} \leq 84.4$ A1A1A1 N3

Note: *Award A1 for 68.8, A1 for 84.4, A1 for giving answer as an interval.*

(ii) evidence of appropriate approach (M1)
e.g. $P(-1.5 \le z \le 1.5)$, $P(68.76 < y < 84.44)$
P(qualify) = 0.866 A1 N2

(d) recognizing conditional probability (M1)
e.g. $P(A | B) = \frac{P(A \cap B)}{P(B)}$
P(woman and qualify) = 0.25 × 0.7 (A1)
P(woman | qualify) = $\frac{0.25 \times 0.7}{0.866}$ A1
P(woman | qualify) = 0.202 A1 N3
[15]

6. (a) evidence of attempt to find $P(X \le 475)$ (M1)
e.g. $P(Z \le 1.25)$
$P(X \le 475) = 0.894$ A1 N2

(b) evidence of using the complement (M1)
e.g. 0.73, $1 - p$
$z = 0.6128$ (A1)
setting up equation (M1)
e.g. $\frac{a-450}{20} = 0.6128$
$a = 462$ A1 N3
[6]

7. (a) evidence of using mid-interval values (5, 15, 25, 35, 50, 67.5, 87.5) (M1)
$\sigma = 19.8$ (cm) A2 N3

(b) (i) $Q_1 = 15$, $Q_3 = 40$ (A1)(A1)
$IQR = 25$ (accept any notation that suggests the interval 15 to 40) A1 N3

(ii) **METHOD 1**
60 % have a length less than k (A1)
0.6 × 200 = 120 (A1)
k 30 (cm) A1 N2

METHOD 2

$0.4 \times 200 = 80$ (A1)
$200 - 80 = 120$ (A1)
$k = 30$ (cm) A1 N2

(c) $l < 20$ cm $\Rightarrow$ 70 fish (M1)

P(small) = $\frac{70}{200}$ (= 0.35) A1 N2

(d)

Cost $*X*	4	10	12
P($X = x$)	**0.35**	0.565	**0.085**

A1A1 N2

(e) correct substitution (of their p values) into formula for E(X) (A1)
e.g. $4 \times 0.35 + 10 \times 0.565 + 12 \times 0.085$

E(X) = 8.07 (accept $8.07) A1 N2

[15]

8. (a) E(X) = 2 A1 N1

(b) evidence of appropriate approach involving binomial (M1)
e.g. $\binom{10}{3}(0.2)^3$, $(0.2)^3(0.8)^7$, $X \sim \mathrm{B}(10, 0.2)$
P(X = 3) = 0.201 A1 N2

(c) **METHOD 1**

$\mathrm{P}(X \le 3) = 0.10737 + 0.26844 + 0.30199 + 0.20133$ (= 0.87912...) (A1)
evidence of using the complement (seen anywhere) (M1)
e.g. 1 – any probability, $\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \le 3)$

$\mathrm{P}(X > 3) = 0.121$ A1 N2

METHOD 2

recognizing that $P(X > 3) = P(X \geq 4)$ (M1)
e.g. summing probabilities from $X = 4$ to $X = 10$

correct expression or values (A1)

e.g. $\sum_{r=4}^{10} \binom{10}{r} (0.2)^{10-r} (0.8)^r$

$0.08808 + 0.02642 + 0.005505 + 0.000786 + 0.0000737 + 0.000004 + 0.0000001$

$P(X > 3) = 0.121$ A1 N2

[6]

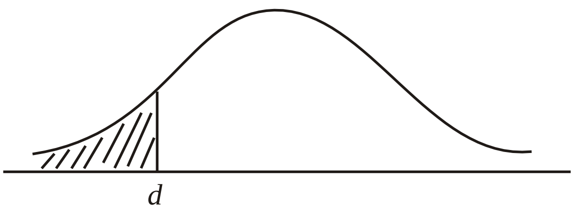
9. $X \sim N(7, 0.5^2)$

(a) (i) $z = 2$ (M1)
$P(X < 8) = P(Z < 2) = 0.977$ A1 N2

(ii) evidence of appropriate approach (M1)
e.g. symmetry, $z = -2$
$P(6 < X < 8) = 0.954$ (tables 0.955) A1 N2

Note: *Award M1A1(AP) if candidates refer to 2 standard deviations from the mean, leading to 0.95.*

(b) (i)



A1A1 N2

Note: *Award A1 for d to the left of the mean, A1 for area to the left of d shaded.*

(ii) $z = -1.645$ (A1)

$\frac{d-7}{0.5} = -1.645$ (M1)

$d = 6.18$ A1 N3

(c) $Y \sim \mathrm{N}(\mu, 0.5^2)$

$\mathrm{P}(Y < 5) = 0.2$ (M1)

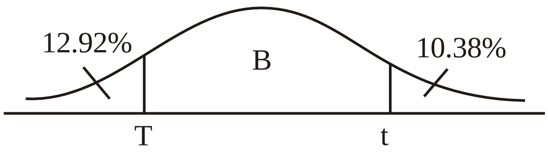
$z = -0.84162...$ A1

$$\frac{5-\mu}{0.5} = -0.8416$$ (M1)

$\mu = 5.42$ A1 N3

[13]

10. (a)



A1A1 N2

Notes: *Award A1 for three re.g.ions, (may be shown by lines or shading) A1 for clear labelling of two re.g.ions (may be shown by percentages or cate.g.ories).*

r and t need not be labelled, but if they are, they may be interchanged.

(b) **METHOD 1**

$\mathrm{P}(X < r) = 0.1292$ (A1)

$r = 6.56$ A1 N2

$1 - 0.1038$ (= 0.8962) (may be seen later) A1

$\mathrm{P}(X < t) = 0.8962$ (A1)

$t = 7.16$ A1 N2

METHOD 2

finding z-values $-1.130...$, $1.260...$ A1A1

evidence of setting up one standardized equation (M1)

e.g. $\frac{r-6.84}{0.25} = -1.13\ldots,\ t = 1.260 \times 0.25 + 6.84$

$r = 6.56$, $t = 7.16$ A1A1 N2N2

[7]

11. $X \sim N(\mu, \sigma^2)$

$P(X > 90) = 0.15$ **and** $P(X < 40) = 0.12$ (M1)

Finding standardized values 1.036, –1.175 A1A1

Setting up the equations $1.036 = \frac{90-\mu}{\sigma}, -1.175 = \frac{40-\mu}{\sigma}$ (M1)

$\mu = 66.6, \sigma = 22.6$ A1A1 N2N2

[6]

12. (a) evidence of valid approach involving A and B (M1)

e.g. $P(A \cap \text{pass}) + P(B \cap \text{pass})$, tree diagram

correct expression (A1)

e.g. $P(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9$

$P(\text{pass}) = 0.84$ A1 N2 3

(b) evidence of recognizing complement (seen anywhere) (M1)

e.g. $P(B) = x$, $P(A) = 1 - x$, $1 - P(B)$, $100 - x$, $x + y = 1$

evidence of valid approach (M1)

e.g. $0.8(1 - x) + 0.9x$, $0.8x + 0.9y$

correct expression A1

e.g. $0.87 = 0.8(1 - x) + 0.9x$, $0.8 \times 0.3 + 0.9 \times 0.7 = 0.87$, $0.8x + 0.9y = 0.87$

70 % from B A1 N2 4

[7]

13. (a) three correct pairs A1A1A1 N3 3

e.g. (2, 4), (3, 3), (4, 2), $R2G4$, $R3G3$, $R4G2$

(b) $p = \frac{1}{16}, q = \frac{2}{16}, r = \frac{2}{16}$ A1A1A1 N3 3

(c) let X be the number of times the sum of the dice is 5

evidence of valid approach (M1)

e.g. $X \sim \mathrm{B}(n, p)$, tree diagram, 5 sets of outcomes produce a win

one correct parameter (A1)

e.g. $n = 4$, $p = 0.25$, $q = 0.75$

Fred wins prize is $\mathrm{P}(X \geq 3)$ (A1)

appropriate approach to find probability M1

e.g. complement, summing probabilities, using a CDF function

correct substitution (A1)

e.g. $1 - 0.949\ldots$, $1 - \frac{243}{256}$, $0.046875 + 0.00390625$, $\frac{12}{256} + \frac{1}{256}$

probability of winning = 0.0508 $\left(\frac{13}{256}\right)$ A1 N3 6

[12]

14. (a) 36 outcomes (seen anywhere, even in denominator) (A1)

valid approach of listing ways to get sum of 5, showing at least two pairs (M1)
e.g. (1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2) , lattice diagram

P(prize) = $\frac{4}{36}\left(=\frac{1}{9}\right)$ A1 N3

(b) recognizing binomial probability (M1)

e.g. $\mathrm{B}\left(8, \frac{1}{9}\right)$, binomial pdf, $\binom{8}{3}\left(\frac{1}{9}\right)^3\left(\frac{8}{9}\right)^5$

P(3 prizes) = 0.0426 A1 N2

[5]

15. (a) (i) valid approach (M1)

e.g. np, $5 \times \frac{1}{5}$

$\mathrm{E}(X) = 1$ A1 N2

(ii) evidence of appropriate approach involving binomial (M1)
e.g. $X \sim \mathrm{B}\left(5, \frac{1}{5}\right)$

recognizing that Mark needs to answer 3 **or more** questions correctly (A1)
e.g. $\mathrm{P}(X \geq 3)$

valid approach M1
e.g. $1 - \mathrm{P}(X \leq 2)$, $\mathrm{P}(X = 3) + \mathrm{P}(X = 4) + \mathrm{P}(X = 5)$

P(pass) = 0.0579 A1 N3

(b) (i) evidence of summing probabilities to 1 (M1)
e.g. $0.67 + 0.05 + (a + 2b) + \ldots + 0.04 = 1$

some simplification that clearly leads to required answer
e.g. $0.76 + 4a + 2b = 1$ A1

$4a + 2b = 0.24$ AG N0

(ii) correct substitution into the formula for expected value (A1)
e.g. $0(0.67) + 1(0.05) + \ldots + 5(0.04)$

some simplification (A1)
e.g. $0.05 + 2a + 4b + \ldots + 5(0.04) = 1$

correct equation A1
e.g. $13a + 5b = 0.75$

evidence of solving (M1)

$a = 0.05$, $b = 0.02$ A1A1 N4

(c) attempt to find probability Bill passes (M1)
e.g. $\mathrm{P}(Y \geq 3)$

correct value 0.19 A1
Bill (is more likely to pass) A1 N0

[17]

16. $A \sim \mathrm{N}(46, 10^2)$ $B \sim \mathrm{N}(\mu, 12^2)$

(a) $\mathrm{P}(A > 60) = 0.0808$ A2 N2

(b) correct approach (A1)
e.g. $\mathrm{P}\left(Z < \frac{60-\mu}{12}\right) = 0.85$, sketch

$\frac{60-\mu}{12} = 1.036\ldots$ (A1)
$\mu = 47.6$ A1 N2

(c) (i) route A A1 N1

(ii) **METHOD 1**

$P(A < 60) = 1 - 0.0808 = 0.9192$ A1

valid reason R1

e.g. probability of A getting there on time is greater than probability of B

$0.9192 > 0.85$ N2

METHOD 2

$P(B > 60) = 1 - 0.85 = 0.15$ A1

valid reason R1

e.g. probability of A getting there late is less than probability of B

$0.0808 < 0.15$

N2

(d) (i) let X be the number of days when the van arrives before 07:00

$P(X = 5) = (0.85)^5$ (A1)

$= 0.444$ A1 N2

(ii) **METHOD 1**

evidence of adding correct probabilities (M1)

e.g. $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

correct values $0.1382 + 0.3915 + 0.4437$ (A1)

$P(X \geq 3) = 0.973$ A1 N3

METHOD 2

evidence of using the complement (M1)

e.g. $P(X \geq 3) = 1 - P(X \leq 2)$, $1 - p$

correct values $1 - 0.02661$ (A1)

$P(X \geq 3) = 0.973$ A1 N3

[13]

17. **METHOD 1**

for independence $P(A \cap B) = P(A) \times P(B)$ (R1)

expression for $P(A \cap B)$, indicating $P(B) = 2P(A)$ (A1)

e.g. $P(A) \times 2P(A)$, $x \times 2x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

correct substitution A1

e.g. $0.52 = x + 2x - 2x^2$, $0.52 = P(A) + 2P(A) - 2P(A)P(A)$

correct solutions to the equation (A2)

e.g. 0.2, 1.3 (accept the single answer 0.2)

$P(B) = 0.4$ A1 N6

METHOD 2

for independence $P(A \cap B) = P(A) \times P(B)$ (R1)

expression for $P(A \cap B)$, indicating $P(A) = \frac{1}{2}P(B)$ (A1)

e.g. $P(B) \times \frac{1}{2}P(B)$, $x \times \frac{1}{2}x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

correct substitution A1

e.g. $0.52 = 0.5x + x - 0.5x^2$, $0.52 = 0.5P(B) + P(B) - 0.5P(B)P(B)$

correct solutions to the equation (A2)

e.g. 0.4, 2.6 (accept the single answer 0.4)

$P(B) = 0.4$ (accept $x = 0.4$ if x set up as $P(B)$) A1 N6

[7]

18. (a) evidence of binomial distribution (may be seen in parts (b) or (c)) (M1)

e.g. np, 100×0.04

mean = 4 A1 N2

(b) $P(X = 6) = \binom{100}{6}(0.04)^6(0.96)^{94}$ (A1)

$= 0.105$ A1 N2

(c) for evidence of appropriate approach (M1)

e.g. complement, $1 - P(X = 0)$

$P(X = 0) = (0.96)^{100} = 0.01687...$ (A1)

$P(X \geq 1) = 0.983$ A1 N2

[7]

19. (a) evidence of using binomial probability (M1)

e.g. $P(X = 2) = \binom{7}{2}(0.18)^2(0.82)^5$

$P(X = 2) = 0.252$ A1 N2

(b) **METHOD 1**

evidence of using the complement M1
e.g. $1 - (P(X \le 1))$
$P(X \le 1) = 0.632$ (A1)
$P(X \ge 2) = 0.368$ A1 N2

METHOD 2

evidence of attempting to sum probabilities M1
e.g. P(2 heads) + P(3 heads) + ... + P(7 heads), 0.252 + 0.0923 + …

correct values for each probability (A1)
e.g. 0.252 + 0.0923 + 0.0203 + 0.00267 + 0.0002 + 0.0000061
$P(X \ge 2) = 0.368$ A1 N2

[5]

20. (a) evidence of approach (M1)

e.g. finding 0.84…, using $\frac{23.7 - 21}{\sigma}$

correct working (A1)

e.g. $0.84... = \frac{23.7 - 21}{\sigma}$, graph

$\sigma = 3.21$ A1 N2

(b) (i) evidence of attempting to find $P(X < 25.4)$ (M1)
e.g. using $z = 1.37$
$P(X < 25.4) = 0.915$ A1 N2

(ii) evidence of recognizing symmetry (M1)
e.g. $b = 21 - 4.4$, using $z = -1.37$
$b = 16.6$ A1 N2

[7]

21. (a) $X \sim B(100, 0.02)$
$E(X) = 100 \times 0.02 = 2$ A1 N1

(b) $P(X = 3) = \binom{100}{3}(0.02)^3\ (0.98)^{97}$ (M1)

$= 0.182$ A1 N2

(c) **METHOD 1**

$P(X > 1) = 1 - P(X \le 1) = 1 - (P(X = 0) + P(X = 1))$ M1

$= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ (M1)

$= 0.597$ A1 N2

METHOD 2

$P(X > 1) = 1 - P(X \le 1)$ (M1)

$= 1 - \ \ 0.40327$ (A1)

$= 0.597$ A1 N2

Note: *Award marks as follows for finding P(X ≥ 1), if working shown.*

$P(X \ge 1)$ A0

$= 1 - P(X \le 2) = 1 - 0.67668$ M1(FT)

$= 0.323$ A1(FT) N0

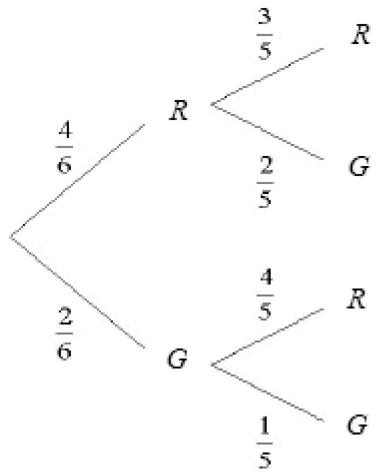
[6]

22. (a) Using $E(X) = \sum_{0}^{2} x\, P(X = x)$ (M1)

Substituting correctly $E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}$ A1

$= 0.8$ A1 N2

(b) (i)



A1A1A1 N3

Note: *Award A1 for each complementary pair of probabilities,*
i.e. $\frac{4}{6}$ and $\frac{2}{6}$, $\frac{3}{5}$ and $\frac{2}{5}$, $\frac{4}{5}$ and $\frac{1}{5}$.

(ii) $P(Y = 0) = \frac{2}{5}\times\frac{1}{5} = \frac{2}{30}$ A1

$P(Y = 1) = P(RG) + P(GR)\left(= \frac{4}{6}\times\frac{2}{5} + \frac{2}{6}\times\frac{4}{5}\right)$ M1

$= \frac{16}{30}$ A1

$P(Y = 2) = \frac{4}{6}\times\frac{3}{5} = \frac{12}{30}$ (A1)

For forming a distribution M1

y	0	1	2
$P(Y = y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

N4

(c) P(Bag A) = $\frac{2}{6}\left(=\frac{1}{3}\right)$ (A1)

P(Bag B) = $\frac{4}{6}\left(=\frac{2}{3}\right)$ (A1)

For summing $P(A \cap RR)$ and $P(B \cap RR)$ (M1)

Substituting correctly $P(RR) = \frac{1}{3}\times\frac{1}{10} + \frac{2}{3}\times\frac{12}{30}$ A1

= 0.3 A1 N3

(d) For recognising that $P(1 \text{ or } 6 \mid RR) = P(A \mid RR) = \frac{P(A \cap RR)}{P(RR)}$ (M1)

$= \frac{1}{30} \div \frac{27}{90}$ A1

$= 0.111$ A1 N2

[19]

23. (a) $P(H < 153) = 0.705 \Rightarrow z = 0.538(836...)$ (A1)

Standardizing $\frac{153-\mu}{5}$ (A1)

Setting up **their** equation $0.5388... = \frac{153-\mu}{5}$ M1

$\mu = 150.30\ldots$

$= 150$ (to 3sf) A1 N3

(b) $Z = \frac{153-\mu}{5} = 1.138...$ (accept 1.14 from $\mu = 150.3$, or 1.2 from $\mu = 150$) (A1)

$P(Z > 1.138) = 0.128$ (accept 0.127 from $z = 1.14$, or 0.115 from $z = 1.2$) A1 N2

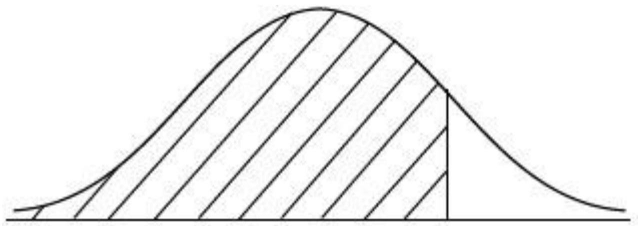
[6]

24. (a) 0.0668 A2 N2

(b) Using the standardized value 1.645 (A1)

$k = 26.1$ kg A1 N2

(c)

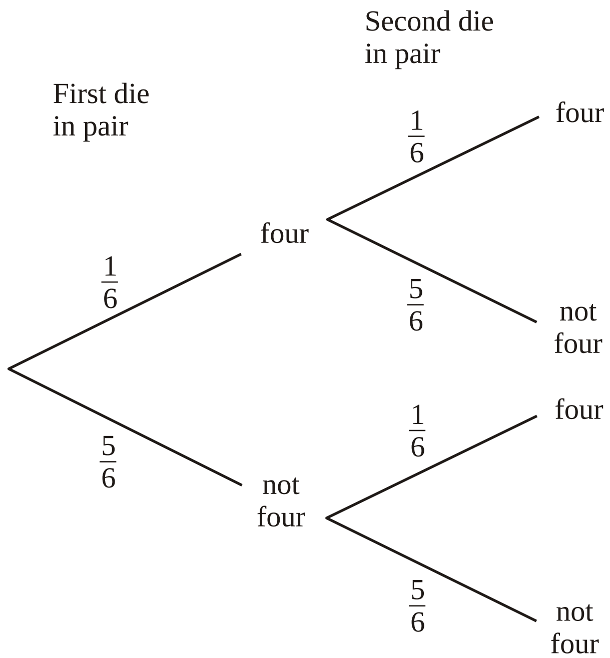


A1A1 N2

Note: *Award A1 for vertical line to right of the mean, A1 for shading to left of* ***their*** *vertical line.*

[6]

25. (a)



A1A1A1 N3

Note: *Award A1 for* ***each pair*** *of complementary probabilities.*

(b) $\text{P}(E) = \frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}\ \left(=\frac{5}{36}+\frac{5}{36}\right)$ (A2)

$= \frac{10}{36}\ \left(=\frac{5}{18}\text{ or } 0.278\right)$ A1 N3

(c) Evidence of recognizing the binomial distribution (M1)

eg $X \sim \mathrm{B}\left(5, \frac{5}{18}\right)$ or $p = \frac{5}{18}, q = \frac{13}{18}$

$\mathrm{P}(X = 3) = \binom{5}{3}\left(\frac{5}{18}\right)^3\left(\frac{13}{18}\right)^2$ (or other evidence of correct setup) (A1)

$= 0.112$ A1 N3

(d) **METHOD 1**

Evidence of using the complement M1
eg $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$
Correct value $1 - 0.865$ (A1)

$= 0.135$ A1 N2

METHOD 2

Evidence of adding correct probabilities M1
eg $\mathrm{P}(X \geq 3) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4) + \mathrm{P}(X = 5)$
Correct values $0.1118 + 0.02150 + 0.001654$ (A1)

$= 0.135$ A1 N2

[12]

26. (a) $\mathrm{P}(F \cup S) = 1 - 0.14\ (= 0.86)$ (A1)

Choosing an appropriate formula (M1)

eg $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

Correct substitution

eg $\mathrm{P}(F \cap S) = 0.93 - 0.86$ A1

$\mathrm{P}(F \cap S) = 0.07$ AG N0

Notes: *There are several valid approaches. Award (A1)(M1)A1 for relevant working using any appropriate strategy eg formula, Venn Diagram, or table.*

Award no marks for the incorrect solution

$\mathrm{P}(F \cap S) = 1 - \mathrm{P}(F) + \mathrm{P}(S) = 1 - 0.93 = 0.07$

(b) Using conditional probability (M1)

eg $P(F \mid S)$ $\left(= \dfrac{P(F \cap S)}{P(S)}\right)$

$P(F \mid S) = \dfrac{0.07}{0.62}$ (A1)

$= 0.113$ A1 N3

(c) F and S are **not** independent A1 N1

EITHER

If independent $P(F \mid S) = P(F)$, $0.113 \neq 0.31$ R1R1 N2

OR

If independent $P(F \cap S) = P(F)\,P(S)$, $0.07 \neq 0.31 \times 0.62$ (= 0.1922) R1R1 N2

(d) Let $P(F) = x$

$P(S) = 2P(F)$ $(= 2x)$ (A1)

For independence $P(F \cap S) = P(F)P(S)$ $(= 2x^2)$ (R1)

Attempt to set up a quadratic equation (M1)

eg $P(F \cup S) = P(F)P(S) - P(F)P(S)$, $0.86 = x + 2x - 2x^2$

$2x^2 - 3x + 0.86 = 0$ A2

$x = 0.386$, $x = 1.11$ (A1)

$P(F) = 0.386$ (A1) N5

[16]

27. *Note:* *Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.*

$W \sim N(2.5, 0.3^2)$

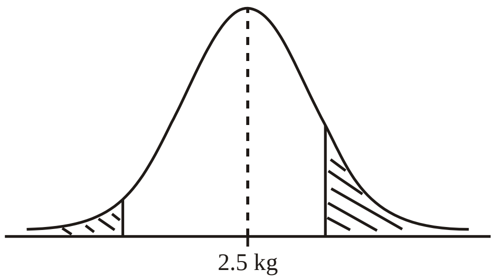
(a) (i) $z = -1.67$ (accept 1.67) (A1)

$P(W < 2) = 0.0478$ (accept answers between 0.0475 and 0.0485) A1 N2

(ii) $z = 1$ (A1)

$P(W > 2.8) = 0.159$ A1 N2

(iii)



A1A1 N2

Note: *Award A1 for a vertical line to left of mean andshading to left, A1 for vertical line to right of mean and shading to right.*

(iv) Evidence of appropriate calculation M1

eg $1 - (0.047790 + 0.15866)$, $0.8413 - 0.0478$

$P = 0.7936$ AG N0

Note: *The final value may vary depending on what level of accuracy is used.*

Accept their value in subsequent parts.

(b) (i) $X \sim B(10, 0.7935...)$

Evidence of calculation M1

eg $P(X = 10) = (0.7935...)^{10}$

$P(X = 10) = 0.0990$ (3 sf) A1 N1

(ii) **METHOD 1**

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

$P(X \leq 6) = 0.1325...$ (or $P(X = 1) + ... + P(X = 6)$) (A1)

evidence of using the complement (M1)

eg $P(X \geq 7) = 1 - P(X \leq 6)$, $P(X \geq 7) = 1 - P(X < 7)$

$P(X \geq 7) = 0.867$ A1 N3

METHOD 2

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

For adding terms from $P(X = 7)$ to $P(X = 10)$ (M1)

$P(X \geq 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030$ (A1)

$= 0.867$ A1 N3

[13]

28. (a) $z = \frac{180 - 160}{20} = 1$ (A1)

$\phi(1) = 0.8413$ (A1)

P(height > 180) = 1 – 0.8413

= 0.159 A1 N3

(b) $z = -1.1800$ (A1)

Setting up equation $-1.18 = \frac{d - 160}{20}$ (M1)

$d = 136$ A1 N3

[6]

29.

***Notes**: Accept any suitable notation, as long as thecandidate's intentions are clear.*

The following symbols will be used in the markscheme.

Girls' height G ~ N(155, 10^2), boys' height B ~ N(160, 12^2)

Height H, Female F, Male M.

(a) $P(G > 170) = 1 - P(G < 170)$ (A1)

$P(G > 170) = P\left(Z < \frac{170-155}{10}\right)$ (A1)

$P(G > 170) = 1 - \Phi(1.5) = 1 - 0.9332$

$= 0.0668$ A1 N3

(b) $z = -1.2816$ (A1)

Correct calculation (*eg* $x = 155 + -1.282 \times 10$) (A1)

$x = 142$ A1 N3

(c) Calculating one variable (A1)

eg $P(B < r) = 0.95$, $z = 1.6449$

$r = 160 + 1.645(12) = 179.74$

$= 180$ A1 N2

Any valid calculation for the second variable, including use of symmetry (A1)

eg $P(B < q) = 0.05$, $z = -1.6449$

$q = 160 - 1.645(12) = 140.26$

$= 140$ A1 N2

***Note**: Symbols are not required in parts (d) and (e).*

(d) $P(M \cap (B > 170)) = 0.4 \times 0.2020$, $P(F \cap (G > 170)) = 0.6 \times 0.0668$ (A1)(A1)

$P(H > 170) = 0.0808 + 0.04008$ A1

$= 0.12088 = 0.121$ (3 sf) A1 N2

(e) $\mathrm{P}(F \mid H > 170) = \dfrac{\mathrm{P}(F \cap (H > 170))}{\mathrm{P}(H > 170)}$ (M1)

$= \dfrac{0.60 \times 0.0668}{0.121} \quad \left(= \dfrac{0.0401}{0.121} \text{ or } \dfrac{0.04008}{0.1208}\right)$ A1

$= 0.332$ A1 N1

[17]

30. METHOD 1 Use of the GDC

(a) Evidence of using the binomial facility, M1

that is set up with $P = \dfrac{1}{2}$ and $n = 5$.

$\mathrm{P}(X = 3) = 0.3125 \quad \left(0.313, \dfrac{5}{16}\right)$ A2 N2

(b) Evidence of set up, with $1 - \mathrm{P}(X = 0)$ M1

$= 0.969 \quad \left(= \dfrac{31}{32}\right)$ A2 N2

METHOD 2 Use of the formula

(a) Evidence of binomial formula (M1)

$\mathrm{P}(X = 3) = \dbinom{5}{3}\left(\dfrac{1}{2}\right)^5$ A1

$= \dfrac{5}{16} \quad (= 0.313)$ A1 N2

(b) **METHOD 1**

P(at least one head) $= 1 - \text{P}(X = 0)$ (M1)

$= 1 - \left(\frac{1}{2}\right)^5$ A1

$= \frac{31}{32} \ (= 0.969)$ A1 N2

METHOD 2

P(at least one head) $= \text{P}(X = 1) + \text{P}(X = 2) + \text{P}(X = 3) + \text{P}(X = 4) + \text{P}(X = 5)$ (M1)

$= 0.15625 + 0.3125 + 0.3125 + 0.15625 + 0.03125$ A1

$= 0.969$ A1 N2

[6]

31. $X \sim \text{N}(\mu, \sigma^2)$, $\text{P}(X < 3) = 0.2$, $\text{P}(X > 8) = 0.1$

$\text{P}(X < 8) = 0.9$ (M1)

Attempt to set up equations (M1)

$\frac{3-\mu}{\sigma} = -0.8416, \quad \frac{8-\mu}{\sigma} = 1.282$ A1A1

$3 - \mu = -0.8416\sigma$

$8 - \mu = 1.282\sigma$

$5 = 2.1236\sigma$

$\sigma = 2.35, \quad \mu = 4.99$ A1A1 N4

[6]

32. (a) $X \sim \text{B}(100, 0.02)$

$E(X) = 100 \times 0.02 = 2$ A1 1

(b) $P(X = 3) = \binom{100}{3}(0.02)^3(0.98)^{97}$ (M1)

$= 0.182$ A1 2

(c) **METHOD 1**

$\text{P}(X > 1) = 1 - \text{P}(X \le 1) = 1 - (\text{P}(X = 0) + \text{P}(X = 1))$ M1

$= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ (M1)

$= 0.597$ A1 2

METHOD 2

$P(X > 1) = 1 – P(X \le 1)$ (M1)
$= 1 – 0.40327$ (A1)
$= 0.597$ A1 2

Note*: Award marks as follows for finding P(X > 1), if working shown.*

$P(X \ge 1)$ A0
$= 1 – P(X<2) = 1 – 0.67668$ M1(ft)
$= 0.323$ A1(ft) 2

[6]

33. $X \sim N(\mu, \sigma^2)$, $P(X > 90) = 0.15$ **and** $P(X < 40) = 0.12$ (M1)
Finding standardized values 1.036, –1.175 A1A1

Setting up the equations $1.036 = \frac{90-\mu}{\sigma}$, $-1.175 = \frac{40-\mu}{\sigma}$ (M1)

$\mu = 66.6$, $\sigma = 22.6$ A1A1

[6]

34. (i) $P(X > 3\,200) = P(Z > 0.4)$ (M1)

$= 1 - 0.6554 = 34.5\,\%\ (= 0.345)$ (A1) (N2)

(ii) $P(2\,300 < X < 3\,300) = P(-1.4 < Z < 0.6)$ (M1)

$= 0.4192 + 0.2257$

$= 0.645$ (A1)

$P(\text{both}) = (0.645)^2 = 0.416$ (A1) (N2)

(iii) $0.7422 = P(Z < 0.65)$ (A1)

$\frac{d - 3\,000}{500} = 0.65$ (A1)

$d = \$\,3\,325\ (= \$\,3\,330 \text{ to } 3 \text{ s.f.})$ (Accept \$3325.07) (A1) (N3)

[8]

35. (a) $z = \frac{185-170}{20} = 0.75$ (M1)(A1)

$P(Z < 0.75) = 0.773$ (A1) (N3)

(b) $z = –0.47$ (may be implied) (A1)

$-0.47 = \frac{d-170}{20}$ (M1)

$d = 161$ (A1) (N3)

[6]

36. (a) (i) $a = -1$ (A1)

$b = 0.5$ (A1)

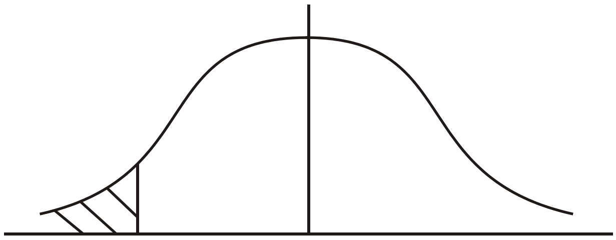
(ii) (a) 0.841 (A2)

(b) $0.6915 - 0.1587$ (or $0.8413 - 0.3085$) (M1)

$= 0.533$ (3 sf)

(A1) (N2) 6

(b) (i) Sketch of normal curve (A1)(A1)



(ii) $c = 0.647$ (A2) 4

[10]

37. **Method 1**

$b^2 - 4ac = 9 - 4k$ (M1)

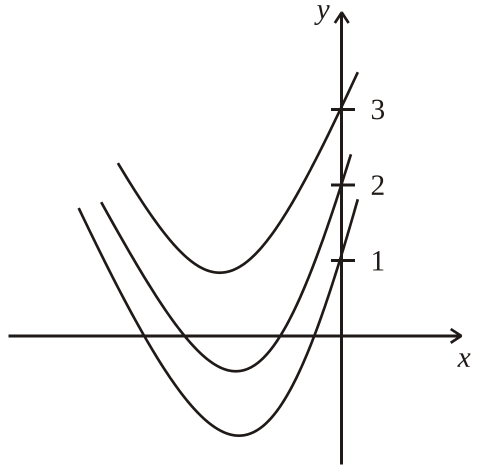
$9 - 4k > 0$ (M1)

$2.25 > k$ (A1)

crosses the x-axis if $k = 1$ **or** $k = 2$ (A1)(A1)

probability = $\frac{2}{7}$ (A1) (C6)

Method 2



(M2)(M1)

Note: *Award (M2) for one (relevant) curve; (M1) for a second one.*

$k = 1$ **or** $k = 2$ (G1)(G1)

probability = $\frac{2}{7}$ (A1) (C6)

[6]

38. X ~ N $(80, 8^2)$

(a) $P(X < 72) = P(Z < -1)$ (M1)

$= 1 - 0.8413$

$= 0.159$ (A1)

OR

$P(X < 72) = 0.159$ (G2) 2

(b) (i) $P(72 < X < 90) = P(-1 < Z < 1.25)$ (M1)

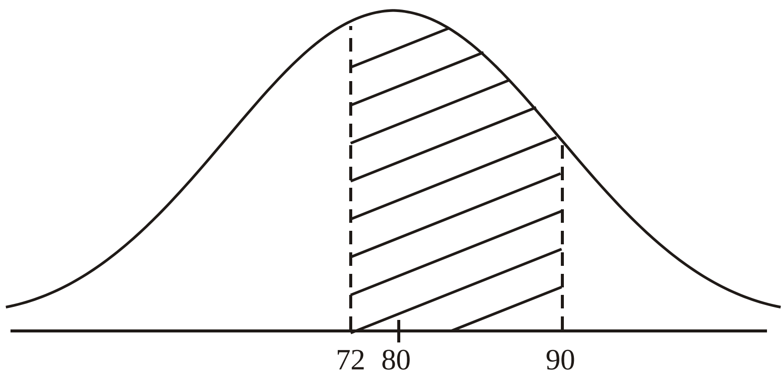
$= 0.3413 + 0.3944$ (A1)

$= 0.736$ (A1)

OR

$P(72 < X < 90) = 0.736$ (G3)

(ii)



(A1)(A1) 5

Note: *Award (A1) for a normal curve and (A1) for the shaded area, which should not be symmetrical.*

(c) 4% fail in less than x months

$\Rightarrow x = 80 - 8 \times \Phi^{-1}(0.96)$ (M1)

$= 80 - 8 \times 1.751$ (A1)

$= 66.0$ months (A1)

OR

$x = 66.0$ months (G3) 3

[10]

39. (a) $P(M \geq 350) = 1 - P\ (M < 350) = 1 - P\left(Z < \frac{350 - 310}{30}\right)$ (M1)

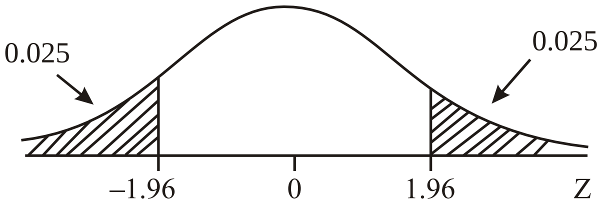
$= 1 - P(Z < 1.333) = 1 - 0.9088$

$= 0.0912$ (accept 0.0910 to 0.0920) (A1)

OR

$P(M \geq 350) = 0.0912$ (G2)

(b)



P(Z < 1.96) = 1 – 0.025 = 0.975 (A1)
1.96 (30) = 58.8 (M1)
310 – 58.8 < M < 310 + 58.8 ⇒ a = 251, b = 369 (A1)

OR

251 < M < 369 (G3)

Note: *Award (G1) if only one of the end points is correct.*

[5]

40. (a) (These answers may be obtained from a calculator or by finding z in each case and the corresponding area.)

$M \sim N$ (750, 625)

(i) P (M < 740 g) = 0.345 (G2)

OR

z = –0.4 P(z < –0.4) = 0.345 (A1)(A1)

(ii) P (M > 780 g) = 0.115 (G2)

OR

z = 1.2 P(z > 1.2) = l – 0.885 = 0.115 (A1)(A1)

(iii) P(740 < M < 780) = 0.540 (G1)

OR

1 – (0.345 + 0.115) = 0.540 (A1) 5

(b) Independent events

Therefore, P (both < 740) = 0.345^2 (M1)
= 0.119 (A1) 2

(c) 70% have mass < 763 g (G1)
Therefore, 70% have mass of at least 750 – 13
x = 737 g (A1) 2

[9]

41. ***Note:*** *Where accuracy is not specified, accept answers with greater than 3 sf accuracy, provided they are correct as far as 3 sf*

(a) $z = \dfrac{197 - 187.5}{9.5} = 1.00$ (M1)

$P(Z > 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$

$= 0.159$ (3 sf) (A1)

$= 15.9\%$ (A1)

OR

$P(H > 197) = 0.159$ (G2)

$= 15.9\%$ (A1) 3

(b) Finding the 99th percentile

$\Phi(a) = 0.99 \Rightarrow a = 2.327$ (accept 2.33) (A1)

$\Rightarrow$ 99% of heights under $187.5 + 2.327(9.5) = 209.6065$ (M1)

$= 210$ (3 sf) (A1)

OR

99% of heights under 209.6 = 210 cm (3 sf) (G3)

Height of standard doorway = 210 + 17 = 227 cm (A1) 4

[7]

42. (a) Let X be the random variable for the IQ.

$X \sim N(100, 225)$

$P(90 < X < 125) = P(-0.67 < Z < 1.67)$ (M1)

$= 0.701$

70.1 percent of the population (accept 70 percent). (A1)

OR

$P(90 < X < 125) = 70.1\%$ (G2) 2

(b) $P(X \geq 125) = 0.0475$ (or 0.0478) (M1)

P(both persons having IQ ≥ 125) $= (0.0475)^2$ (or $(0.0478)^2$) (M1)

$= 0.00226$ (or 0.00228) (A1) 3

(c) Null hypothesis (H_0): mean IQ of people with disorder is 100 (M1)
Alternative hypothesis (H_1): mean IQ of people with disorder is less than 100 (M1)

$$P(\bar{X} < 95.2) = P\left(Z < \left(\frac{95.2 - 100}{\frac{15}{\sqrt{25}}}\right)\right) = P(Z < -1.6) = 1 - 0.9452$$

$= 0.0548$ (A1)

The probability that the sample mean is 95.2 and the null hypothesis true is 0.0548 > 0.05. Hence the evidence is not sufficient. (R1) 4

[9]

43. (a) $Z = \frac{25 - 25.7}{0.50} = -1.4$ (M1)

$P(Z < -1.4) = 1 - P(Z < 1.4)$
$= 1 - 0.9192$
$= 0.0808$ (A1)

OR

$P(W < 25) = 0.0808$ (G2) 2

(b) $P(Z < -a) = 0.025 \Rightarrow P(Z < a) = 0.975$
$\Rightarrow a = 1.960$ (A1)

$\frac{25 - \mu}{0.50} = -1.96 \Rightarrow \mu = 25 + 1.96\ (0.50)$ (M1)
$= 25 + 0.98 = 25.98$ (A1)
$= 26.0$ (3 sf) (AG)

OR

$\frac{25.0 - 26.0}{0.50} = -2.00$ (M1)
$P(Z < -2.00) = 1 - P(Z < 2.00)$
$= 1 - 0.9772 = 0.0228$ (A1)
≈ 0.025 (A1)

OR

$\mu = 25.98$ (G2)
$\Rightarrow$ mean $= 26.0$ (3 sf) (A1)(AG) 3

(c) Clearly, by symmetry $\mu = 25.5$ (A1)

$Z = \dfrac{25.0 - 25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma$ (M1)

$\Rightarrow \sigma = 0.255$ kg (A1) 3

(d) On average, $\dfrac{\text{cement saving}}{\text{bag}} = 0.5$ kg (A1)

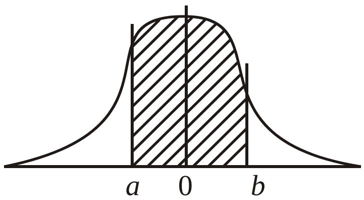
$\dfrac{\text{cost saving}}{\text{bag}} = 0.5(0.80) = \$0.40$ (M1)

To save \$5000 takes $\dfrac{5000}{0.40} = 12500$ bags (A1) 3

[11]

44. (a) Let X be the lifespan in hours

$X \sim N(57, 4.4^2)$



(i) $a = -0.455$ (3 sf) (A1)

$b = 0.682$ (3 sf) (A1)

(ii) (a) $P(X > 55) = P(Z > -0.455)$

$= 0.675$ (A1)

(b) $P(55 \le X \le 60) = P\left(\dfrac{2}{4.4} \le Z \le \dfrac{3}{4.4}\right)$

$\approx P(0.455 \le Z \le 0.682)$

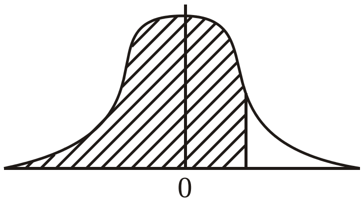
$\approx 0.6754 + 0.752 - 1$ (A1)

$= 0.428$ (3sf) (A1)

OR

$P(55 \le X \le 60) = 0.428$ (3 sf) (G2) 5

(b) 90% have died ⇒ shaded area = 0.9 (M1)



(A1)

Hence $t = 57 + (4.4 \times 1.282)$ (M1)
$= 57 + 5.64$ (A1)
$= 62.6$ hours (A1)

OR $t = 62.6$ hours (G3) 5

[10]

45. (a) ***Note:*** *Candidates using tables may get slightly different answers, especially if they do not interpolate. Accept these answers.*

$P(\text{speed} > 50) = 0.3 = 1 - \Phi\left(\frac{50-\mu}{10}\right)$ (A1)

Hence, $\frac{50-\mu}{10} = \Phi^{-1}(0.7)$ (M1)

$\mu = 50 - 10\Phi^{-1}(0.7)$ (M1)
$= 44.75599 \ldots\ldots = 44.8$ km/h (3 sf) (accept 44.7) (AG) 3

(b) H_1: "the mean speed has been reduced by the campaign". (A1) 1

(c) One-tailed; because H_1 involves only "<". (A2) 2

(d) For a one-tailed test at 5% level, critical region is
$Z < \mu_m - 1.64\sigma_m$ *(accept $-1.65\sigma_m$)* (M1)

Now, $\mu_m = \mu = 44.75...$; $\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$ *(allow ft)* (A1)

So test statistic is $44.75... -1.64 \times 2 = 41.47$ (A1)

Now $41.3 < 41.47$ so reject H_0, yes. (A1) 4

[10]

46. (a) Area $A = 0.1$ (A1) 1

(b) **EITHER** Since $p(X \geq 12) = p(X \leq 8)$, (M1)
then 8 and 12 are symmetrically disposed around the mean. (M1)(R1)

Thus mean $= \frac{8+12}{2}$ (M1)

$= 10$ (A1)

Notes: *If a candidate says simply "by symmetry $\mu = 10$" with no further explanation award [3 marks] (M1, A1, R1). As a full explanation is requested award an additional (A1) for saying since $p(X < 8) = p(X > 12)$ and another (A1) for saying that the normal curve is symmetric.*

OR $p(X \geq 12) = 0.1 \Rightarrow p\left(Z \geq \frac{12-\mu}{\sigma}\right) = 0.1$ (M1)

$\Rightarrow p\left(Z \leq \frac{12-\mu}{\sigma}\right) = 0.9$

$p(X \leq 8) = 0.1 \Rightarrow p\left(Z \leq \frac{8-\mu}{\sigma}\right) = 0.1$

$\Rightarrow p\left(Z \leq \frac{\mu-8}{\sigma}\right) = 0.9$ (A1)

So $\frac{12-\mu}{\sigma} = \frac{\mu-8}{\sigma}$ (M1)

$\Rightarrow 12 - \mu = \mu - 8$ (M1)

$\Rightarrow \mu = 10$ (A1) 5

(c) $\Phi\left(\frac{12-10}{\sigma}\right) = 0.9$ (A1)(M1)(A1)

Note: *Award (A1) for $\left(\frac{12-10}{\sigma}\right)$, (M1) for standardizing, and (A1) for 0.9.*

$\Rightarrow \frac{2}{\sigma} = 1.282$ (or 1.28) (A1)

$\sigma = \frac{2}{1.282}\left(\text{or} \frac{2}{1.28}\right)$ (A1)

$= 1.56$ (3 sf) (AG) 5

Note: *Working backwards from $\sigma = 1.56$ to show it leads the given data should receive a maximum of [3 marks] if done correctly.*

(d) $p\ (X \le 11) = p\left(Z \le \frac{11-10}{1.561}\right)$ (or 1.56) (M1)(A1)

Note: *Award (M1) for standardizing and (A1) for* $\left(\frac{11-10}{1.561}\right)$.

$= p\ (Z \le 0.6407)$ (or 0.641 or 0.64) (A1)
$= \Phi(0.6407)$ (M1)
$= 0.739$ (3 sf) (A1) 5

[16]

47. (a) p (4 heads) = $\binom{8}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{8-4}$ (M1)

$= \frac{8\times7\times6\times5}{1\times2\times3\times4}\times\left(\frac{1}{2}\right)^8$

$= \frac{70}{256} \cong 0.273$ (3 sf) (A1) 2

(b) p (3 heads) = $\binom{8}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{8-3} = \frac{8\times7\times6}{1\times2\times3}\times\left(\frac{1}{2}\right)^8$

$= \frac{56}{256} \cong 0.219$ (3 sf) (A1) 1

(c) p (5 heads) = p (3 heads) (by symmetry) (M1)
p (3 or 4 or 5 heads) = p (4) + 2p (3) (M1)
$= \frac{70+2\times56}{256} = \frac{182}{256}$
≈ 0.711 (3 sf) (A1) 3

[6]