1. (a) \( \sigma = 3 \)  
   evidence of attempt to find \( P(X \leq 24.5) \)  
   \( e.g. \ z = 1.5, \ \frac{24.5 - 20}{3} \)  
   \( P(X \leq 24.5) = 0.933 \)  
   A1 N3 3

(b) (i)  
   Note: Award A1 with shading that clearly extends to right of the mean, A1 for any correct label, either \( k \), area or their value of \( k \)

(ii) \( z = 1.03(64338) \)  
   attempt to set up an equation  
   \( e.g. \ \frac{k - 20}{3} = 1.0364 \)  
   \( k - 20 \)  
   \( k = 23.1 \)  
   A1 N3 5

2. (a) correct substitution into formula for \( E(X) \)  
   \( e.g. \ 0.05 \times 240 \)  
   \( E(X) = 12 \)  
   A1 N2 2

(b) evidence of recognizing binomial probability (may be seen in part (a))  
   \( e.g. \ \binom{240}{15}(0.05)^{15}(0.95)^{225}, X \sim B(240,0.05) \)  
   \( P(X = 15) = 0.0733 \)  
   A1 N2 2
(c) \( P(X \leq 9) = 0.236 \) \hspace{1cm} (A1)  
 evidence of valid approach \hspace{1cm} (M1)  
e.g. using complement, summing probabilities  
\( P(X \geq 10) = 0.764 \) \hspace{1cm} A1 N3 3  

3. (a) symmetry of normal curve \hspace{1cm} (M1)  
e.g. \( P(X < 25) = 0.5 \)  
\( P(X > 27) = 0.2 \) \hspace{1cm} A1 N2 2  

(b) METHOD 1  
finding standardized value \hspace{1cm} (A1)  
e.g. \[ \frac{27 - 25}{\sigma} \]  
evidence of complement \hspace{1cm} (M1)  
e.g. 1-\( p \), \( P(X < 27) \), 0.8  
finding \( z \)-score \hspace{1cm} (A1)  
e.g. \( z = 0.84 \ldots \)  
attempt to set up equation involving the standardized value \hspace{1cm} M1  
e.g. \[ 0.84 = \frac{27 - 25}{\sigma}, 0.84 = \frac{X - \mu}{\sigma} \]  
\( \sigma = 2.38 \) \hspace{1cm} A1 N3 5  

METHOD 2  
set up using normal CDF function and probability \hspace{1cm} (M1)  
e.g. \( P(25 < X < 27) = 0.3, P(X < 27) = 0.8 \)  
correct equation \hspace{1cm} A2  
e.g. \( P(25 < X < 27) = 0.3, P(X > 27) = 0.2 \)  
attempt to solve the equation using GDC \hspace{1cm} (M1)  
e.g. solver, graph, trial and error (more than two trials must be shown)  
\( \sigma = 2.38 \) \hspace{1cm} A1 N3 5  

4. (a) evidence of recognizing binomial probability (may be seen in (b) or (c)) (M1)
   e.g. probability = \( \binom{7}{4}(0.9)^4(0.1)^3, X \sim B(7, 0.9) \), complementary probabilities
   \[ \text{probability} = 0.0230 \]
   A1 N2

(b) correct expression (A1A1 N2)
   e.g. \( \binom{7}{4}p^4(1-p)^3, 35p^4(1-p)^3 \)

   __Note: Award A1 for binomial coefficient \( \binom{7}{3} \), A1 for \( p^4(1-p)^3 \).__

(c) evidence of attempting to solve their equation (M1)
   e.g. \( \binom{7}{4}p^4(1-p)^3 = 0.15 \), sketch
   \[ p = 0.356, 0.770 \]
   A1A1 N3

5. (a) evidence of appropriate approach (M1)
   e.g. \( 1 - 0.85 \), diagram showing values in a normal curve
   \[ \text{P}(w \geq 82) = 0.15 \]
   A1 N2

(b) (i) \( z = -1.64 \) (A1 N1)

(ii) evidence of appropriate approach (M1)
   e.g. \( -1.64 = \frac{x - \mu}{\sigma}, \frac{68 - 76.6}{\sigma} \)
   correct substitution (A1)
   e.g. \( -1.64 = \frac{68 - 76.6}{\sigma} \)
   \( \sigma = 5.23 \) (A1 N1)

(c) (i) \( 68.8 \leq \text{weight} \leq 84.4 \) (A1A1A1 N3)

   __Note: Award A1 for 68.8, A1 for 84.4, A1 for giving answer as an interval.__
(ii) evidence of appropriate approach
\[ e.g. P(-1.5 \leq z \leq 1.5), P(68.76 < y < 84.44) \]
\[ P(\text{qualify}) = 0.866 \]
\[ \text{A1 N2} \]

(d) recognizing conditional probability
\[ e.g. P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]
\[ P(\text{woman and qualify}) = 0.25 \times 0.7 \]
\[ P(\text{woman} \mid \text{qualify}) = \frac{0.25 \times 0.7}{0.866} \]
\[ P(\text{woman} \mid \text{qualify}) = 0.202 \]
\[ \text{A1 N3} \]

6. (a) evidence of attempt to find \( P(X \leq 475) \)
\[ e.g. P(Z \leq 1.25) \]
\[ P(X \leq 475) = 0.894 \]
\[ \text{A1 N2} \]

(b) evidence of using the complement
\[ e.g. 0.73, 1 - p \]
\[ z = 0.6128 \]
\[ \text{setting up equation} \]
\[ e.g. \frac{a - 450}{20} = 0.6128 \]
\[ a = 462 \]
\[ \text{A1 N3} \]

7. (a) evidence of using mid-interval values (5, 15, 25, 35, 50, 67.5, 87.5)
\[ \sigma = 19.8 \text{ (cm)} \]
\[ \text{A2 N3} \]

(b) (i) \( Q_1 = 15, Q_3 = 40 \)
\[ IQR = 25 \text{ (accept any notation that suggests the interval 15 to 40)} \]
\[ \text{A1 N3} \]

(ii) METHOD 1
\[ 60\% \text{ have a length less than } k \]
\[ 0.6 \times 200 = 120 \]
\[ k \text{ 30 (cm)} \]
\[ \text{A1 N2} \]
METHOD 2

\[0.4 \times 200 = 80\]  \hspace{1cm} (A1)
\[200 - 80 = 120\]  \hspace{1cm} (A1)
\[k = 30\] (cm)  \hspace{1cm} A1 \ N2

(c) \[l < 20\] cm \[\Rightarrow 70\] fish\hspace{1cm} (M1)
\[P(\text{small}) = \frac{70}{200} = 0.35\]  \hspace{1cm} A1 \ N2

(d)

<table>
<thead>
<tr>
<th>Cost $X</th>
<th>4</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = x)</td>
<td>0.35</td>
<td>0.565</td>
<td>0.085</td>
</tr>
</tbody>
</table>

\hspace{7cm} A1A1 \ N2

(e) correct substitution (of their \(p\) values) into formula for \(E(X)\) \hspace{1cm} (A1)
\[e.g. \ 4 \times 0.35 + 10 \times 0.565 + 12 \times 0.085\]
\[E(X) = 8.07\] (accept $8.07) \hspace{1cm} A1 \ N2

8. (a) \[E(X) = 2\] \hspace{1cm} A1 \ N1

(b) evidence of appropriate approach involving binomial \hspace{1cm} (M1)
\[e.g. \ \binom{10}{3}(0.2)^3(0.8)^7, \ X \sim B(10, 0.2)\]
\[P(X = 3) = 0.201\] \hspace{1cm} A1 \ N2

(c) **METHOD 1**
\[P(X \leq 3) = 0.10737 + 0.26844 + 0.30199 + 0.20133 = 0.87912...\] \hspace{1cm} (A1)
\[e.g. 1 \text{ - any probability, } P(X > 3) = 1 - P(X \leq 3)\] \hspace{1cm} (M1)
\[P(X > 3) = 0.121\] \hspace{1cm} A1 \ N2
METHOD 2

recognizing that \( P(X > 3) = P(X \geq 4) \)  
\( e.g. \) summing probabilities from \( X = 4 \) to \( X = 10 \)

correct expression or values  
\( e.g. \sum_{r=4}^{10} \binom{10}{r} (0.2)^{10-r}(0.8)^r \)

\[ 0.08808 + 0.02642 + 0.005505 + 0.000786 + 0.0000737 + 0.000004 + 0.0000001 \]

\[ P(X > 3) = 0.121 \]

9. \( X \sim N (7, 0.5^2) \)

(a)  
(i) \( z = 2 \)  
\( P(X < 8) = P(Z < 2) = 0.977 \)

(ii) evidence of appropriate approach  
\( e.g. \) symmetry, \( z = -2 \)

\( P(6 < X < 8) = 0.954 \) (tables 0.955)

**Note:** Award M1A1(AP) if candidates refer to 2 standard deviations from the mean, leading to 0.95.

(b)  
(i)

![Normal Distribution](image)

\( d \)

**Note:** Award A1 for \( d \) to the left of the mean, A1 for area to the left of \( d \) shaded.

(ii) \( z = -1.645 \)  
\[ \frac{d - 7}{0.5} = -1.645 \]  
\[ d = 6.18 \]
(c) \( Y \sim N(\mu, 0.5^2) \)
\[ P(Y < 5) = 0.2 \]  
\[ z = -0.84162... \]  
\[ \frac{5 - \mu}{0.5} = -0.8416 \]  
\[ \mu = 5.42 \]

10. (a) 

![Diagram showing 12.92% and 10.38% between T and B.](image)

Notes: Award A1 for three regions, (may be shown by lines or shading) A1 for clear labelling of two regions (may be shown by percentages or categories).

\( r \) and \( t \) need not be labelled, but if they are, they may be interchanged.

(b) **METHOD 1**

\[ P(X < r) = 0.1292 \]  
\[ r = 6.56 \]  
\[ 1 - 0.1038 (= 0.8962) \] (may be seen later)

\[ P(X < t) = 0.8962 \]  
\[ t = 7.16 \]  

**METHOD 2**

finding \( z \)-values \(-1.130..., 1.260...\)

A1A1

evidence of setting up one standardized equation

\[ e.g. \quad \frac{r - 6.84}{0.25} = -1.13..., \quad t = 1.260 \times 0.25 + 6.84 \]

\[ r = 6.56, \ t = 7.16 \]  
A1A1 N2N2
11. \( X \sim N(\mu, \sigma^2) \)
\[ P(X > 90) = 0.15 \ \text{and} \ P(X < 40) = 0.12 \quad (M1) \]
Finding standardized values 1.036, \(-1.175\)  
\[ \frac{90 - \mu}{\sigma}, \frac{40 - \mu}{\sigma} \]  
\( \mu = 66.6, \ \sigma = 22.6 \)  
\[ \text{A1A1 N2N2} \]  
\[ [6] \]

12. (a) evidence of valid approach involving \( A \) and \( B \)  
\[ e.g. \ P(A \cap \ \text{pass}) + P(B \cap \ \text{pass}), \ \text{tree diagram} \]  
\[ \text{correct expression} \]  
\[ e.g. \ P(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9 \]  
\[ P(\text{pass}) = 0.84 \]  
\[ \text{A1 N2 3} \]

(b) evidence of recognizing complement (seen anywhere)  
\[ e.g. \ P(B) = x, \ P(A) = 1 - x, \ 1 - P(B), \ 100 - x, \ x + y = 1 \]  
\[ \text{evidence of valid approach} \]  
\[ e.g. \ 0.8(1 - x) + 0.9x, \ 0.8x + 0.9y \]  
\[ \text{correct expression} \]  
\[ e.g. \ 0.87 = 0.8(1 - x) + 0.9x, \ 0.8 \times 0.3 + 0.9 \times 0.7 = 0.87, \ 0.8x + 0.9y = 0.87 \]  
\[ 70 \% \ \text{from} \ B \]  
\[ \text{A1 N2 4} \]  
\[ [7] \]

13. (a) three correct pairs  
\[ e.g. \ (2, \ 4), \ (3, \ 3), \ (4, \ 2), \ R2G4, \ R3G3, \ R4G2 \]  
\[ \text{A1A1A1 N3 3} \]

(b) \[ p = \frac{1}{16}, \ q = \frac{2}{16}, \ r = \frac{2}{16} \]  
\[ \text{A1A1A1 N3 3} \]
(c) Let $X$ be the number of times the sum of the dice is 5

**Evidence of valid approach (M1)**

*E.g.* $X \sim B(n, p)$, tree diagram, 5 sets of outcomes produce a win

**One correct parameter (A1)**

*E.g.* $n = 4, p = 0.25, q = 0.75$

Fred wins prize is $P(X \geq 3)$

**Appropriate approach to find probability (M1)**

*E.g.* complement, summing probabilities, using a CDF function

**Correct substitution (A1)**

*E.g.* $1 - 0.949\ldots, 1 - \frac{243}{256}, 0.046875 + 0.00390625 + \frac{12}{256} + \frac{1}{256}$

Probability of winning $= 0.0508 \left( \frac{13}{256} \right)$

---

**14.**

(a) 36 outcomes (seen anywhere, even in denominator) (A1)

**Valid approach of listing ways to get sum of 5, showing at least two pairs (M1)**

*E.g.* (1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2) , lattice diagram

$P(\text{prize}) = \frac{4}{36} \left( = \frac{1}{9} \right)$

**A1**

(b) Recognizing binomial probability (M1)

*E.g.* $B \left( 8, \frac{1}{9} \right)$, binomial pdf, $\binom{8}{3} \left( \frac{1}{9} \right)^3 \left( \frac{8}{9} \right)^5$

$P(3 \text{ prizes}) = 0.0426$

---

**15.**

(a) (i) Valid approach (M1)

*E.g.* $np, 5 \times \frac{1}{5}$

$E(X) = 1$

**A1**
(ii) evidence of appropriate approach involving binomial
\[ e.g. \ X \sim B \left( 5, \frac{1}{5} \right) \]
recognizing that Mark needs to answer 3 or more questions correctly
\[ e.g. \ P(X \geq 3) \]
valid approach
\[ e.g. \ 1 - P(X \leq 2), P(X = 3) + P(X = 4) + P(X = 5) \]
P(pass) = 0.0579

(b) (i) evidence of summing probabilities to 1
\[ e.g. \ 0.67 + 0.05 + (a + 2b) + ... + 0.04 = 1 \]
some simplification that clearly leads to required answer
\[ e.g. \ 0.76 + 4a + 2b = 1 \]
\[ 4a + 2b = 0.24 \]

(ii) correct substitution into the formula for expected value
\[ e.g. \ 0(0.67) + 1(0.05) + ... + 5(0.04) \]
some simplification
\[ e.g. \ 0.05 + 2a + 4b + ... + 5(0.04) = 1 \]
correct equation
\[ e.g. \ 13a + 5b = 0.75 \]
evidence of solving
\[ a = 0.05, \ b = 0.02 \]

(c) attempt to find probability Bill passes
\[ e.g. \ P(Y \geq 3) \]
correct value 0.19
Bill (is more likely to pass)

16. \[ A \sim N(46, \ 10^2) \ B \sim N(\mu, \ 12^2) \]
(a) \[ P(A > 60) = 0.0808 \]

(b) correct approach
\[ e.g. \ P \left( Z < \frac{60 - \mu}{12} \right) = 0.85, \ sketch \]
\[ \frac{60 - \mu}{12} = 1.036... \]
\[ \mu = 47.6 \]

(c) (i) route A
(ii) **METHOD 1**

\[ P(A < 60) = 1 - 0.0808 = 0.9192 \]  
valid reason  
\textit{e.g.} probability of A getting there on time is greater than probability of B  
\[ 0.9192 > 0.85 \]  
\[ A1 \]  
\[ R1 \]  
\[ N2 \]

**METHOD 2**

\[ P(B > 60) = 1 - 0.85 = 0.15 \]  
valid reason  
\textit{e.g.} probability of A getting there late is less than probability of B  
\[ 0.0808 < 0.15 \]  
\[ A1 \]  
\[ R1 \]  
\[ N2 \]

(d) (i) let \( X \) be the number of days when the van arrives before 07:00

\[ P(X = 5) = (0.85)^5 \]  
\[ = 0.444 \]  
\[ A1 \]  
\[ N2 \]

(ii) **METHOD 1**

\[ \text{evidence of adding correct probabilities} \]  
\textit{e.g.} \( P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) \)  
\text{correct values} 0.1382 + 0.3915 + 0.4437  
\[ P(X \geq 3) = 0.973 \]  
\[ A1 \]  
\[ N3 \]

**METHOD 2**

\[ \text{evidence of using the complement} \]  
\textit{e.g.} \( P(X \geq 3) = 1 - P(X \leq 2), 1 - p \)  
\text{correct values} 1 - 0.02661  
\[ P(X \geq 3) = 0.973 \]  
\[ A1 \]  
\[ N3 \]
17. **METHOD 1**

for independence $P(A \cap B) = P(A) \times P(B)$ \hspace{1cm} (R1)

expression for $P(A \cap B)$, indicating $P(B) = 2P(A)$ \hspace{1cm} (A1)

e.g. $P(A) \times 2P(A) \times x \times 2x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ \hspace{1cm} (M1)

correct substitution \hspace{1cm} A1

e.g. $0.52 = x + 2x - 2x^2$, $0.52 = P(A) + 2P(A) - 2P(A)P(A)$

correct solutions to the equation \hspace{1cm} (A2)

e.g. $0.2, 1.3$ (accept the single answer 0.2)

$P(B) = 0.4$ \hspace{1cm} A1 \hspace{1cm} N6

**METHOD 2**

for independence $P(A \cap B) = P(A) \times P(B)$ \hspace{1cm} (R1)

expression for $P(A \cap B)$, indicating $P(A) = \frac{1}{2} P(B)$ \hspace{1cm} (A1)

e.g. $P(B) \times \frac{1}{2} P(B) \times x \times \frac{1}{2} x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ \hspace{1cm} (M1)

correct substitution \hspace{1cm} A1

e.g. $0.52 = 0.5x + x - 0.5x^2$, $0.52 = 0.5P(B) + P(B) - 0.5P(B)P(B)$

correct solutions to the equation \hspace{1cm} (A2)

e.g. $0.4, 2.6$ (accept the single answer 0.4)

$P(B) = 0.4$ (accept $x = 0.4$ if $x$ set up as $P(B)$) \hspace{1cm} A1 \hspace{1cm} N6

18. (a) evidence of binomial distribution (may be seen in parts (b) or (c)) \hspace{1cm} (M1)

e.g. $np$, $100 \times 0.04$

mean = 4 \hspace{1cm} A1 \hspace{1cm} N2

(b) \hspace{1cm} $P(X = 6) = \binom{100}{6} (0.04)^6 (0.96)^{94}$ \hspace{1cm} (A1)

\hspace{2cm} = 0.105 \hspace{1cm} A1 \hspace{1cm} N2

(c) for evidence of appropriate approach \hspace{1cm} (M1)

e.g. complement, $1 - P(X = 0)$

$P(X = 0) = (0.96)^{100} = 0.01687...$ \hspace{1cm} (A1)

$P(X \geq 1) = 0.983$ \hspace{1cm} A1 \hspace{1cm} N2
19. (a) evidence of using binomial probability \[(M1)\]
\[e.g. \ P(X = 2) = \binom{7}{2} (0.18)^2 (0.82)^5\]
\[P(X = 2) = 0.252 \ \ A1 \ N2\]

(b) METHOD 1

evidence of using the complement \[(M1)\]
\[e.g. \ 1 - (P(X \leq 1))\]
\[P(X \leq 1) = 0.632 \ \ (A1)\]
\[P(X \geq 2) = 0.368 \ \ A1 \ N2\]

METHOD 2

evidence of attempting to sum probabilities \[(M1)\]
\[e.g. \ P(2 \text{ heads}) + P(3 \text{ heads}) + \ldots + P(7 \text{ heads}), 0.252 + 0.0923 + \ldots\]
correct values for each probability \[(A1)\]
\[e.g. \ 0.252 + 0.0923 + 0.0203 + 0.00267 + 0.0002 + 0.0000061\]
\[P(X \geq 2) = 0.368 \ \ A1 \ N2\]

20. (a) evidence of approach \[(M1)\]
\[e.g. \ \text{finding 0.84…}, \text{using } \frac{23.7 - 21}{\sigma}\]
correct working \[(A1)\]
\[e.g. \ 0.84... = \frac{23.7 - 21}{\sigma}, \text{graph}\]
\[\sigma = 3.21 \ \ A1 \ N2\]

(b) (i) evidence of attempting to find \(P(X < 25.4)\) \[(M1)\]
\[e.g. \ \text{using } z = 1.37\]
\[P(X < 25.4) = 0.915 \ \ A1 \ N2\]

(ii) evidence of recognizing symmetry \[(M1)\]
\[e.g. \ b = 21 - 4.4, \text{using } z = -1.37\]
\[b = 16.6 \ \ A1 \ N2\]

21. (a) \(X \sim B(100, 0.02)\)
\[E(X) = 100 \times 0.02 = 2 \ \ A1 \ N1\]
(b) \(P(X = 3) = \binom{100}{3}(0.02)^3(0.98)^{97}\) \hspace{1cm} (M1)
\[= 0.182\] \hspace{1cm} A1 N2

(c) **METHOD 1**
\[P(X > 1) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1))\] \hspace{1cm} M1
\[= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})\] \hspace{1cm} (M1)
\[= 0.597\] \hspace{1cm} A1 N2

**METHOD 2**
\[P(X > 1) = 1 - P(X \leq 1)\] \hspace{1cm} (M1)
\[= 1 - 0.40327\] \hspace{1cm} (A1)
\[= 0.597\] \hspace{1cm} A1 N2

*Note: Award marks as follows for finding \(P(X \geq 1)\), if working shown.*

\[P(X \geq 1)\] \hspace{1cm} A0
\[= 1 - P(X \leq 2) = 1 - 0.67668\] \hspace{1cm} M1(FT)
\[= 0.323\] \hspace{1cm} A1(FT) N0

22. (a) Using \(E(X) = \sum_0^2 xP(X = x)\) \hspace{1cm} (M1)

Substituting correctly \(E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}\) \hspace{1cm} A1
\[= 0.8\] \hspace{1cm} A1 N2
(b)  

(i) 

\[
\begin{align*}
\text{Note: Award A1 for each complementary pair of probabilities,} \\
i.e. \frac{4}{6} \text{ and } \frac{2}{5}, \frac{3}{5} \text{ and } \frac{2}{5}, \frac{4}{5} \text{ and } \frac{1}{5}.
\end{align*}
\]

(ii) \( P(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30} \)  

\[
P(Y = 1) = P(RG) + P(GR) = \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5}
\]

\[
= \frac{16}{30} = \frac{8}{15}
\]

\[
P(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30} = \frac{2}{5}
\]

For forming a distribution

\[
\begin{array}{c|c|c|c}
y & 0 & 1 & 2 \\
P(Y = y) & \frac{2}{30} & \frac{16}{30} & \frac{12}{30}
\end{array}
\]

(c) \( P(\text{Bag A}) = \frac{2}{6} \left( \frac{1}{3} \right) = \frac{1}{9} \)  

\( P(\text{Bag B}) = \frac{4}{6} \left( \frac{2}{3} \right) = \frac{4}{9} \)

For summing \( P(A \cap RR) \) and \( P(B \cap RR) \)

Substituting correctly \( P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30} = \frac{1}{30} + \frac{12}{30} = \frac{13}{30} \)

\(= 0.3 \)

(d) For recognising that \( P(1 \text{ or } 6 \mid RR) = P(A \mid RR) = \frac{P(A \cap RR)}{P(RR)} \)
\[ \frac{1}{30} \div \frac{27}{90} = 0.111 \quad \text{A1} \quad \text{N2} \]

23. (a) \( P(H < 153) = 0.705 \Rightarrow z = 0.538(836...) \) \quad \text{(A1)}

Standardizing \( \frac{153-\mu}{5} \) \quad \text{(A1)}

Setting up their equation \( 0.5388... = \frac{153-\mu}{5} \) \quad \text{M1}

\( \mu = 150.30... \)

\( = 150 \) (to 3sf) \quad \text{A1} \quad \text{N3}

(b) \( Z = \frac{153-\mu}{5} = 1.138... \) \quad \text{(accept 1.14 from} \mu = 150.3, \text{or 1.2 from} \mu = 150) \quad \text{(A1)}

\( P(Z > 1.138) = 0.128 \) \quad \text{(accept 0.127 from} z = 1.14, \text{or 0.115 from} z = 1.2) \quad \text{A1} \quad \text{N2} \]

24. (a) 0.0668 \quad \text{A2} \quad \text{N2}

(b) Using the standardized value 1.645 \quad \text{(A1)}

\( k = 26.1 \text{ kg} \) \quad \text{A1} \quad \text{N2}
Note: Award A1 for vertical line to right of the mean, A1 for shading to left of their vertical line.

25. (a) 

First die in pair

Second die in pair

\[
\begin{array}{c}
\text{four} \\
\frac{1}{6} \\
\frac{5}{6} \\
\frac{5}{6} \\
\frac{5}{6}
\end{array}
\]

\[
\begin{array}{c}
\text{not four} \\
\frac{1}{6} \\
\frac{5}{6} \\
\frac{5}{6} \\
\frac{5}{6}
\end{array}
\]

Note: Award A1 for each pair of complementary probabilities.

(b) \[P(E) = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \left(\frac{5}{36} + \frac{5}{36}\right)\] 

\[= \frac{10}{36} \left(\frac{5}{18} \text{ or } 0.278\right)\]  

A1 A1 A1 N3
(c) Evidence of recognizing the binomial distribution (M1)

\[
X \sim B(5, \frac{5}{18}) \text{ or } p = \frac{5}{18}, q = \frac{13}{18}
\]

\[
P(X = 3) = \binom{5}{3} \left( \frac{5}{18} \right)^3 \left( \frac{13}{18} \right)^2 \quad \text{(or other evidence of correct setup)} \quad \text{(A1)}
\]

\[
= 0.112 \quad \text{A1 N3}
\]

(d) METHOD 1

Evidence of using the complement M1

\[
eg \text{ eg } P(X \geq 3) = 1 - P(X \leq 2)
\]

Correct value \(1 - 0.865\) (A1)

\[
= 0.135 \quad \text{A1 N2}
\]

METHOD 2

Evidence of adding correct probabilities M1

\[
eg \text{ eg } P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)
\]

Correct values \(0.1118 + 0.02150 + 0.001654\) (A1)

\[
= 0.135 \quad \text{A1 N2}
\]

[12]

26. (a) \(P(F \cup S) = 1 - 0.14 (= 0.86)\) (A1)

Choosing an appropriate formula (M1)

\[
eg \text{ eg } P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

Correct substitution

\[
eg \text{ eg } P(F \cap S) = 0.93 - 0.86 \quad \text{A1}
\]

\[
P(F \cap S) = 0.07 \quad \text{AG N0}
\]

Notes: There are several valid approaches. Award (A1)(M1)A1 for relevant working using any appropriate strategy eg formula, Venn Diagram, or table.

Award no marks for the incorrect solution

\[
P(F \cap S) = 1 - P(F) + P(S) = 1 - 0.93 = 0.07
\]
(b) Using conditional probability

\[ P(F \mid S) = \frac{P(F \cap S)}{P(S)} \]

\[ P(F \mid S) = \frac{0.07}{0.62} = 0.113 \quad \text{(A1)} \]

(c) \( F \) and \( S \) are not independent

**EITHER**

If independent \( P(F \mid S) = P(F) \), 0.113 ≠ 0.31

**OR**

If independent \( P(F \cap S) = P(F) \cdot P(S) \), 0.07 ≠ 0.31 \times 0.62 (≈ 0.1922)

(d) Let \( P(F) = x \)

\[ P(S) = 2P(F) (= 2x) \quad \text{(A1)} \]

For independence \( P(F \cap S) = P(F) \cdot P(S) (= 2x^2) \)

Attempt to set up a quadratic equation

\[ \text{eg } P(F \cup S) = P(F)P(S) - P(F)P(S), 0.86 = x + 2x - 2x^2 \]

\[ 2x^2 - 3x + 0.86 = 0 \]

\[ x = 0.386, x = 1.11 \quad \text{(A1)} \]

\[ P(F) = 0.386 \quad \text{(A1) N5} \]
27. **Note:** Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.

\[ W \sim N(2.5, 0.3^2) \]

(a) (i) \( z = -1.67 \) (accept 1.67) \hfill (A1)

\[ P(W < 2) = 0.0478 \] (accept answers between 0.0475 and 0.0485) \hfill A1 N2

(ii) \( z = 1 \) \hfill (A1)

\[ P(W > 2.8) = 0.159 \] \hfill A1 N2

(iii)

\[ \text{2.5 kg} \]

\[ \text{Note: Award A1 for a vertical line to left of mean and shading to left, A1 for vertical line to right of mean and shading to right.} \]

(iv) Evidence of appropriate calculation \hfill M1

eg 1 \(- (0.047790 + 0.15866), 0.8413 - 0.0478\n
\[ P = 0.7936 \] \hfill AG N0

**Note:** The final value may vary depending on what level of accuracy is used.

Accept their value in subsequent parts.
(b) (i) \( X \sim B(10, 0.7935...) \)

Evidence of calculation

\( eg \) \( P(X = 10) = (0.7935...)^{10} \)

\( P(X = 10) = 0.0990 \) (3 sf)  

M1

(ii) **METHOD 1**

Recognizing \( X \sim B(10, 0.7935...) \) (may be seen in (i))  

\( P(X \leq 6) = 0.1325... \) (or \( P(X = 1) + ... + P(X = 6) \))  

(A1)

Evidence of using the complement  

\( eg \) \( P(X \geq 7) = 1 - P(X \leq 6), P(X \geq 7) = 1 - P(X < 7) \)

\( P(X \geq 7) = 0.867 \)  

A1 N3

**METHOD 2**

Recognizing \( X \sim B(10, 0.7935...) \) (may be seen in (i))  

For adding terms from \( P(X = 7) \) to \( P(X = 10) \)  

\( P(X \geq 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030 \)  

(A1)

\( = 0.867 \)  

A1 N3

---

28. (a) \( z = \frac{180 - 160}{20} = 1 \)  

(A1)

\( \phi(1) = 0.8413 \)  

(A1)

\( P(\text{height} > 180) = 1 - 0.8413 \)  

\( = 0.159 \)  

A1 N3

(b) \( z = -1.1800 \)  

(A1)

Setting up equation \(-1.18 = \frac{d - 160}{20} \)  

(M1)

\( d = 136 \)  

A1 N3
Notes: Accept any suitable notation, as long as the candidate’s intentions are clear.
The following symbols will be used in the markscheme.

Girls’ height $G \sim N(155, 10^2)$, boys’ height $B \sim N(160, 12^2)$

Height $H$, Female $F$, Male $M$.

(a) $P(G > 170) = 1 - P(G < 170)$ (A1)

$$P(G > 170) = P \left( Z < \frac{170 - 155}{10} \right)$$ (A1)

$$P(G > 170) = 1 - \Phi (1.5) = 1 - 0.9332 = 0.0668$$ A1 N3

(b) $z = -1.2816$ (A1)

Correct calculation (eg $x = 155 + (-1.282 \times 10)$) (A1)

$x = 142$ A1 N3

(c) Calculating one variable (A1)

eg $P(B < r) = 0.95$, $z = 1.6449$

$r = 160 + 1.645(12) = 179.74$

$= 180$ A1 N2

Any valid calculation for the second variable, including use of symmetry (A1)

eg $P(B < q) = 0.05$, $z = -1.6449$

$q = 160 - 1.645(12) = 140.26$

$= 140$ A1 N2

Note: Symbols are not required in parts (d) and (e).

(d) $P(M \cap (B > 170)) = 0.4 \times 0.2020$, $P(F \cap (G > 170)) = 0.6 \times 0.0668$ (A1)(A1)

$$P(H > 170) = 0.0808 + 0.04008$$ A1

$= 0.12088 = 0.121$ (3 sf) A1 N2
(e) \[ P(F \mid H > 170) = \frac{P(F \cap (H > 170))}{P(H > 170)} \]  

\[ = \frac{0.60 \times 0.0668}{0.121} = \left( \frac{0.0401}{0.121} \right) \text{ or } \left( \frac{0.04008}{0.1208} \right) \]  

\[ = 0.332 \]  

A1 N1

[17]

30. **METHOD 1 Use of the GDC**

(a) Evidence of using the binomial facility, M1

that is set up with \( P = \frac{1}{2} \) and \( n = 5 \).

\[ P(X = 3) = 0.3125 \]  

\[ = \binom{5}{3} \left( \frac{1}{2} \right)^3 \left( \frac{1}{2} \right)^2 \]  

\[ = 0.3125 \]  

A2 N2

(b) Evidence of set up, with \( 1 - P(X = 0) \) M1

\[ = 0.969 \left( \frac{31}{32} \right) \]  

A2 N2

**METHOD 2 Use of the formula**

(a) Evidence of binomial formula (M1)

\[ P(X = 3) = \binom{5}{3} \left( \frac{1}{2} \right)^3 \left( \frac{1}{2} \right)^2 \]  

\[ = \frac{5}{16} (= 0.313) \]  

A1 N2
(b) **METHOD 1**

\[ P(\text{at least one head}) = 1 - P(X = 0) \]  
\[ = 1 - \left( \frac{1}{2} \right)^5 \]  
\[ = \frac{31}{32} = 0.969 \]  

**METHOD 2**

\[ P(\text{at least one head}) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \]  
\[ = 0.15625 + 0.3125 + 0.3125 + 0.15625 + 0.03125 \]  
\[ = 0.969 \]

31. \( X \sim N(\mu, \sigma^2), P(X < 3) = 0.2, P(X > 8) = 0.1 \)

\[ P(X < 8) = 0.9 \]  

Attempt to set up equations

\[ \frac{3 - \mu}{\sigma} = -0.8416, \quad \frac{8 - \mu}{\sigma} = 1.282 \]  

\[ 3 - \mu = -0.8416\sigma \]  

\[ 8 - \mu = 1.282\sigma \]  

\[ 5 = 2.1236\sigma \]  

\[ \sigma = 2.35, \quad \mu = 4.99 \]  

32. (a) \( X \sim B(100, 0.02) \)

\[ E(X) = 100 \times 0.02 = 2 \]  

(b) \( P(X = 3) = \binom{100}{3}(0.02)^3(0.98)^{97} \)  
\[ = 0.182 \]  

(c) **METHOD 1**

\[ P(X > 1) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1)) \]  
\[ = 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99}) \]  
\[ = 0.597 \]  

**METHOD 2**
P(X > 1) = 1 – P(X ≤ 1) (M1)
= 1 – 0.40327 (A1)
= 0.597 A1 2

Note: Award marks as follows for finding P(X > 1), if working shown.

P(X ≥ 1)
= 1 – P(X < 2) = 1 – 0.67668 M1(ft)
= 0.323 A1(ft) 2

33. \(X \sim N(\mu, \sigma^2)\), P(X > 90) = 0.15 and P(X < 40) = 0.12 (M1)
Finding standardized values 1.036, –1.175 A1A1
Setting up the equations 1.036 = \(\frac{90 - \mu}{\sigma}\), –1.175 = \(\frac{40 - \mu}{\sigma}\) (M1)
\(\mu = 66.6, \quad \sigma = 22.6\) A1A1

34. (i) P(X > 3 200) = P(Z > 0.4) (M1)
= 1 – 0.6554 = 34.5% (= 0.345) (A1) (N2)

(ii) P(2 300 < X < 3 300) = P(–1.4 < Z < 0.6) (M1)
= 0.4192 + 0.2257
= 0.645 (A1)
P(both) = (0.645)^2 = 0.416 (A1) (N2)

(iii) 0.7422 = P(Z < 0.65) (A1)
\(\frac{d - 3 000}{500} = 0.65\) (A1)
d = $3 325 (= $3 330 to 3 s.f.) (Accept $3325.07) (A1) (N3)

35. (a) \(z = \frac{185 - 170}{20} = 0.75\) (M1)(A1)
P(Z < 0.75) = 0.773 (A1) (N3)

(b) \(z = -0.47\) (may be implied) (A1)
\(-0.47 = \frac{d - 170}{20}\) (M1)
d = 161 (A1) (N3)
36. (a) (i) $a = -1$  
$b = 0.5$  
(ii) (a) $0.841$  
(b) $0.6915 - 0.1587$ (or $0.8413 - 0.3085$)  
$= 0.533$ (3 sf)  
(A1) (M1) (N2)  
6
(b) (i) Sketch of normal curve  
(A1)(A1)
(ii) $c = 0.647$  
(A2)  
4

37. **Method 1**

\[ b^2 - 4ac = 9 - 4k \]  
\[ 9 - 4k > 0 \]  
\[ 2.25 > k \]  
(crosses the x-axis if $k = 1$ or $k = 2$)  
(A1)(A1)

\[ \text{probability} = \frac{2}{7} \]  
(A1) (C6)
Method 2

\[ \begin{align*}
\text{Note: } & \text{ Award (M2) for one (relevant) curve;} \\
& \text{(M1) for a second one.}
\end{align*} \]

\[ k = 1 \text{ or } k = 2 \]

probability = \( \frac{2}{7} \)

38. \( X \sim N(80, 8^2) \)

(a) \( P(X < 72) = P(Z < -1) \)

\[ = 1 - 0.8413 \]
\[ = 0.159 \]

\( P(X < 72) = 0.159 \)  \( \text{(G2) } 2 \)

(b) (i) \( P(72 < X < 90) = P(-1 < Z < 1.25) \)

\[ = 0.3413 + 0.3944 \]
\[ = 0.736 \]

\( P(72 < X < 90) = 0.736 \)  \( \text{(G3)} \)
(ii)  

Note: Award (A1) for a normal curve and (A1) for the shaded area, which should not be symmetrical.

(c) 4% fail in less than $x$ months

\[ x = 80 - 8 \times \Phi^{-1}(0.96) \]
\[ = 80 - 8 \times 1.751 \]  
\[ = 66.0 \text{ months} \]  

OR

\[ x = 66.0 \text{ months} \]

[10]

39. (a) \( P(M \geq 350) = 1 - P(M < 350) = 1 - P \left( Z < \frac{350 - 310}{30} \right) \)

\[ = 1 - P(Z < 1.333) = 1 - 0.9088 \]
\[ = 0.0912 \text{ (accept 0.0910 to 0.0920)} \]

OR

\[ P(M \geq 350) = 0.0912 \]

(G2)
40. (a) (These answers may be obtained from a calculator or by finding \( z \) in each case and the corresponding area.)

\( M \sim N(750, 625) \)

(i) \( P(M < 740) = 0.345 \)  

\[ \text{OR} \]
\[ z = -0.4 \quad P(z < -0.4) = 0.345 \]  

(ii) \( P(M > 780) = 0.115 \)  

\[ \text{OR} \]
\[ z = 1.2 \quad P(z > 1.2) = 1 - 0.885 = 0.115 \]

(iii) \( P(740 < M < 780) = 0.540 \)  

\[ \text{OR} \]
\[ 1 - (0.345 + 0.115) = 0.540 \]

(b) Independent events

Therefore, \( P(\text{both} < 740) = 0.345^2 \)  

\[ = 0.119 \]

(c) 70% have mass < 763 g

Therefore, 70% have mass of at least 750 - 13

\[ x = 737 \text{ g} \]
41. \textbf{Note: Where accuracy is not specified, accept answers with greater than 3 sf accuracy, provided they are correct as far as 3 sf}

(a) \[ z = \frac{197 - 187.5}{9.5} = 1.00 \] (M1)

\[ P(Z > 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587 \]

\[ = 0.159 \text{ (3 sf)} \] (A1)

\[ = 15.9\% \] (A1)

\textbf{OR}

\[ P(H > 197) = 0.159 \] (G2)

\[ = 15.9\% \] (A1)

(b) Finding the 99\textsuperscript{th} percentile

\[ \Phi(a) = 0.99 \Rightarrow a = 2.327 \text{ (accept 2.33)} \] (A1)

\[ \Rightarrow 99\% \text{ of heights under } 187.5 + 2.327(9.5) = 209.6065 \]

\[ = 210 \text{ (3 sf)} \] (A1)

\textbf{OR}

99\% of heights under 209.6 = 210 cm (3 sf) (G3)

Height of standard doorway = 210 + 17 = 227 cm (A1)

4

42. (a) Let \( X \) be the random variable for the IQ.

\( X \sim N(100, 225) \)

\[ P(90 < X < 125) = P(-0.67 < Z < 1.67) \] (M1)

\[ = 0.701 \]

70.1 percent of the population (accept 70 percent). (A1)

\textbf{OR}

\[ P(90 < X < 125) = 70.1\% \] (G2)

2

(b) \[ P(X \geq 125) = 0.0475 \text{ (or 0.0478)} \] (M1)

\[ P(\text{both persons having IQ } \geq 125) = (0.0475)^2 \text{ (or } (0.0478)^2) \] (M1)

\[ = 0.00226 \text{ (or 0.00228)} \] (A1)

3
(c) Null hypothesis (H₀): mean IQ of people with disorder is 100

Alternative hypothesis (H₁): mean IQ of people with disorder is less than 100

\[
P(\bar{X} < 95.2) = P\left(Z < \frac{95.2 - 100}{\frac{15}{\sqrt{25}}}\right) = P(Z < -1.6) = 1 - 0.9452
\]

\[
= 0.0548
\]

The probability that the sample mean is 95.2 and the null hypothesis true is 0.0548 > 0.05. Hence the evidence is not sufficient.

43. (a) \[
Z = \frac{25 - 25.7}{0.50} = -1.4
\]

\[
P(Z < -1.4) = 1 - P(Z < 1.4)
\]

\[
= 1 - 0.9192
\]

\[
= 0.0808
\]

OR

\[
P(W < 25) = 0.0808
\]

(b) \[
P(Z < -a) = 0.025 \Rightarrow P(Z < a) = 0.975
\]

\[
\Rightarrow a = 1.960
\]

\[
\frac{25 - \mu}{0.50} = -1.96 \Rightarrow \mu = 25 + 1.96 (0.50)
\]

\[
= 25 + 0.98 = 25.98
\]

\[
= 26.0 (3 \text{ sf})
\]

OR

\[
\frac{25.0 - 26.0}{0.50} = -2.00
\]

\[
P(Z < -2.00) = 1 - P(Z < 2.00)
\]

\[
= 1 - 0.9772 = 0.0228
\]

\[
\approx 0.025
\]

OR

\[
\mu = 25.98
\]

\[
\Rightarrow \text{mean} = 26.0 (3 \text{ sf})
\]
(c) Clearly, by symmetry \( \mu = 25.5 \)  

\[
Z = \frac{25.0 - 25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma
\]

\[\Rightarrow \sigma = 0.255 \text{ kg}\]  

(A1)  

(M1)  

(A1)  

(d) On average, \( \frac{\text{cement saving}}{\text{bag}} = 0.5 \) kg  

\[
\frac{\text{cost saving}}{\text{bag}} = 0.5(0.80) = $0.40
\]

To save $5000 takes  

\[
\frac{5000}{0.40} = 12500 \text{ bags}
\]

(A1)  

On average,  

\[
\text{bag saving = 0.5 kg}
\]

(A1)  

\[
\text{bag saving cost} = 0.5(0.80) = $0.40
\]

(M1)  

(A1)  

(3)

44.  

(a) Let \( X \) be the lifespan in hours

\[X \sim N(57, 4.4^2)\]

(i)  

\( a = -0.455 \) (3 sf)  

\( b = 0.682 \) (3 sf)  

(A1)  

(A1)

(ii)  

(a) \( P(X > 55) = P(Z > -0.455) = 0.675 \)

(A1)

(b) \( P(55 \leq X \leq 60) = P\left(\frac{2}{4.4} \leq Z \leq \frac{3}{4.4}\right)\)

\[\approx P(0.455 \leq Z \leq 0.682)\]

\[\approx 0.6754 + 0.752 - 1\]

\[= 0.428 \text{ (3sf)}\]  

(A1)

OR

\( P(55 \leq X \leq 60) = 0.428 \) (3 sf)  

(G2)  

5
(b) 90% have died \(\Rightarrow\) shaded area = 0.9  

![Graph showing shaded area]

\[Hence \quad t = 57 + (4.4 \times 1.282)\]
\[= 57 + 5.64\]
\[= 62.6 \text{ hours}\]

OR \(t = 62.6 \text{ hours}\)

45. (a) \textbf{Note: Candidates using tables may get slightly different answers, especially if they do not interpolate. Accept these answers.}

\[P(\text{speed} > 50) = 0.3 = 1 - \Phi\left(\frac{50 - \mu}{10}\right)\]

Hence, \(\frac{50 - \mu}{10} = \Phi^{-1}(0.7)\)

\[\mu = 50 - 10\Phi^{-1}(0.7)\]
\[= 44.7599 \ldots \approx 44.8 \text{ km/h (3 sf) (accept 44.7)}\]

(b) \(H_1:\) “the mean speed has been reduced by the campaign”.

(c) One-tailed; because \(H_1\) involves only “<”.

(d) For a one-tailed test at 5% level, critical region is \(Z < \mu_m - 1.64\sigma_m\) (accept \(-1.65\sigma_m\))

Now, \(\mu_m = \mu = 44.75\ldots\); \(\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2\) (allow ft)

So test statistic is \(44.75\ldots - 1.64 \times 2 = 41.47\)

Now 41.3 < 41.47 so reject \(H_0\), yes.

46. (a) Area \(A = 0.1\)
(b) EITHER Since \( p(X \geq 12) = p(X \leq 8) \), then 8 and 12 are symmetrically disposed around the mean.

Thus mean = \( \frac{8 + 12}{2} = 10 \) (M1) (A1)

Notes: If a candidate says simply “by symmetry \( \mu = 10 \)” with no further explanation award [3 marks] (M1, A1, R1). As a full explanation is requested award an additional (A1) for saying since \( p(X < 8) = p(X > 12) \) and another (A1) for saying that the normal curve is symmetric.

OR \( p(X \geq 12) = 0.1 \Rightarrow p \left( Z \geq \frac{12 - \mu}{\sigma} \right) = 0.1 \) (M1)

\[ \Rightarrow p \left( Z \leq \frac{12 - \mu}{\sigma} \right) = 0.9 \]

\( p(X \leq 8) = 0.1 \Rightarrow p \left( Z \leq \frac{8 - \mu}{\sigma} \right) = 0.1 \)

\[ \Rightarrow p \left( Z \leq \frac{\mu - 8}{\sigma} \right) = 0.9 \] (A1)

So \( \frac{12 - \mu}{\sigma} = \frac{\mu - 8}{\sigma} \) (M1)

\[ \Rightarrow 12 - \mu = \mu - 8 \] (M1)

\[ \Rightarrow \mu = 10 \] (A1)

(c) \( \Phi \left( \frac{12 - 10}{\sigma} \right) = 0.9 \) (A1)(M1)(A1)

Note: Award (A1) for \( \frac{12 - 10}{\sigma} \), (M1) for standardizing, and (A1) for 0.9.

\[ \Rightarrow \frac{2}{\sigma} = 1.282 \text{ (or 1.28)} \] (A1)

\[ \sigma = \frac{2}{1.282} \left( \text{or } \frac{2}{1.28} \right) \] (A1)

= 1.56 (3 sf) (AG)

Note: Working backwards from \( \sigma = 1.56 \) to show it leads the given data should receive a maximum of [3 marks] if done correctly.
(d) \[ p(X \leq 11) = \Phi\left(\frac{11-10}{1.561}\right) \quad \text{(or 1.56)} \] (M1)(A1)

Note: Award (M1) for standardizing and (A1) for \( \Phi\left(\frac{11-10}{1.561}\right) \).

\[ = p(Z \leq 0.6407) \quad \text{(or 0.641 or 0.64)} \] (A1)
\[ = \Phi(0.6407) \] (M1)
\[ = 0.739 \quad \text{(3 sf)} \] (A1) 5

47. (a) \[ p(4 \text{ heads}) = \binom{8}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{8-4} \] (M1)
\[ = \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \times \left(\frac{1}{2}\right)^8 \]
\[ = \frac{70}{256} \approx 0.273 \quad \text{(3 sf)} \] (A1) 2

(b) \[ p(3 \text{ heads}) = \binom{8}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{8-3} = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \left(\frac{1}{2}\right)^8 \]
\[ = \frac{56}{256} \approx 0.219 \quad \text{(3 sf)} \] (A1) 1

(c) \[ p(5 \text{ heads}) = p(3 \text{ heads}) \text{ (by symmetry)} \] (M1)
\[ p(3 \text{ or 4 or 5 heads}) = p(4) + 2p(3) \] (M1)
\[ = \frac{70 + 2 \times 56}{256} = \frac{182}{256} \]
\[ = 0.711 \quad \text{(3 sf)} \] (A1) 3

[6]