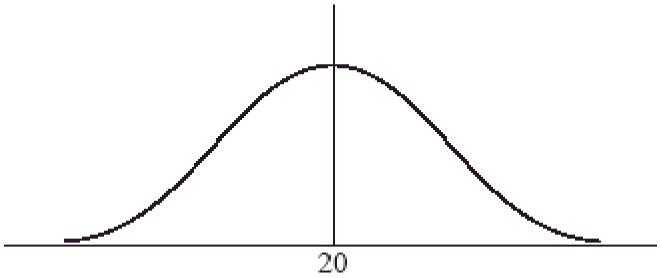
1. A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $\mathrm{P}(X \leq 24.5)$.

(3)

(b) Let $\mathrm{P}(X \leq k) = 0.85$.

(i) Represent this information on the following diagram.



(ii) Find the value of k.

(5)
(Total 8 marks)

2. A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box.

(2)

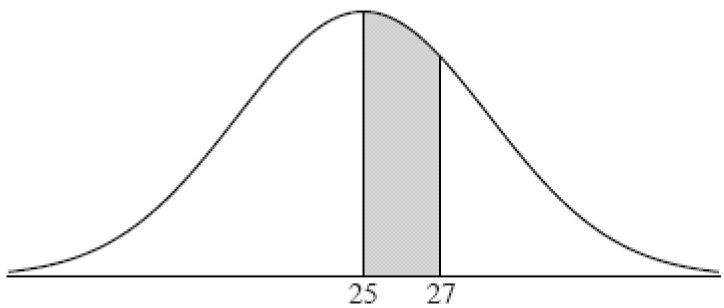
(b) Find the probability that there are 15 brown eggs in the box.

(2)

(c) Find the probability that there are at least 10 brown eggs in the box.

(3)
(Total 7 marks)

3. Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

(a) Find P(X > 27).

(2)

(b) Find the standard deviation of X.

(5)
(Total 7 marks)

4. Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games.

(2)

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games.

(2)

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15.

(3)
(Total 7 marks)

5. The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg.

(2)

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights.

(4)

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

(5)

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.

(4)
(Total 15 marks)

6. A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $P(X \leq 475)$.

(2)

(b) Given that $P(X > a) = 0.27$, find a.

(4)
(Total 6 marks)

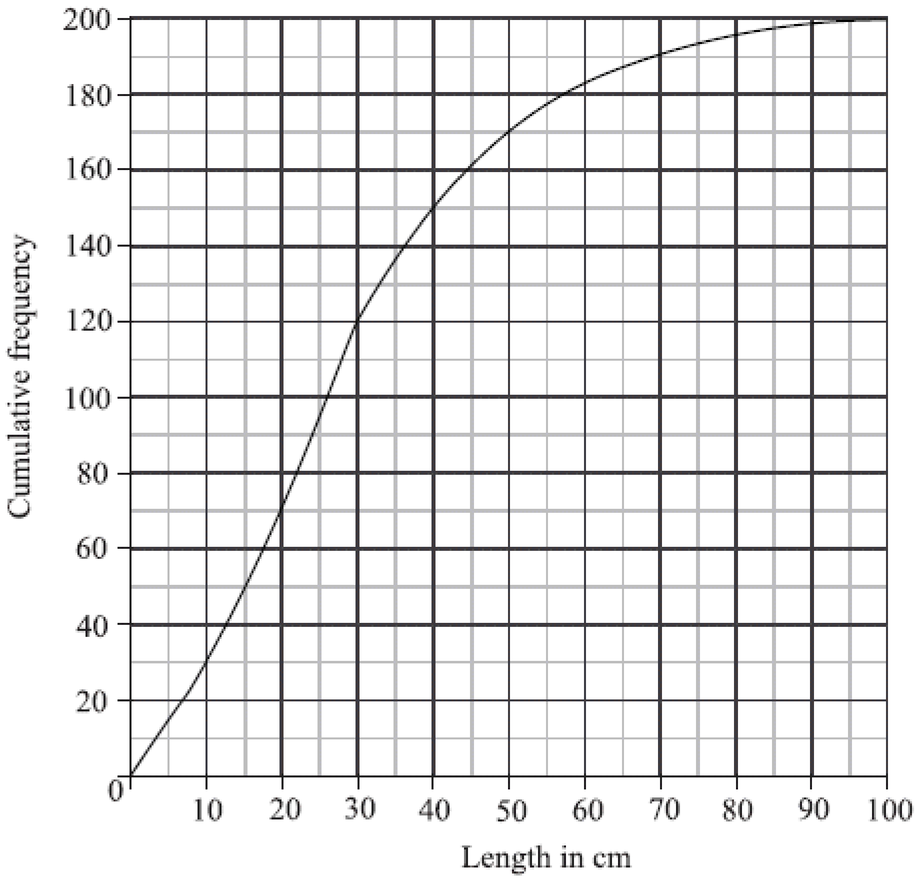
7. A fisherman catches 200 fish to sell. He measures the lengths, l cm of these fish, and the results are shown in the frequency table below.

Length l cm	$0 \le l < 10$	$10 \le l < 20$	$20 \le l < 30$	$30 \le l < 40$	$40 \le l < 60$	$60 \le l < 75$	$75 \le l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish.

(3)

(b) A cumulative frequency diagram is given below for the lengths of the fish.



Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 40 % of the fish have a length more than k cm, find the value of k.

(6)

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.
Medium fish have a length greater than or equal to 20 cm but less than 60 cm.
Large fish have a length greater than or equal to 60 cm.

(c) Write down the probability that a fish is small.

(2)

The cost of a small fish is \$4, a medium fish \$10, and a large fish \$12.

(d) Copy and complete the following table, which gives a probability distribution for the cost \$$X$.

Cost \$*X*	4	10	12
$\mathrm{P}(X = x)$		0.565	

(2)

(e) Find $\mathrm{E}(X)$.

(2)
(Total 15 marks)

8. A multiple choice test consists of ten questions. Each question has five answers.
Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a) Find the expected number of questions Jose answers correctly.

(1)

(b) Find the probability that Jose answers exactly three questions correctly.

(2)

(c) Find the probability that Jose answers more than three questions correctly.

(3)
(Total 6 marks)

9. A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

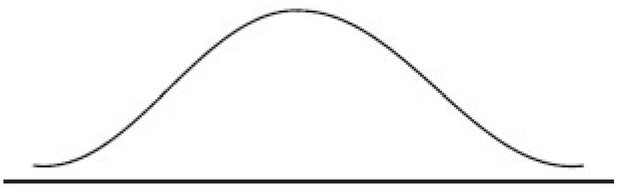
(i) weighs less than 8 g;

(ii) weighs between 6 g and 8 g.

(4)

(b) Five percent of the biscuits in the box weigh less than d grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



(ii) Find the value of d.

(5)

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weigh less than 5 g.

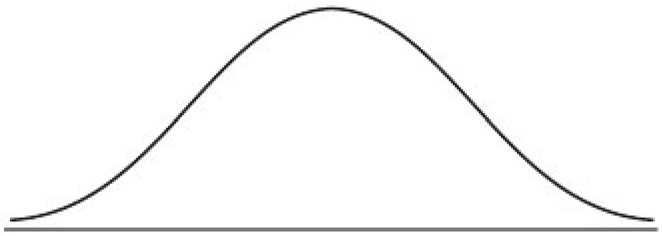
Find the value of μ.

(4)
(Total 13 marks)

10. The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92% are in category A.
The tallest 10.38% are in category C.
All the other plants are in category B with heights between *r* cm and *t* cm.

(a) Complete the following diagram to represent this information.



(2)

(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of *r* and of *t*.

(5)
(Total 7 marks)

11. The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than 90 km h^{-1} and 12 % of them at speeds less than 40 km h^{-1}. Find μ and σ.

(Total 6 marks)

12. A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection.

(3)

(b) The company would like the probability that a box passes inspection to be 0.87. Find the percentage of boxes that should be made by machine B to achieve this.

(4)
(Total 7 marks)

13. Two fair 4-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

(a) List the pairs of scores that give a sum of 6.

(3)

The probability distribution for the sum of the scores on the two dice is shown below.

Sum	2	3	4	5	6	7	8
Probability	p	q	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	r	$\frac{1}{16}$

(b) Find the value of p, of q, and of r.

(3)

Fred plays a game. He throws two fair 4-sided dice four times. He wins a prize if the sum is 5 on three or more throws.

(c) Find the probability that Fred wins a prize.

(6)
(Total 12 marks)

14. Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

(a) Jan tosses the two dice once. Find the probability that she wins a prize.

(3)

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes.

(2)
(Total 5 marks)

15. A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.

(a) (i) Find $E(X)$.

(ii) Find the probability that Mark passes the test.

(6)

Bill also takes the test. Let Y be the number of questions that Bill answers correctly.
The following table is the probability distribution for Y.

y	0	1	2	3	4	5
P($Y = y$)	0.67	0.05	$a + 2b$	$a - b$	$2a + b$	0.04

(b) (i) Show that $4a + 2b = 0.24$.

(ii) Given that $E(Y) = 1$, find a and b.

(8)

(c) Find which student is more likely to pass the test.

(3)
(Total 17 marks)

16. A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.

(a) For Route A, find the probability that the journey takes **more** than 60 minutes.

(2)

(b) For Route B, the probability that the journey takes **less** than 60 minutes is 0.85. Find the value of μ.

(3)

(c) The van sets out at 06:00 and needs to arrive before 07:00.

(i) Which route should it take?

(ii) Justify your answer.

(3)

(d) On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that

(i) it arrives before 07:00 on all five days;

(ii) it arrives before 07:00 on at least three days.

(5)
(Total 13 marks)

17. Consider the independent events A and B. Given that $\mathrm{P}(B) = 2\mathrm{P}(A)$, and $\mathrm{P}(A \cup B) = 0.52$, find $\mathrm{P}(B)$.

(Total 7 marks)

18. A factory makes switches. The probability that a switch is defective is 0.04.
The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample.

(2)

(b) Find the probability that there are exactly six defective switches in the sample.

(2)

(c) Find the probability that there is at least one defective switch in the sample.

(3)
(Total 7 marks)

19. The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads.

(2)

(b) Find the probability of obtaining **at least** two heads.

(3)
(Total 5 marks)

20. The scores of a test given to students are normally distributed with a mean of 21.
80 % of the students have scores less than 23.7.

(a) Find the standard deviation of the scores.

(3)

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than b.

(b) (i) Find the probability the student has a score less than 25.4.

(ii) Find the value of b.

(4)
(Total 7 marks)

21. A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample.

(1)

(b) Find the probability that three calculators are faulty.

(2)

(c) Find the probability that more than one calculator is faulty.

(3)
(Total 6 marks)

22. Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X.

X	0	1	2
$\mathrm{P}(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate $\mathrm{E}(X)$, the mean number of red balls chosen.

(3)

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for Y, where Y is the number of red balls chosen.

(8)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(5)

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die.

(3)
(Total 19 marks)

23. The heights of boys at a particular school follow a normal distribution with a standard deviation of 5 cm. The probability of a boy being shorter than 153 cm is 0.705.

(a) Calculate the mean height of the boys.

(b) Find the probability of a boy being taller than 156 cm.

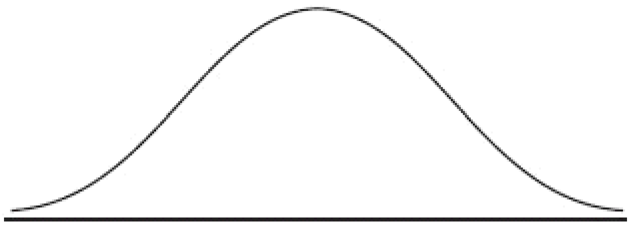
(Total 6 marks)

24. The weights of a group of children are normally distributed with a mean of 22.5 kg and a standard deviation of 2.2 kg.

(a) Write down the probability that a child selected at random has a weight more than 25.8 kg.

(b) Of the group 95% weigh less than k kilograms. Find the value of *k*.

(c) The diagram below shows a normal curve.



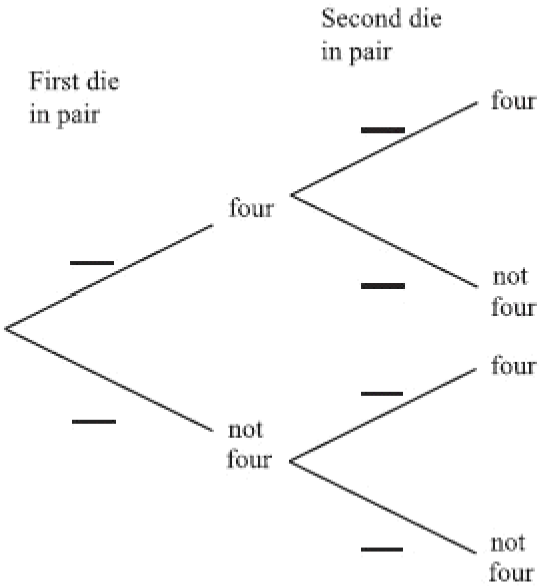
On the diagram, shade the region that represents the following information:

87% of the children weigh less than 25 kg

(Total 6 marks)

25. A pair of fair dice is thrown.

(a) Copy and complete the tree diagram below, which shows the possible outcomes.



(3)

Let E be the event that **exactly** one four occurs when the pair of dice is thrown.

(b) Calculate P(E).

(3)

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs **exactly** three times in the five throws.

(3)

(d) Calculate the probability that event E occurs **at least** three times in the five throws.

(3)
(Total 12 marks)

26. Two restaurants, *Center* and *New*, sell fish rolls and salads.

Let F be the event a customer chooses a fish roll.
Let S be the event a customer chooses a salad.
Let N be the event a customer chooses neither a fish roll nor a salad.

In the *Center* restaurant $P(F) = 0.31$, $P(S) = 0.62$, $P(N) = 0.14$.

(a) Show that $P(F \cap S) = 0.07$.

(3)

(b) Given that a customer chooses a salad, find the probability the customer also chooses a fish roll.

(3)

(c) Are F and S independent events? Justify your answer.

(3)

At *New* restaurant, $P(N) = 0.14$. Twice as many customers choose a salad as choose a fish roll. Choosing a fish roll is **independent** of choosing a salad.

(d) Find the probability that a fish roll is chosen.

(7)
(Total 16 marks)

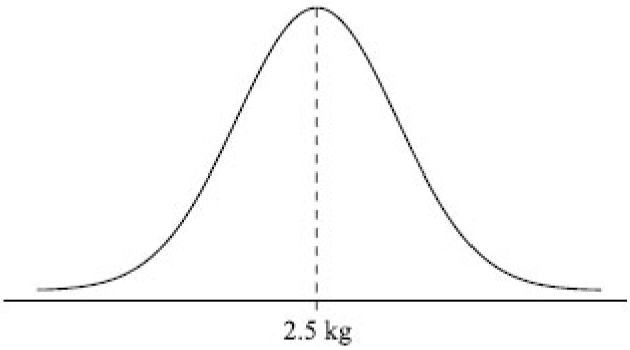
27. The weights of chickens for sale in a shop are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

(a) A chicken is chosen at random.

(i) Find the probability that it weighs less than 2 kg.

(ii) Find the probability that it weighs more than 2.8 kg.

(iii) Copy the diagram below. Shade the areas that represent the probabilities from parts (i) and (ii).



(iv) **Hence** show that the probability that it weighs between 2 kg and 2.8 kg is 0.7936 (to four significant figures).

(7)

(b) A customer buys 10 chickens.

(i) Find the probability that all 10 chickens weigh between 2 kg and 2.8 kg.

(ii) Find the probability that at least 7 of the chickens weigh between 2 kg and 2.8 kg.

(6)
(Total 13 marks)

28. The heights of a group of students are normally distributed with a mean of 160 cm and a standard deviation of 20 cm.

(a) A student is chosen at random. Find the probability that the student's height is greater than 180 cm.

(b) In this group of students, 11.9% have heights less than d cm. Find the value of d.

(Total 6 marks)

29. In a large school, the heights of all fourteen-year-old students are measured.

The heights of the girls are normally distributed with mean 155 cm and standard deviation 10 cm.

The heights of the boys are normally distributed with mean 160 cm and standard deviation 12 cm.

(a) Find the probability that a girl is taller than 170 cm.

(3)

(b) Given that 10% of the girls are shorter than x cm, find x.

(3)

(c) Given that 90% of the boys have heights between q cm and r cm where q and r are symmetrical about 160 cm, and $q < r$, find the value of q and of r.

(4)

In the group of fourteen-year-old students, 60% are girls and 40% are boys.
The probability that a girl is taller than 170 cm was found in part (a).
The probability that a boy is taller than 170 cm is 0.202.

A fourteen-year-old student is selected at random.

(d) Calculate the probability that the student is taller than 170 cm.

(4)

(e) Given that the student is taller than 170 cm, what is the probability the student is a girl?

(3)
(Total 17 marks)

30. A fair coin is tossed five times. Calculate the probability of obtaining

(a) exactly three heads;

(b) at least one head.

(Total 6 marks)

31. The heights of certain flowers follow a normal distribution. It is known that 20% of these flowers have a height less than 3 cm and 10% have a height greater than 8 cm.

Find the value of the mean μ and the standard deviation σ.

(Total 6 marks)

32. A factory makes calculators. Over a long period, 2% of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample.

(b) Find the probability that three calculators are faulty.

(c) Find the probability that more than one calculator is faulty.

(Total 6 marks)

33. The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15% of the cars travelled at speeds greater than 90 km h^{-1} and 12% of them at speeds less than 40 km h^{-1}. Find μ and σ.

(Total 6 marks)

34. Residents of a small town have savings which are normally distributed with a mean of \$3000 and a standard deviation of \$500.

(i) What percentage of townspeople have savings greater than \$3200?

(ii) Two townspeople are chosen at random. What is the probability that **both** of them have savings between \$2300 and \$3300?

(iii) The percentage of townspeople with savings less than d dollars is 74.22%. Find the value of d.

(Total 8 marks)

35. The heights, H, of the people in a certain town are normally distributed with mean 170 cm and standard deviation 20 cm.

(a) A person is selected at random. Find the probability that his height is less than 185 cm.

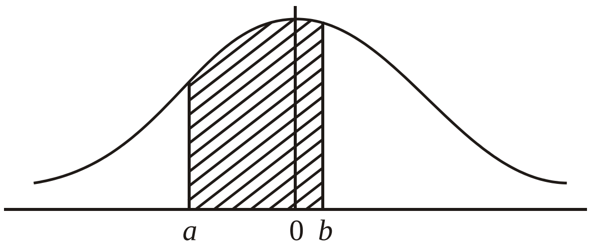
(3)

(b) Given that P ($H > d$) = 0.6808, find the value of d.

(3)
(Total 6 marks)

36. Reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds.

(a) The graph below is that of the **standard** normal curve. The shaded area represents the probability that the reaction time of a person chosen at random is between 0.70 and 0.79 seconds.



(i) Write down the value of a and of b.

(ii) Calculate the probability that the reaction time of a person chosen at random is

(a) greater than 0.70 seconds;

(b) between 0.70 and 0.79 seconds.

(6)

Three percent (3%) of the population have a reaction time less than c seconds.

(b) (i) Represent this information on a diagram similar to the one above. Indicate clearly the area representing 3%.

(ii) Find c.

(4)
(Total 10 marks)

37. A family of functions is given by

$f(x) = x^2 + 3x + k$, where $k \in \{1, 2, 3, 4, 5, 6, 7\}$.

One of these functions is chosen at random. Calculate the probability that the curve of this function crosses the x-axis.

Working:

Answer:

....

(Total 6 marks)

38. A company manufactures television sets. They claim that the lifetime of a set is normally distributed with a mean of 80 months and standard deviation of 8 months.

(a) What proportion of television sets break down in less than 72 months?

(2)

(b) (i) Calculate the proportion of sets which have a lifetime between 72 months and 90 months.

(ii) Illustrate this proportion by appropriate shading in a sketch of a normal distribution curve.

(5)

(c) If a set breaks down in less than x months, the company replace it free of charge. They replace 4% of the sets. Find the value of x.

(3)

(Total 10 marks)

39. It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more.

(2)

(b) The probability that the mass of a lion lies between a and b is 0.95, where a and b are symmetric about the mean. Find the value of a and of b.

(3)
(Total 5 marks)

40. The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.

(a) Find the probability that a packet chosen at random has mass

(i) less than 740 g;

(ii) at least 780 g;

(iii) between 740 g and 780 g.

(5)

(b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g?

(2)

(c) The mass of 70% of the packets is more than x grams. Find the value of x.

(2)
(Total 9 marks)

41. In a country called *Tallopia*, the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

(a) What percentage of adults in *Tallopia* have a height greater than 197 cm?

(3)

(b) A standard doorway in *Tallopia* is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm.

(4)
(Total 7 marks)

42. Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.

(a) What percentage of the population has an IQ between 90 and 125?

(2)

(b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125?

(3)

(c) The mean IQ of a random group of 25 persons suffering from a certain brain disorder was found to be 95.2. Is this sufficient evidence, at the 0.05 level of significance, that people suffering from the disorder have, on average, a lower IQ than the entire population? State your null hypothesis and your alternative hypothesis, and explain your reasoning.

(4)
(Total 9 marks)

43. Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.

(a) What is the probability a bag selected at random will weigh less than 25.0 kg?

(2)

In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5% of the total, the mean is increased without changing the standard deviation.

(b) Show that the increased mean is 26.0 kg.

(3)

It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5% of bags be under 25 kg and that only 2.5% of bags be over 26 kg.

(c) Calculate the mean and standard deviation that satisfy these requirements.

(3)

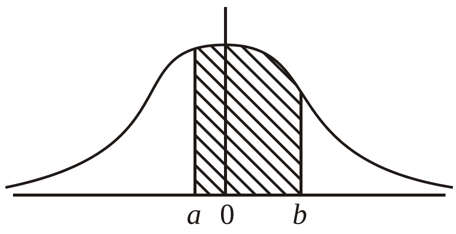
The cost of the new machine is $5000. Cement sells for $0.80 per kg.

(d) Compared to the cost of operating with a 26 kg mean, how many bags must be filled in order to recover the cost of the new equipment?

(3)
(Total 11 marks)

44. The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

(a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



(i) Write down the values of a and b.

(2)

(ii) Find the probability that the lifespan of an insect of this species is

(a) more than 55 hours;

(1)

(b) between 55 and 60 hours.

(2)

(b) 90% of the insects die after t hours.

(i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%.

(2)

(ii) Find the value of t.

(3)
(Total 10 marks)

45. An urban highway has a speed limit of 50 km h^{-1}. It is known that the speeds of vehicles travelling on the highway are normally distributed, with a standard deviation of l0 km h^{-1}, and that 30% of the vehicles using the highway exceed the speed limit.

(a) Show that the mean speed of the vehicles is approximately 44.8 km h^{-1}.

(3)

The police conduct a "Safer Driving" campaign intended to encourage slower driving, and want to know whether the campaign has been effective. It is found that a sample of 25 vehicles has a mean speed of 41.3 km h^{-1}.

(b) Given that the null hypothesis is

H_0: the mean speed has been unaffected by the campaign

State H_1, the alternative hypothesis.

(1)

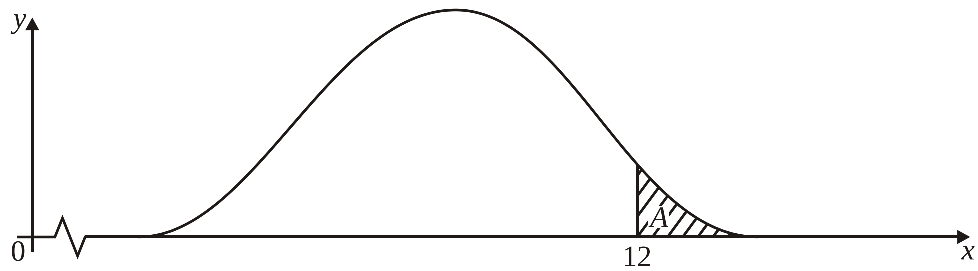
(c) State whether a one-tailed or two-tailed test is appropriate for these hypotheses, and explain why.

(2)

(d) Has the campaign had significant effect at the 5% level?

(4)
(Total 10 marks)

46. The graph shows a normal curve for the random variable X, with mean μ and standard deviation σ.



It is known that $p\ (X \geq 12) = 0.1$.

(a) The shaded region A is the region under the curve where $x \geq 12$. Write down the area of the shaded region A.

(1)

It is also known that $p\ (X \leq 8) = 0.1$.

(b) Find the value of μ, explaining your method in full.

(5)

(c) Show that $\sigma = 1.56$ to an accuracy of three significant figures.

(5)

(d) Find $p\ (X \leq 11)$.

(5)
(Total 16 marks)

47. A fair coin is tossed eight times. Calculate

(a) the probability of obtaining exactly 4 heads;

(2)

(b) the probability of obtaining exactly 3 heads;

(1)

(c) the probability of obtaining 3, 4 or 5 heads.

(3)
(Total 6 marks)