

# P.7 Binomial Probability

Probability is used to make inferences and predictions.

How are probabilities computed?

Why is the computation of probabilities useful?

Ratio

Set Theory

Sample Space

Fundamental Probability

Independent Events

Dependent Events

Combined Events

Mutually Exclusive Events

Conditional Events

Venn Diagrams

Tree Diagrams

Lattice Diagrams

Imagine rolling a single die four times.

What is the probability of getting...

- no 5s?  $\left(\frac{5}{6}\right)^4$
- exactly one 5?  $\binom{4}{1}\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^1$
- exactly two 5s?  $\binom{4}{2}\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2$
- exactly three 5s?  $\binom{4}{3}\left(\frac{5}{6}\right)^1\left(\frac{1}{6}\right)^3$
- four 5s?  $\left(\frac{1}{6}\right)^4$

Binomial probability measures the probability of a series of events under the following conditions:

- Each event is identical
- Each event is independent
- Each event has only two outcomes

Some examples of this kind of probability:

- answering multiple choice questions correctly or incorrectly
- making or missing free throws, or pitching strikes or balls
- winning or losing a game
- a product being defective or not

The formula for binomial probability is:

$$\rightarrow X \sim B(n, p) \Rightarrow P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$$

*(Note: In the original image, the binomial coefficient  $\binom{n}{r}$  is circled in green, with a green arrow pointing to it from the label  $nCr$  written above.)*

where

$X$  is a binomial event

$n$  is the number of trials

$r$  is the number of successes

$p$  is the probability of success

$n - r$  is the number of failures

$1 - p$  is the probability of failure

Imagine rolling a single die four times.

What is the probability of getting...

- no 5s?
- exactly one 5?
- exactly two 5s?
- exactly three 5s?
- four 5s?

You should always write out this formula for each problem.

You can then evaluate it on your calculator using

$2^{\text{nd}}$  → DISTR → binompdf

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0:5/12 DRAW
8:1/X^2cdf(
9: Fpdf(
0: Fcdf(
B: binompdf(
→ B: binomcdf(
C: Poissonpdf(
D: Poissoncdf(
```

```
binompdf
 trials:
 P:
 x value:
 Paste
```

1. In an examination hall, it is known that 15% of the desks are wobbly.

a) What is the probability that exactly one desk will be wobbly in a row of six desks?

$$\binom{6}{1} (0.15)^1 (0.85)^5 = 0.399$$

b) What is the probability that more than one desk will be wobbly in a row of six?

$$\begin{aligned} 1 - \left[ \binom{6}{0} (0.15)^0 (0.85)^6 + \binom{6}{1} (0.15)^1 (0.85)^5 \right] \\ = 1 - 0.776 \\ = \boxed{0.224} \end{aligned}$$

2. The probability that a telephone line is engaged at a company switchboard is 0.25. The switchboard has 10 lines. Find the probability that

a) one half of the lines are engaged.

$$\binom{10}{5} (0.25)^5 (0.75)^5 = 0.0584$$

b) at least one line is engaged.

$$1 - \left[ \binom{10}{0} (0.25)^0 (0.75)^{10} \right]$$
$$= 1 - 0.0563 = \boxed{0.944}$$



3. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

a) Find the probability that there is at least one defective lamp in the sample.

$$1 - \binom{30}{0} (0.05)^0 (0.95)^{30} \\ = 1 - 0.215 = \boxed{0.785}$$

b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps.

$$P(\leq 2) = 0.812 - 0.215 = 0.597 \\ \frac{0.597}{0.785} = \boxed{0.761}$$

The formula for binomial probability for exactly  $r$  winners out of  $n$  trials is:

$${}_n C_r \cdot P(\text{success})^r \cdot P(\text{failure})^{n-r}$$

What is the probability of guessing the correct answer to exactly 7 out of 10 multiple choice questions if each question has 5 choices?

The formula for binomial probability for exactly  $r$  winners out of  $n$  trials is:

$${}_n C_r \cdot P(\text{success})^r \cdot P(\text{failure})^{n-r}$$

Buzzy makes about 75% of her volleyball spikes. What is the probability she will make exactly 6 of the next 10 spikes?

The formula for binomial probability for exactly  $r$  winners out of  $n$  trials is:

$${}_n C_r \cdot P(\text{success})^r \cdot P(\text{failure})^{n-r}$$

What is the probability of guessing the correct answer to at least 7 out of 10 multiple choice questions if each question has 5 choices?

The formula for binomial probability for exactly  $r$  winners out of  $n$  trials is:

$${}_n C_r \cdot P(\text{success})^r \cdot P(\text{failure})^{n-r}$$

Jelle makes about 75% of his volleyball spikes. What is the probability he will make at least 1 of his next 10 spikes?

Next Time: Review  
Monday: TEST Probability

## Attachments

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