Real life!!! Mr. Steponic's friend Krysta is tired of sorting the socks for her four kids. She now throws them all in one basket and lets them sort it out.
Here's her recent post on FB:

"So. . . let's say there are 83 socks in my sock basket (because there would never be an even number, for goodness' sake), and 20 of them are identical in my attempt to increase the probability of a matching pair, and 13 of them will be ignored because they are colored or patterned in some way, what is the probability that my child will select two that match? Knowing my kids, the answer is somewhere around zero."

What, exactly, is the probability that Krysta's daughter will pull out a matching pair of socks?

P.4 Homework Answers:

1. Mutually exclusive: $\frac{10}{13} \approx 0.769$

2. Inclusive: $\frac{115}{200} = 0.575$

3. Mutually exclusive and dependent: P(SS)+P(RR) =
$\left(\frac{13}{52} * \frac{12}{51}\right) + \left(\frac{26}{52} * \frac{25}{51}\right) = \frac{31}{102} \approx 0.304$

4. Inclusive: $x = \frac{1}{15} \approx 0.0667$

5. Mutually exclusive, so $P(X \cap Y) = 0$
Mutually exclusive, so $P(X \cup Y) = \frac{19}{21} \approx 0.905$

6. The events are mutually exclusive if $P(S \cap T) = 0$. So see if it works...

7. Inclusive: 0.11

8. a. $\frac{19}{37}$ b. $\frac{12}{37}$ c. $\frac{6}{37}$

9. $\frac{5}{6}$

10. $\frac{20}{27}$

Probability of an event A:	$P(A) = \frac{n(A)}{n(U)}$
Complementary events:	$P(A') = 1 - P(A)$
Combined events:	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events:	$P(A \cup B) = P(A) + P(B)$
Independent events:	$P(A \cap B) = P(A)\,P(B)$
Conditional probability:	$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$

$P(A \cap B) = P(A)\,P(B \mid A)$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Given ____B____, find ____P(A)____ .

Find ____P(A)____ given ____B____ .

What we're doing is reducing the sample space to a smaller set [P(B)], then looking for the winners within that sample space only [$P(A \cap B)$].

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

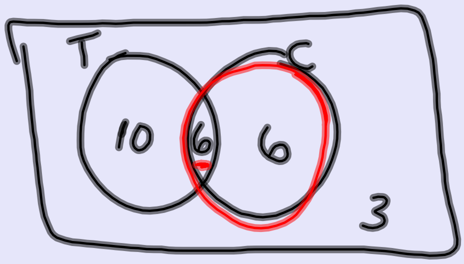
Two dice numbered one to six are rolled. Find the probability of obtaining a sum of five given that the sum is seven or less.

$$P(\text{sum } 5 \mid \text{sum } 7 \text{ or less})$$

$$= \frac{P(\text{sum } 5 \cap 7 \text{ or less})}{P(\text{sum } 7 \text{ or less})} = \frac{4/\cancel{36}}{21/\cancel{36}}$$

$$= \frac{4}{21}$$

In a class of 25 students it is found that 6 of the students play both tennis and chess, 10 play tennis only, and 3 do not participate in any activities. Find the probability that a student selected at random from this group plays tennis, given the student plays chess.

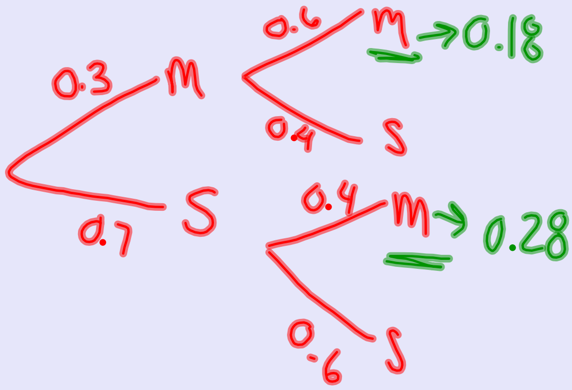


$$P(T|C) = \frac{6}{12}$$

Bag A contains 5 blue and 4 green marbles. Bag B contains 3 yellow, 4 blue, and 2 green marbles. Given you have a green marble, what is the probability it came from Bag A?

$$P(A|G) = \frac{4}{6} = \frac{2}{3}$$

At the basketball game, Amanda got into a two-shot foul situation. She figured her chance of making the first shot was 0.7. If she made the first shot, her chance of making the second shot was 0.6. However, if she missed the first shot, her probability of making the second shot was only 0.4. Given Amanda missed the second shot, find the probability that she made the first shot.



$$P(S^{1st} \mid M_{2nd})$$

$$= \frac{P(S \cap M)}{P(M)}$$

$$= \frac{0.28}{0.46} = \boxed{0.609}$$

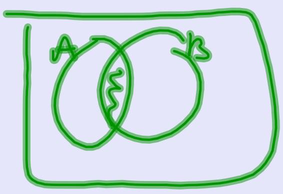
The events A and B are independent. $P(A \cap B) = 0.6$ and $P(B) = 0.8$.

a) $P(A)$

$0.6 = 0.8(P(A))$

$P(A) = \frac{0.6}{0.8}$

$= 0.75$



b) $P(A|B)$

$\frac{P(A \cap B)}{P(B)}$

$= \frac{0.6}{0.8}$

$= 0.75$

c) $P(A|B')$

$\frac{P(A \cap B')}{P(B')}$

$= \frac{0.75 - 0.6}{0.2}$

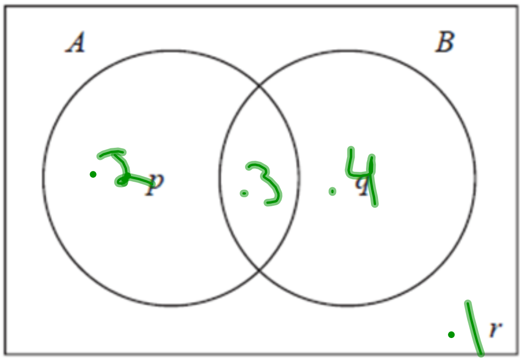
$= 0.75$

5. *[Maximum mark: 6]*

Consider the events A and B, where $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.

b) $\frac{.5-.3}{.3} = \frac{2}{3}$



(a) Write down the value of

(i) p; $.2$

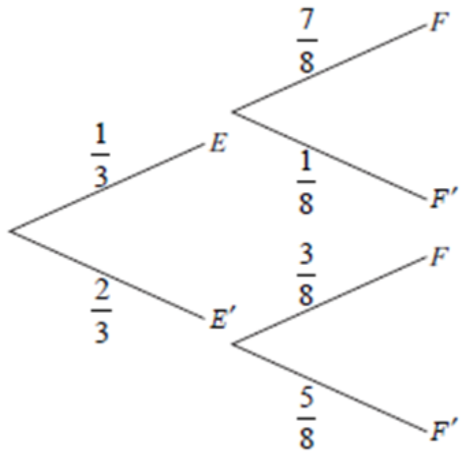
(ii) q; $.4$

(iii) r. $.1$ *[3 marks]*

(b) Find the value of $P(A \mid B')$. *[2 marks]*

(c) Hence, or otherwise, show that the events A and B are **not** independent. *[1 mark]*

José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.
If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.
If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.
Let E be the event "he misses his bus" and F the event "he is late for school".
The information above is shown on the following tree diagram.



(a) Find

(i) $P(E \cap F)$; $\frac{7}{24}$

(ii) $P(F)$. $\frac{13}{24}$ *[4 marks]*

(b) Find the probability that

(i) José misses his bus and is not late for school; $P(E \cap F') = \frac{1}{24}$

(ii) José missed his bus, given that he is late for school. *[5 marks]*

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{7}{24}}{\frac{13}{24}} = \frac{7}{13}$$

P.5 Homework Exercises

1. Dana and Lana are trying to solve a physics problem.
Dana has a 65% chance of solving the problem, and Lana has a 75% chance.
Find the probability that

a. only Lana solves the problem.

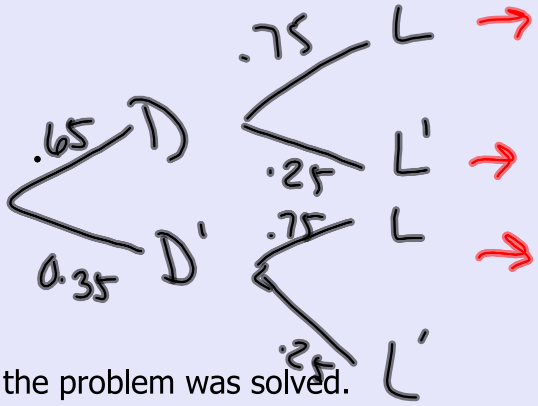
$(0.35)(0.75) = 0.263$

b. Lana solves the problem.

0.75

c. both solve the problem.

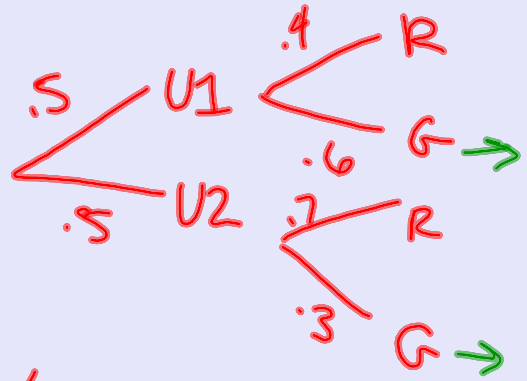
0.488



d. Dana solves the problem, given the problem was solved.

$$P(D \mid \text{solved}) = \frac{0.65}{0.9125} \approx 0.712$$

<u>Homework</u>
9. Urn 1 contains 4 red and 6 green balls while Urn 2 contains 7 red and 3 green balls. An urn is chosen at random and then a ball is chosen from the selected urn.
Draw a tree diagram. Find P(Urn 1|G).



$$P(Urn1|G) = \frac{P(U1 \cap G)}{P(G)} = \frac{0.3}{0.45} = \frac{2}{3}$$

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