P.2 Independent and Dependent Events

Today we will look at the probability of two events both happening. The notation for this is P(A and B) or $P(A \cap B)$.

Tree diagrams are often the easiest way to organize these kinds of events.

Two events are <u>independent</u> if the outcome of the first event does not influence the outcome of the second event. The following formula applies:

$$P(A \cap B) = P(A) \times P(B)$$

1. Find the probability of getting a sum of 5 on the first toss of two dice and a sum of 3 on the second toss.

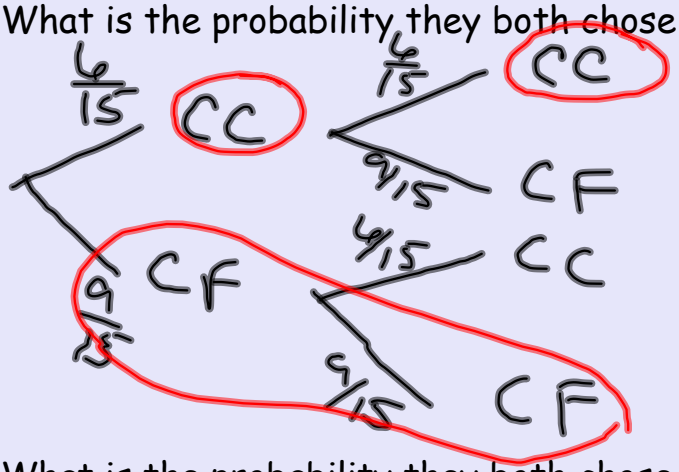
$P(+5) = \frac{4}{36} = \frac{1}{9}$

$P(+3) = \frac{2}{36} = \frac{1}{18}$

$P(A \cap B) = \frac{1}{9} \cdot \frac{1}{18} = \frac{1}{162}$

A shelf contains 9 boxes of Corn Flakes and 6 boxes of Captain Crunch. Ahmed chooses one box at random, then puts it back. A second person does the same thing.

What is the probability they both chose Captain Crunch?



$P(CC \cap CC) = \frac{6}{15} \cdot \frac{6}{15}$

$\frac{36}{225}$

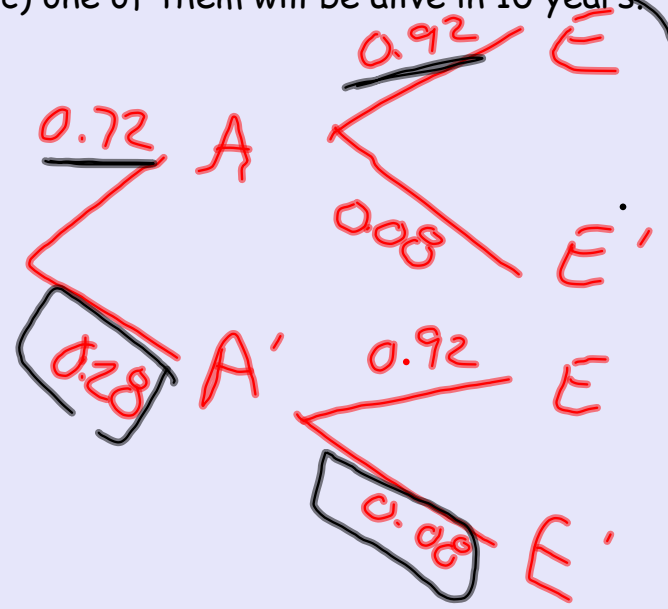
What is the probability they both chose the same cereal?

$P(CF \cap CF) = \frac{9}{15} \cdot \frac{9}{15} = \frac{81}{225}$

$\frac{36}{225} + \frac{81}{225} = \frac{117}{225}$

Andrew is 55, and the probability that he will be alive in 10 years is 0.72. Ellen is 35, and the probability that she will be alive in 10 years is 0.92.

What is the probability that
a) they will both be alive in 10 years;
b) neither will be alive in 10 years;
c) one of them will be alive in 10 years.

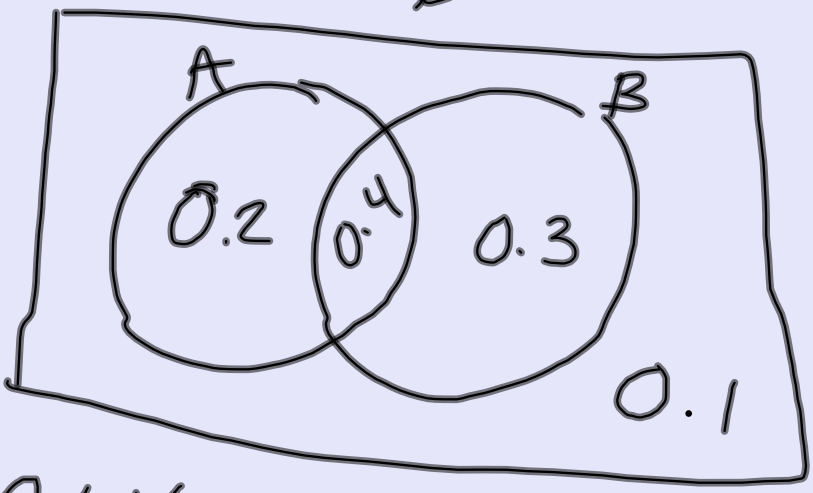


$P(A \cap E) = 0.72 \cdot 0.92 = 0.6624$

$P(A' \cap E') = 0.0224$

$P(A \cap E') + P(A' \cap E) = 0.3152$

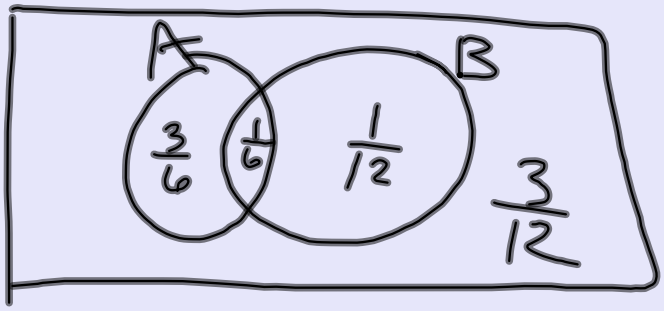
It is given that $P(A) = 0.6$, $P(B) = 0.7$ and $P(A \cap B) = 0.4$. Draw a Venn diagram to represent this information. Are events A and B independent? Why or why not?



$(0.6)(0.7) = 0.42 \neq 0.4$

not independent

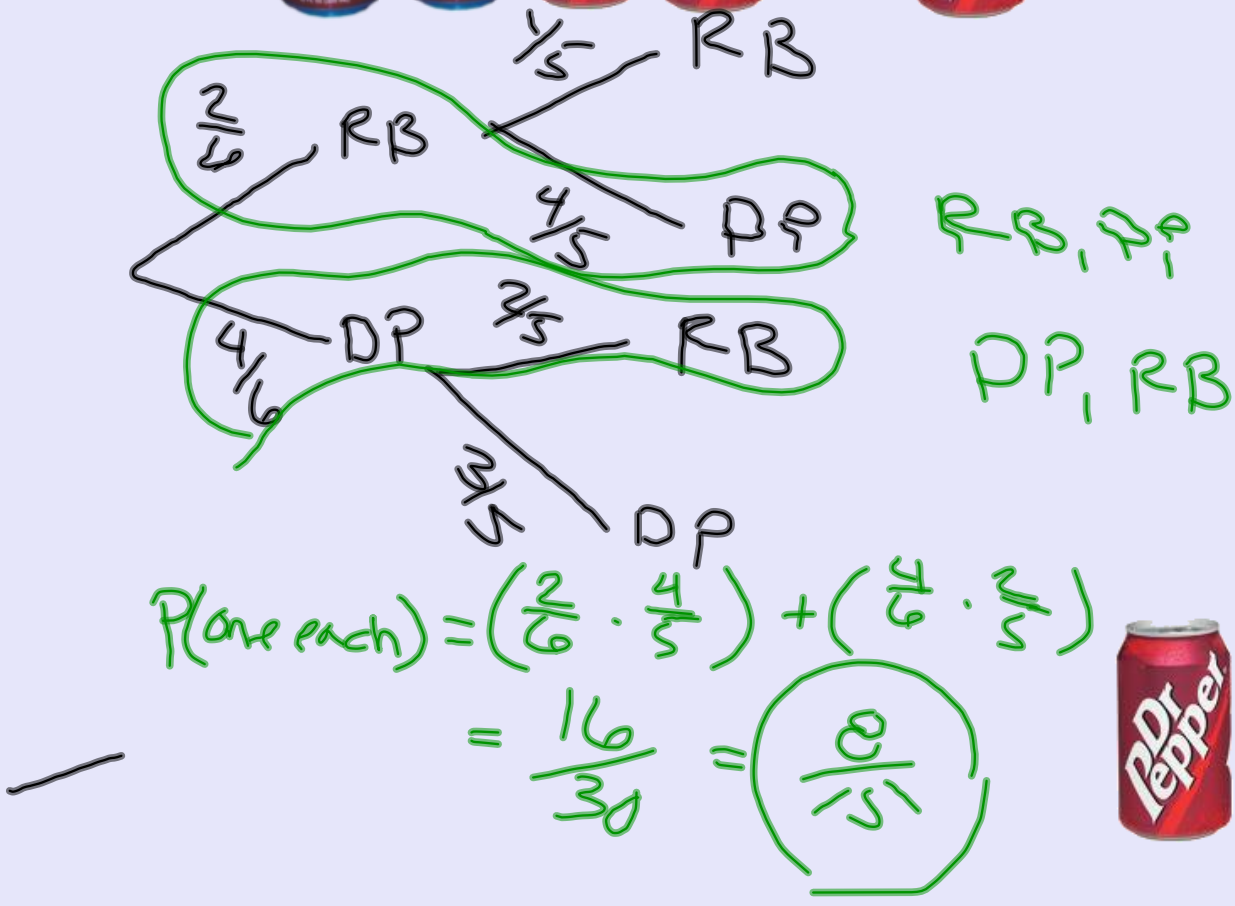
It is given that $P(A) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. Draw a Venn diagram to represent this information. Find P(B) if events A and B are independent.



$$P(B) = \frac{1}{6} \div \frac{2}{3}$$

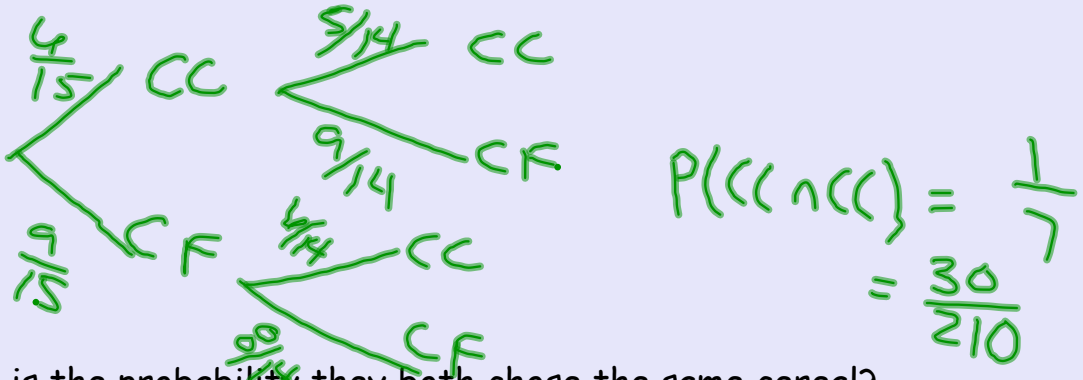
$$= \frac{1}{6} \cdot \frac{3}{2} = \frac{3}{12}$$

There are 2 cans of root beer and 4 cans of Dr. Pepper on the counter. Nada drinks two of them at random. What is the probability that she drank one can of each?



A shelf contains 9 boxes of Corn Flakes and 6 boxes of Captain Crunch. Ahmed chooses one box at random, but does not put it back. A second person does the same thing.

What is the probability they both chose Captain Crunch?

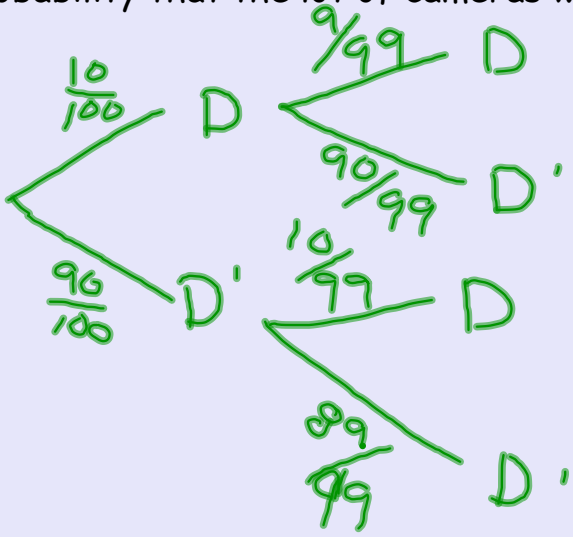


$$P(CC \cap CC) = \frac{1}{7} = \frac{30}{210}$$

What is the probability they both chose the same cereal?

$$\frac{30}{210} + \left(\frac{9}{15} \cdot \frac{8}{14}\right) = \frac{17}{35}$$

A quality-control procedure for testing Ready-Flash disposable cameras consists of choosing two cameras at random from each lot of 100 without replacement. If both cameras are defective, the entire lot is rejected. Typically, 10 cameras of the 100 are defective. Find the probability that the lot of cameras will NOT be rejected.



$P(D \cap D) =$

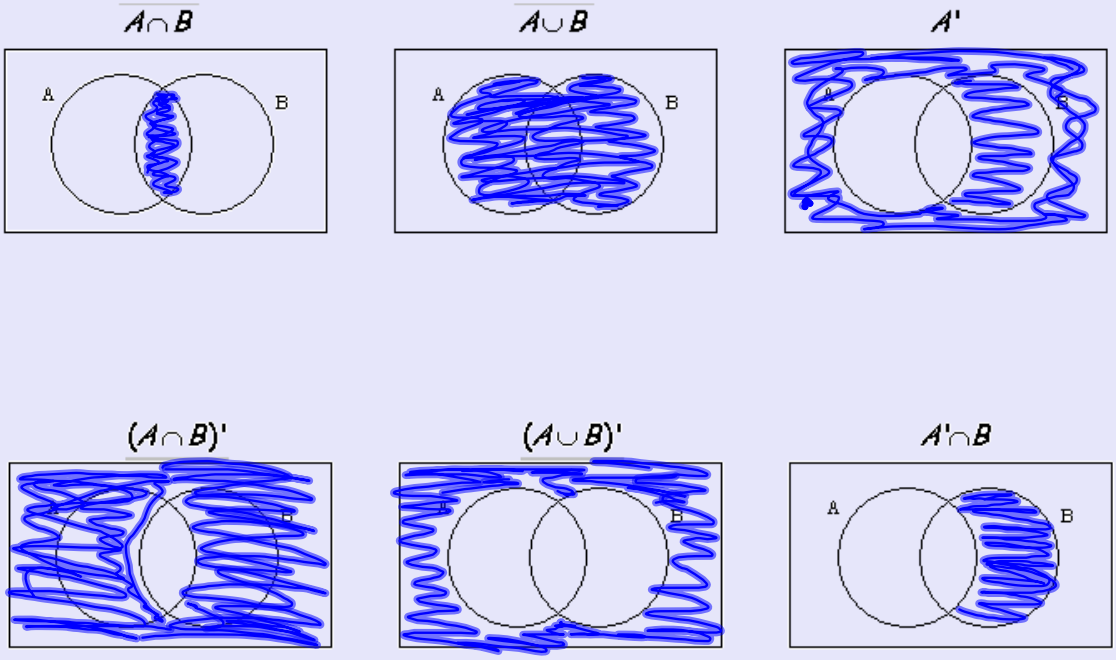
$\frac{10}{100} \cdot \frac{9}{99}$

$\frac{1}{10} \cdot \frac{1}{11} = \frac{1}{110}$

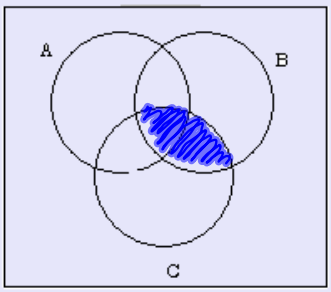
$P(\text{not rejected}) =$

$1 - \frac{1}{110} = \frac{109}{110}$

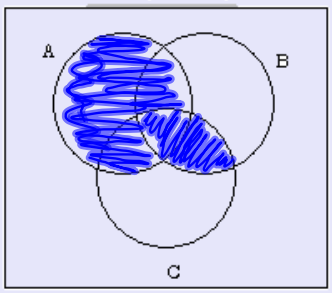
P.3 Basic Set Theory


$A \cap B$
A
B
$A \cup B$
A
B
A'
A
B
$(A \cap B)'$
A
B
$(A \cup B)'$
A
B
$A' \cap B$
A
B

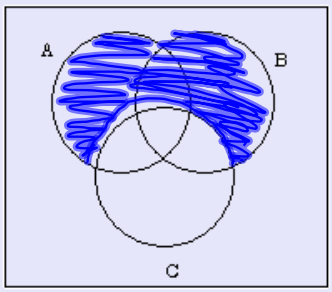
$B \cap C$



$A \cup (B \cap C)$



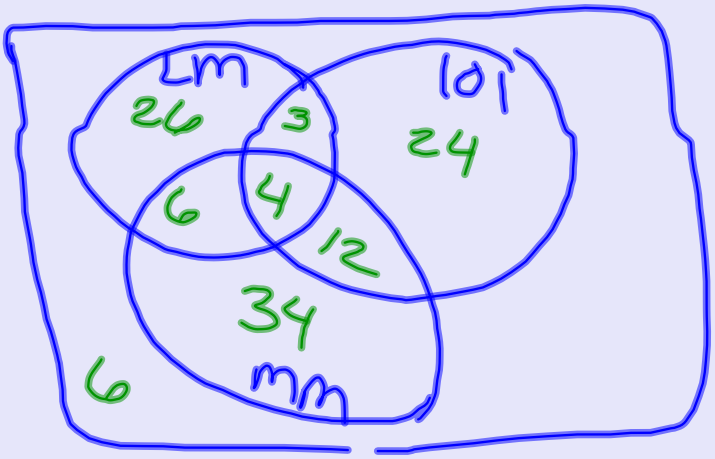
$(A \cup B) \cap C'$



In a survey of children who saw three different shows at Walt Disney World, the following information was gathered:

39 children liked The Little Mermaid
43 children liked 101 Dalmatians
56 children liked Mickey Mouse
7 children liked The Little Mermaid and 101 Dalmatians
10 children liked The Little Mermaid and Mickey Mouse
16 children liked 101 Dalmatians and Mickey Mouse
4 children liked The Little Mermaid, 101 Dalmatians, and Mickey Mouse
6 children did not like any of the shows

How many children were surveyed?



115 children

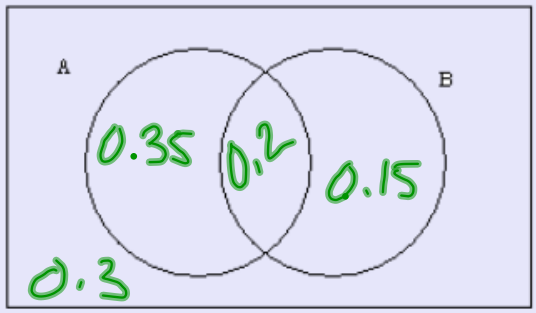
1. Given P(A) = 0.55, $P(A \cup B) = 0.7$, and $P(A \cap B) = 0.2$, find $P(B')$

$P(A \cup B)' = 0.3$

$(0.2 + 0.15)$

$P(B') = 1 - 0.35$

$= 0.65$

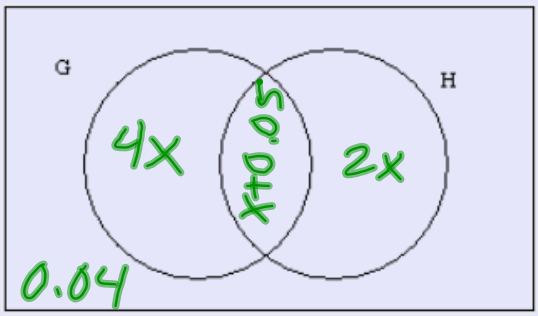


2. Given $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{3}$, and $P(A \cap B) = \frac{1}{2}$, find $P(A \cup B)$

$\frac{18}{30}$ $\frac{20}{30}$ $\frac{1}{30}$

$P(A \cup B) = \frac{23}{30}$

3. Given P(G) = 5(x + 0.01), P(H) = 3x + 0.05, $P(G \cap H)$ = x + 0.05, and $P(G \cup H)'$ = 0.04, find x.



$5x + 0.05 - (x + 0.05)$

$3x + 0.05 - (x + 0.05)$

$4x + 2x + x + 0.05 + 0.04 = 1$

$7x + 0.09 = 1$

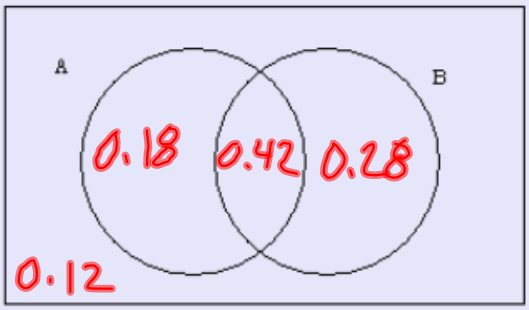
$7x = 0.91$

$x = 0.13$

4. Given that P(A) = 0.6, P(B) = 0.7, and that A and B are independent events, find $P(A \cup B)$, $P(A')$, $P(A' \cap B)$.

Hmmm... What does "independent" mean?
What formula did we learn?

$P(A \cap B) = (0.6)(0.7)$
$= 0.42$



$P(A \cup B) = 0.88$

$P(A') = 0.28 + 0.12 = 0.4$

$P(A' \cap B) = 0.28$

Attachments

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