Discrete Math

Lesson 3.6

The Method of Sealed Bids

Standard DM.4.6
Use fair division techniques to divide continuous objects.

The Method of Sealed Bids

*   Step 1: Bidding

*   Step 2: Allocation

*   Step 3: First Settlement

*   Step 4: Division of the Surplus

*   Step 5 Final Settlement

The Method of Sealed Bids
Example 1: Settling Grandma's Estate

In her last will and testament, Grandma plays a little joke on her four grandchildren (Art, Betty, Carla, and Dave) by leaving just three items—a cabin in the mountains, a vintage 1955 Rolls Royce, and a Picasso painting—with the stipulation that the items must remain with the grandchildren (not sold to outsiders) and must be divided fairly in equal shares among them. How can we possibly resolve this conundrum? The method of sealed bids will give an ingenious and elegant solution.

*   Step 1:   Bidding   Each player makes a bid for each of the items. Bids must be honest assessment of value and done in privacy. Sealed envelopes are used. Here's the results of the bids from the four grandchildren.

The Players' Bids	Art	Betty	Carla	Dave
Cabin	220,000	250,000	211,000	198,000
Rolls Royce	40,000	30,000	47,000	52,000
Picasso	280,000	240,000	234,000	190,000

The Method of Sealed Bids
Example 1: Settling Grandma's Estate

*   Step 2: Allocation Each item will go to the highest bidder for that item. Ties are broken by a coin flip. In this method it is possible for one player to get none of the items and another player to get many or all of the items. Similar to a silent auction--matters who bids the highest.

Who Gets What?

Art: Picasso

Betty: Cabin

Carla: Ø

Dave: Rolls

The Players' Bids				
	Art	Betty	Carla	Dave
Cabin	220,000	250,000	211,000	198,000
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*   Step 3: First Settlement Depending on what items (if any) a player gets in Step 2, he or she will owe money to or be owed money by the estate. First, calculate each player's fair-dollar share (in their own eyes) by adding that player's bids and dividing by the number of players. The fair dollar shares are the baseline for settlements. If the total value of the items the player gets in Step 2 are more than his fair dollar share, he pays the estate the difference. If the total value is less than his fair dollar share, the player gets the difference in cash. How does the settlement work out for each of these players?

Art:

Betty:

Carla:

Dave:

The Method of Sealed Bids
Example 1: Settling Grandma's Estate

*   Step 3: First Settlement

	Art	Betty	Carla	Dave
Cabin	220,000	250,000	211,000	198,000
Rolls Royce	40,000	30,000	47,000	52,000
Picasso	280,000	240,000	234,000	190,000
Total	540,000	520,000	492,000	440,000
Fair-dollar share ÷4	135,000	130,000	123,000	110,000

Art: Picasso 280,000 − 135000 = $145,000 He PAYS

Betty: Cabin (250,000 − 130000) = $120,000 She PAYS

Carla: Nothing   She gets $123000

Dave: Rolls (110,000 − 52000) He gets $58000

The Method of Sealed Bids
Example 1: Settling Grandma's Estate

* Step 3: First Settlement

	Art	Betty	Carla	Dave
Cabin	220,000	250,000	211,000	198,000
Rolls Royce	40,000	30,000	47,000	52,000
Picasso	280,000	240,000	234,000	190,000
Total	540,000	520,000	492,000	440,000
Fair-dollar share	135,000	130,000	123,000	110,000

Art: Gets painting and pays estate $145,000

Betty: Gets cabin and pays estate $120,000

Carla: Receives nothing except $123,000

Dave: Receives Rolls and $58,000

The Method of Sealed Bids
Example 1: Settling Grandma's Estate

*   Step 4: Division of Surplus Add up payments made to the estate and subtract payments made from the estate and discover there is a surplus or deficit. In this case, $145,000+$120,000-$123,000-$58,000= $84,000 surplus!

     Surplus is common money that belongs to the estate and must be divided equally among the players.   So each player here gets $84,000/4 = $21,000.

A   B   C   D   4

                                                                 ESTATE

Art: Gets painting and pays estate $145,000

Betty: Gets cabin and pays estate $120,000

Carla: Receives nothing except $123,000

Dave: Receives Rolls and $58,000   +_____

                                        Surplus:
                        Divided by 4 (each player's share):

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Example 1: Settling Grandma's Estate

*   Step 5: Final Settlement Add each player's share of the surplus to what they previously received in the first settlement (Step 3).

First Settlement	Surplus Share	Final Settlement
Art: Picasso + 145,000	$21,000	Picasso, pays $124000
Betty: Cabin + 120000	$21,000	Cabin, pay $99000
Carla: Nothing paid 123000	$21000	$144,000 paid.
Dave: Rolls, paid 58000	$21000	Rolls, paid $79000

Example 2

EXAMPLE 3.10 | **Splitting Up the House**

Al and Betty are getting a divorce. The only joint property of any value is their house. Rather than hiring attorneys and going to court to figure out how to split up the house, they agree to give the method of sealed bids a try.

Al's bid on the house is $340,000; Betty's bid is $364,000. Their fair-dollar shares of the "estate" are $170,000 and $182,000, respectively. Since Betty is the higher bidder, she gets to keep the house and must pay Al cash for his share. The computation of how much cash Betty pays Al can be done in two steps.

Bidding and Allocation has already occurred. Now do:

Step 3: (First Settlement) Al: no items, gets $170000

B: house, pays $182000 (364000 −182000$)

Step 4: (Division of Surplus)

$182000 - 170000 = \dfrac{12000}{2} \rightarrow \$6{,}000$ each.

Step 5: (Final Settlement)

Al: no item, recieve $176,000

B: house, pay $176,000

The Method of Sealed Bids

Final Thoughts

*   Works well because tweaks basic idea of economics.

*   Each player is simultaneously a buyer and a seller without knowing which one due to sealed bids.

*   Process keeps each player honest and works out to everyone's advantage in the long run.

*   Sealed bids is good settling divorce property and business partnership splitting up. However, rationality is difficult in these situations which are rarely devoid of emotion, spite, hard feelings, etc.

*   Sealed bids will not work, though, unless have:
    1. Each player must have enough money to play the game.
    2. Each player must accept money as a substitute for any item. No item can be considered priceless.

Assignment:

3.6 The Method of Sealed Bids
HW=p. 114-115; # 54, 56, 58(bonus)