

Key

Lone-Divider Method: One divider out of N players, and $N-1$ choosers

Preliminaries: random draw used to determine who will be D
and who will be C_1 and C_2

Step 1:

Division: D divides the cake into 3 shares (not knowing which he will get)

Step 2:

Bidding: C_1 + C_2 independently select their fair share piece(s) in private
(must list all pieces that are $> 1/3$, fair share)

Step 3:

Distribution: $C \rightarrow$ chosen piece
 $U \rightarrow$ unwanted piece

Four partners are dividing a plot of land among themselves using the lone-divider method. After the divider D divides the land into four shares $s_1, s_2, s_3,$ and s_4 , the choosers $C_1, C_2,$ and C_3 submit their bids for these shares.

1. Suppose that the chooser's bids are $C_1 = \{s_3\}, C_2 = \{s_2, s_1\}$ and $C_3 = \{s_3, s_1\}$. Find a fair division of the land. $C_1 - s_3 \quad C_2 - s_2 \quad C_3 - s_1 \quad D - s_4$

2. Suppose that the chooser's bids are $C_1 = \{s_3, s_2\}, C_2 = \{s_1, s_2\}$ and $C_3 = \{s_1, s_3\}$. Find a fair division of the land. Describe one possible fair division of the land.

$$C_1 - s_3 \quad C_2 - s_2 \quad C_3 - s_1 \quad D - s_4$$

3. Describe another possible fair division of the land with bids $C_1 = \{s_3, s_2\}, C_2 = \{s_1, s_2\}$ and $C_3 = \{s_1, s_3\}$.

$$C_1 - s_2 \quad C_2 - s_1 \quad C_3 - s_3 \quad D - s_4$$

4. Suppose that the chooser's bids are $C_1 = \{s_3\}, C_2 = \{s_4, s_1\}$ and $C_3 = \{s_4, s_2\}$. Find a fair division of the land.

$$C_1 - s_3 \quad C_2 - s_1 \quad C_3 - s_4 \quad D - s_2$$

5. Describe another possible fair division of the land with bids $C_1 = \{s_3\}, C_2 = \{s_4, s_1\}$ and $C_3 = \{s_4, s_2\}$.

$$C_1 - s_3 \quad C_2 - s_4 \quad C_3 - s_2 \quad D - s_1$$

6. Describe another possible fair division of the land with bids $C_1 = \{s_3\}, C_2 = \{s_4, s_1\}$ and $C_3 = \{s_4, s_2\}$.

$$C_1 - s_3 \quad C_2 - s_1 \quad C_3 - s_2 \quad D - s_4$$

Four players (Abe, Betty, Cory, and Dana) are dividing a pizza worth \$23.00 among themselves using the lone-divider method. The divider divides into four shares x, y, z, and w. The table shows the value of the four shares in the eyes of each player, but some of the entries in the table are missing.

\$	x	y	z	w
Alex	4.50	6.25	6.25	6.00
Betty	5.75	5.75	5.75	5.75
Cory	5.50	6.25	5.75	5.50
Dana	5.50	5.50	6.75	5.25

$$FS = \frac{23}{4} = \$5.75$$

7. Who was the divider?

Betty

8. Determine each chooser's bid. List the choosers in alphabetical order. Let the first chooser in the alphabetical list be labeled C_1 , let the second be labeled C_2 , and let the third be labeled C_3 . Determine chooser C_1 's bid.

Alex - C_1 : y, z, w

9. Determine chooser C_2 's bid.

Cory - C_2 : y, z

10. Determine chooser C_3 's bid.

Dana - C_3 : z

11. Find a fair division of the pizza.

Alex - w

Cory - y

Dana - z

Betty - x