

Key

Goods: "booty" - the item(s) being divided.

Players: Set of parties with the right to share.

Value System: each player has his own value system to quantify the booty

Rationality: each player rationally seeks to maximize his booty share

Cooperation: participants are willing + accept the rules.

Privacy: players do not have inf. on other players' value system

Symmetry: players have equal rights in sharing the set  $S$ .  
(all entitled to a proportional share)

Fair Division: to divide  $S$  into  $N$  shares fairly

\* Fair Share: a fair share ( $s$ ) if  $s = \frac{1}{N}$ th the total value of  $S$   
in the opinion of  $P$

Carmen and Jorge jointly buy a chocolate-strawberry mousse cake for \$72. Suppose that Carmen values the chocolate cake twice as much as she values strawberry cake.

1. Find the dollar value to Carmen of the chocolate half of the cake.

$$2(24) = \$48$$

2. Find the dollar value to Carmen of the strawberry half of the cake.

$$\$24$$

3. Find the dollar value to Carmen of the slice of strawberry cake shown.

$$\frac{1}{3}(24) = \$8$$

4. Find the dollar value to Carmen of the slice of the chocolate cake shown.

$$\frac{1}{4}(48) = \$12$$



$$2x + x = 72$$

$$x = 24$$

Three players (Alex, Betty, and Cindy) are sharing a cake. Suppose that the cake is divided into three slices (x, y, and z). The table shows the value of x and y to each of the players. The values of z are missing. (The percentage represents the value of the slice as a percent of the value of the entire cake.)

	x	y	z
Alex	40%	34%	26%
Betty	38%	34%	28%
Cindy	32%	36%	32%

5. Which of the three slices are fair to Alex? *x and y*
6. Which of the three slices are fair to Betty? *x and y*
7. Which of the three slices are fair to Cindy? *y only*
8. Is it possible to find a fair division of the cake using x, y, and z as fair shares?

*not possible, no player considers z to be a fair share.*

Four players (Abe, Betty, Cory, and Dana) are sharing a cake. Suppose that the cake is divided into four slices x, y, z, and w. The table gives the value of each slice in the eyes of each of the players.

\$	x	y	z	w		
Abe	1.29	6.84	3.08	3.79	\$15	$s = 3.75$
Betty	4.75	4.75	4.75	4.75	\$19	$s = 4.75$
Cory	3.43	3.27	3.09	3.21	\$13	$s = 3.25$
Dana	2.02	3.84	1.57	1.57	\$9	$s = 2.25$

9. Which of the four slices are fair to Abe? *y + w*
10. Which of the four slices are fair to Betty? *x, y, z, w*
11. Which of the four slices are fair to Cory? *x + y*
12. Which of the four slices are fair to Dana? *y*
13. Fill in the blanks that explain why there is only one possible fair division of the cake using x, y, z, and w as fair shares. Since only slice y is fair to Dana, she has to get that slice. Then, out of the remaining slices only slice x is fair to Cory, so he must get that slice. Then, the only remaining slice that is fair to Abe is slice w, so he must get that slice. Finally, there is only one slice, slice z for Betty to get, so she must get that slice.