Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Friday 11 November 2016 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

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Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x - 5$, for $x \in \mathbb{R}$.

(a) Find $f(8)$. [2]

(b) Find $(g \circ f)(x)$. [2]

(c) Solve $(g \circ f)(x) = 0$. [3]

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2. [Maximum mark: 7]

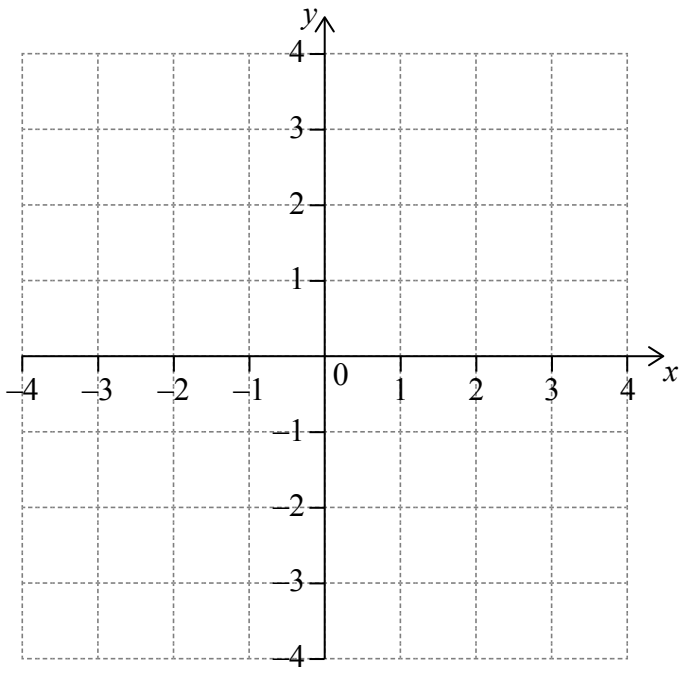
Let $f(x) = 0.225x^3 - 2.7x$, for $-3 \leq x \leq 3$. There is a local minimum point at A.

(a) Find the coordinates of A. [2]

(b) On the following grid,

(i) sketch the graph of f, clearly indicating the point A;

(ii) sketch the tangent to the graph of f at A. [5]



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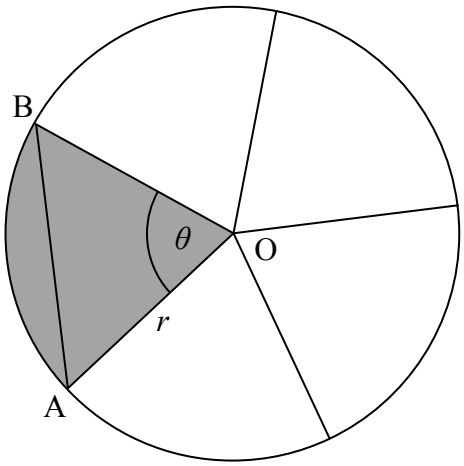
3. [Maximum mark: 7]

The following diagram shows a circle, centre O and radius r mm. The circle is divided into five equal sectors.

diagram not to scale



One sector is OAB, and $\hat{AOB} = \theta$.

(a) Write down the **exact** value of θ in radians. [1]

The area of sector AOB is $20\pi \text{ mm}^2$.

(b) Find the value of r. [3]

(c) Find AB. [3]

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4. [Maximum mark: 6]

Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

(a) Find the value of p and of q. [3]

(b) Hence, find the area of the region enclosed by the graphs of f and g. [3]

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5. [Maximum mark: 6]

The weights, W, of newborn babies in Australia are normally distributed with a mean $3.41\,\text{kg}$ and standard deviation $0.57\,\text{kg}$. A newborn baby has a low birth weight if it weighs less than $w\,\text{kg}$.

(a) Given that $5.3\,\%$ of newborn babies have a low birth weight, find w. [3]

(b) A newborn baby has a low birth weight.
Find the probability that the baby weighs at least $2.15\,\text{kg}$. [3]

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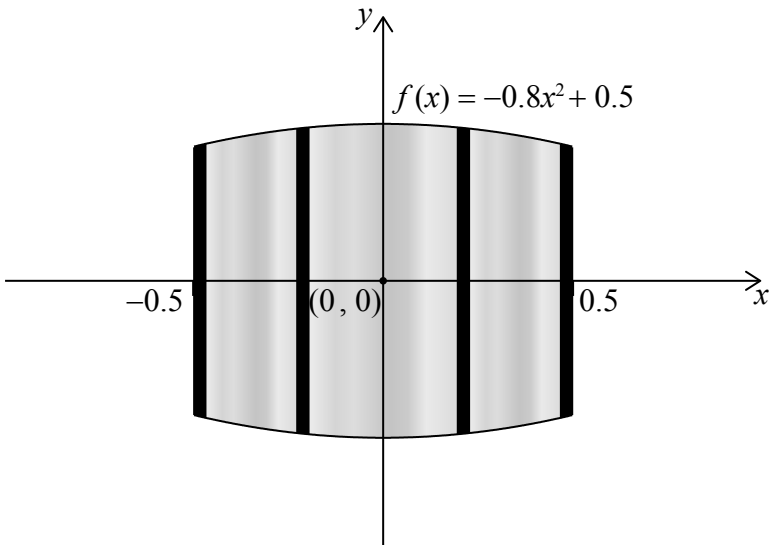
6. [Maximum mark: 6]

All lengths in this question are in metres.

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \leq x \leq 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f, the x-axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the x-axis. This is shown in the following diagram.



(a) Use the model to find the volume of the barrel. [3]

(b) The empty barrel is being filled with water. The volume $V\,\text{m}^3$ of water in the barrel after t minutes is given by $V = 0.8\left(1 - \text{e}^{-0.1t}\right)$. How long will it take for the barrel to be half-full? [3]

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7. [Maximum mark: 6]

A jar contains 5 red discs, 10 blue discs and m green discs. A disc is selected at random and replaced. This process is performed four times.

(a) Write down the probability that the first disc selected is red. [1]

(b) Let X be the number of red discs selected. Find the smallest value of m for which $\text{Var}(X) < 0.6$. [5]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Ten students were surveyed about the number of hours, x, they spent browsing the Internet during week 1 of the school year. The results of the survey are given below.

$$\sum_{i=1}^{10} x_i = 252,\ \sigma = 5 \text{ and median} = 27.$$

(a) Find the mean number of hours spent browsing the Internet. [2]

(b) During week 2, the students worked on a major project and they each spent an additional five hours browsing the Internet. For week 2, write down

(i) the mean;

(ii) the standard deviation. [2]

(c) During week 3 each student spent 5% less time browsing the Internet than during week 1. For week 3, find

(i) the median;

(ii) the variance. [6]

(d) During week 4, the survey was extended to all 200 students in the school. The results are shown in the cumulative frequency graph on the following page.

(i) Find the number of students who spent between 25 and 30 hours browsing the Internet.

(ii) Given that 10% of the students spent more than k hours browsing the Internet, find the maximum value of k. [6]

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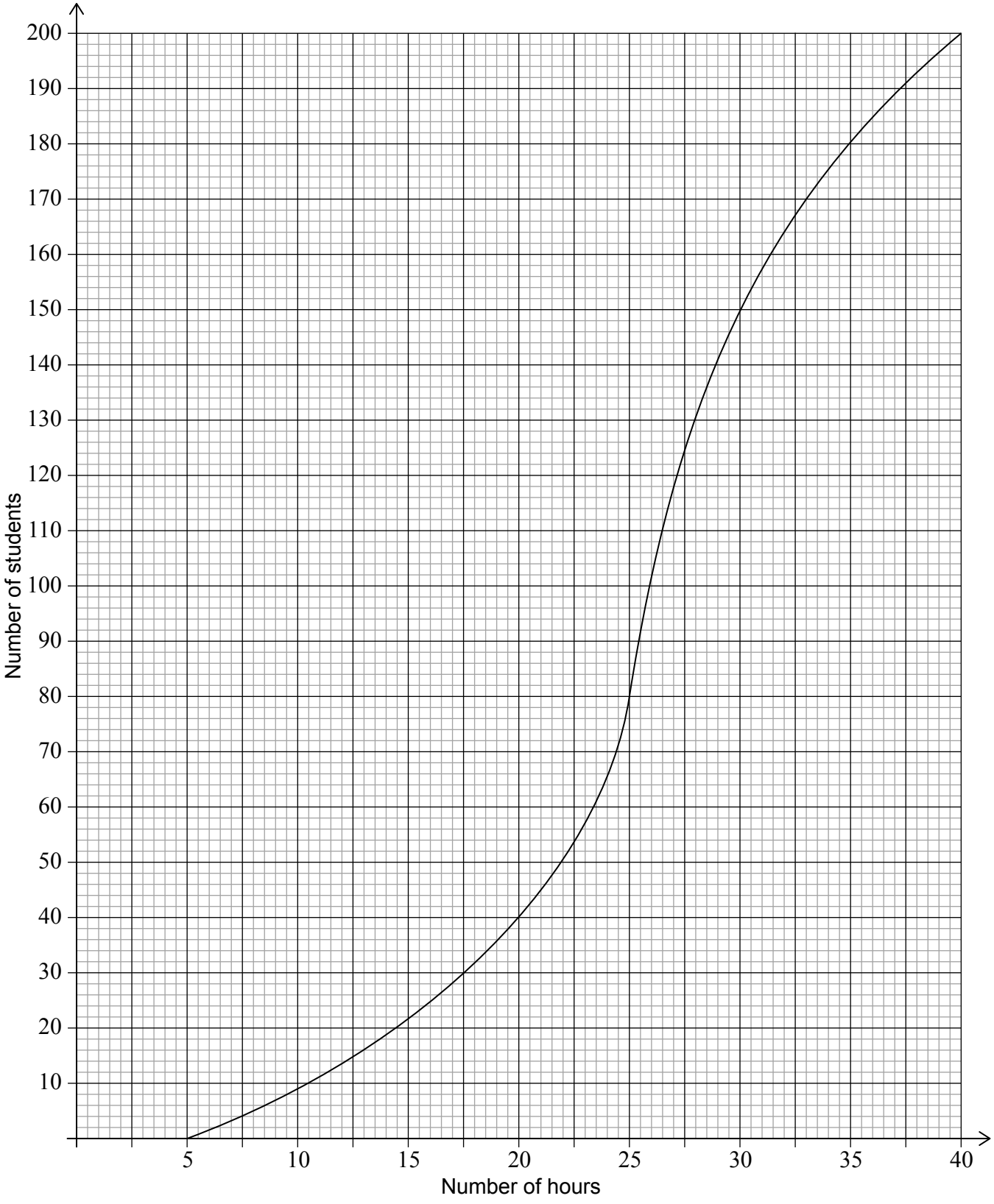
(Question 8 continued)



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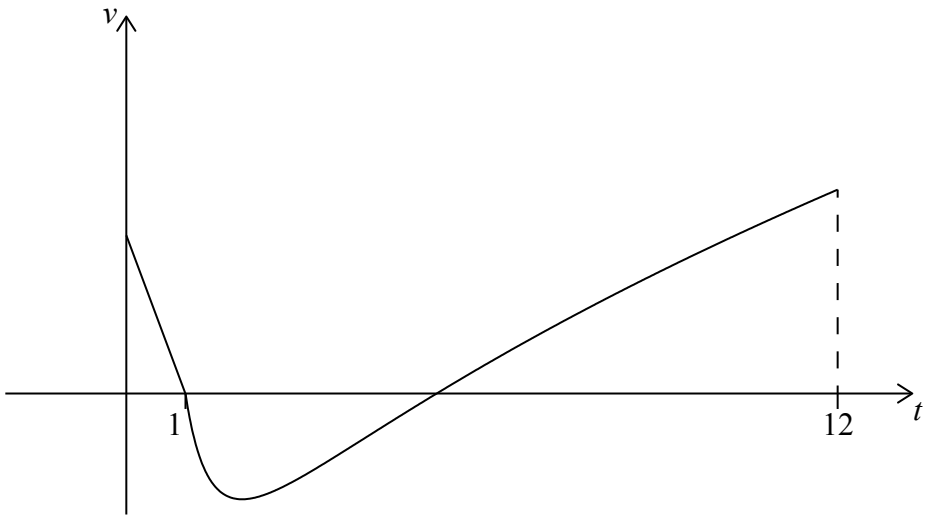
9. [Maximum mark: 14]

A particle P starts from a point A and moves along a horizontal straight line. Its velocity $v \text{ cm s}^{-1}$ after t seconds is given by

$$v(t) = \begin{cases} -2t + 2, \text{ for } 0 \le t \le 1 \\ 3\sqrt{t} + \frac{4}{t^2} - 7, \text{ for } 1 \le t \le 12 \end{cases}$$

The following diagram shows the graph of v.



(a) Find the initial velocity of P. [2]

P is at rest when $t = 1$ and $t = p$.

(b) Find the value of p. [2]

When $t = q$, the acceleration of P is zero.

(c) (i) Find the value of q.

(ii) Hence, find the **speed** of P when $t = q$. [4]

(d) (i) Find the total distance travelled by P between $t = 1$ and $t = p$.

(ii) Hence or otherwise, find the displacement of P from A when $t = p$. [6]

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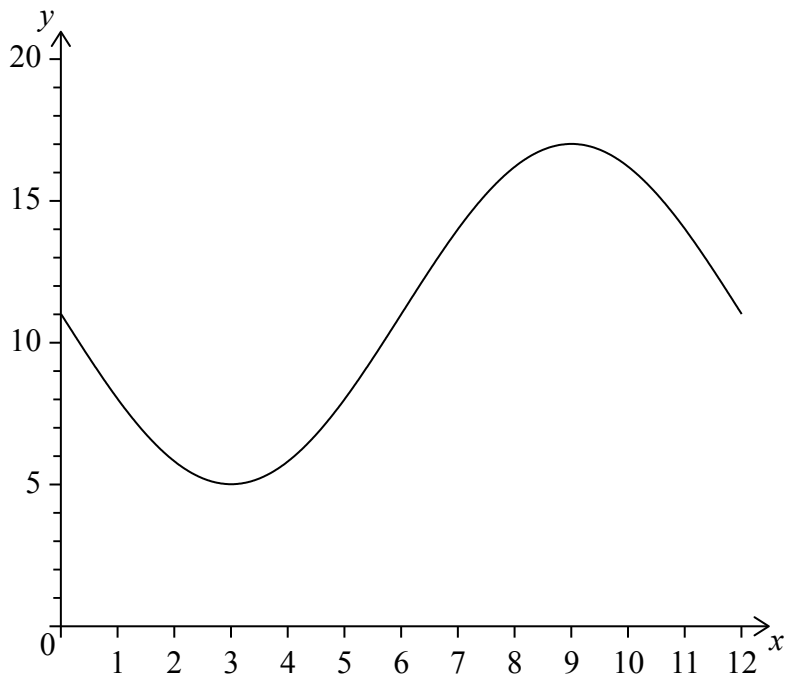
10. [Maximum mark: 15]

The following diagram shows the graph of $f(x) = a\sin bx + c$, for $0 \le x \le 12$.



The graph of f has a minimum point at $(3, 5)$ and a maximum point at $(9, 17)$.

(a) (i) Find the value of c.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a. [6]

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5, 17)$.

(b) (i) Write down the value of k.

(ii) Find $g(x)$. [3]

The graph of g changes from concave-up to concave-down when $x = w$.

(c) (i) Find w.

(ii) Hence or otherwise, find the maximum positive rate of change of g. [6]

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