Key

IB Applications: Simple Harmonic Motion

Key ideas:

1. Find the amplitude, period, midline, and phase shift [if any].
2. Do your best to sketch a simple graph showing maximum and minimum points.
3. When needed, write a trig function to represent the information.
4. Check the equation on your graphing calculator.

Example 1: The height of water at a particular point in a harbor is y meters at time t after low tide: $y = 8 - 6\cos(0.5t)$. Sketch the graph. What are the maximum and minimum heights of the water? How long is it between low tides?

$A = -6$

$P = \frac{2\pi}{1/2} = 4\pi$

$M = +8$

PS = none

max = 14 m

min = 2 m

time between low tides = 4π

Example 2: A model for the height h meters of the tide above a harbor entrance t hours after noon is given by the equation $h = 6 + 4\sin\frac{1}{2}t$. A boat can enter the harbor when the height of the tide is at least 8 meters.

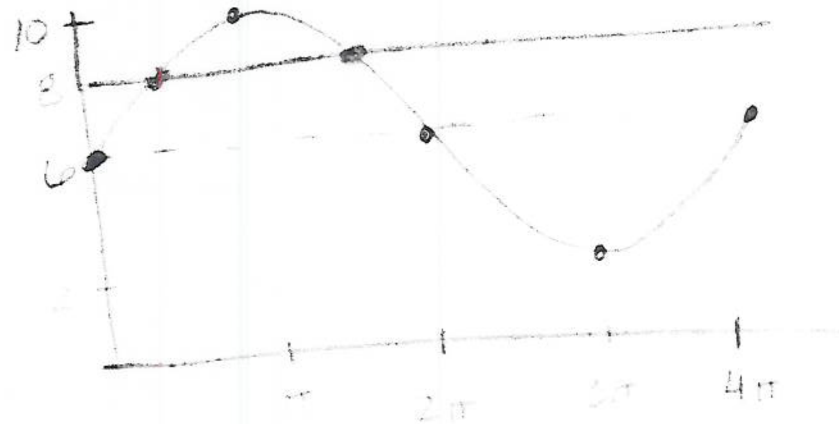
Find the first time after noon that a boat can enter the harbor, and how long it is safe for boats to enter and leave the harbor.

$A = 4$

$Pd = \frac{2\pi}{1/2} = 4\pi$

$M = +6$

$PS = $ none



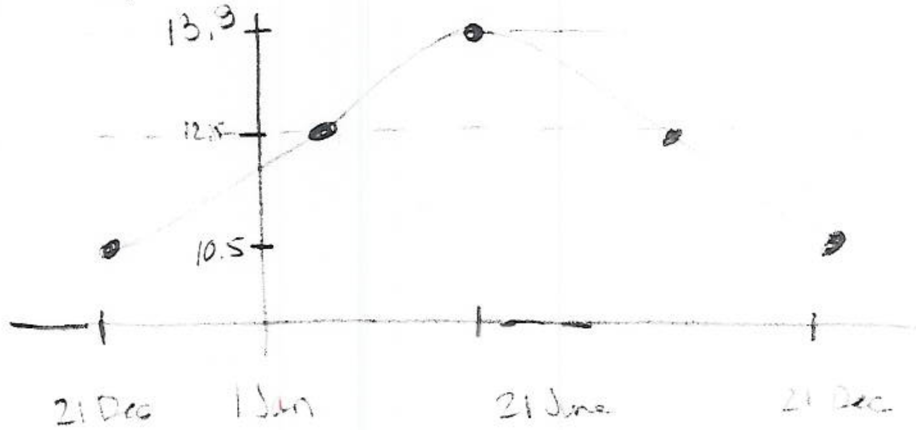
$t = 1.05, 5.24$

1^{st} time = 1:03 pm

$5.24 - 1.05 = 4.19$

safe = 4 hours, 11 minutes

Example 3: The shortest day in Bahrain is 21 December with 10.5 hours of daylight. The longest day is 21 June with 13.8 hours of daylight. Write an equation in the form $h(t) = a\cos b(x + c) + d$ for the hours of daylight where t = 0 represents 1 January.



d = midline = $\frac{13.8 + 10.5}{2}$ = 12.15

a = amp = 13.8 − 12.15 = 1.65

period = 365 $b = \frac{2\pi}{365}$

c = phase shift = +11 days

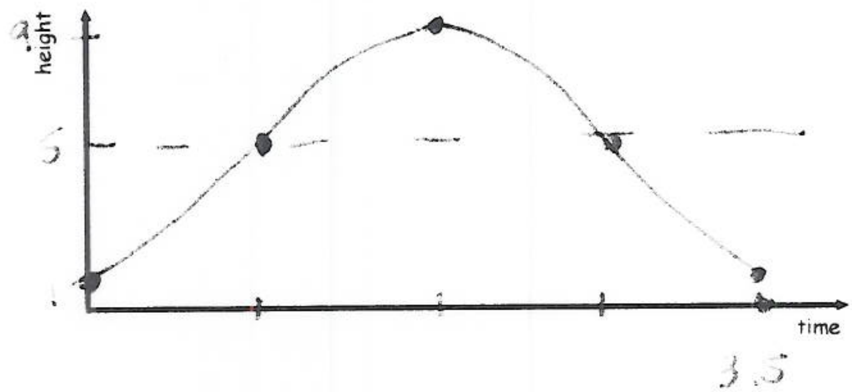
$$h(t) = -1.65\cos\left(\frac{2\pi}{365}(x+11)\right) + 12.15$$

IB Simple Harmonic Motion Homework Problems

1. A signal buoy in the sea floats up and down. During a storm, its height varies from 1 foot to 9 feet, and there are 3.5 seconds from one crest to the next.

   Write an equation to describe the height (h) of the point at any time t, measured in seconds. Assume the height at t = 0 is 1 foot.

   Draw one cycle:



Amplitude: 4

Period: 3.5 $\quad b = \frac{2\pi}{3.5} = 1.795$

Midline: 5

Equation: $-4\cos\left(\frac{2\pi}{3.5}t\right) + 5$

What will be the height of the buoy after 30 seconds?

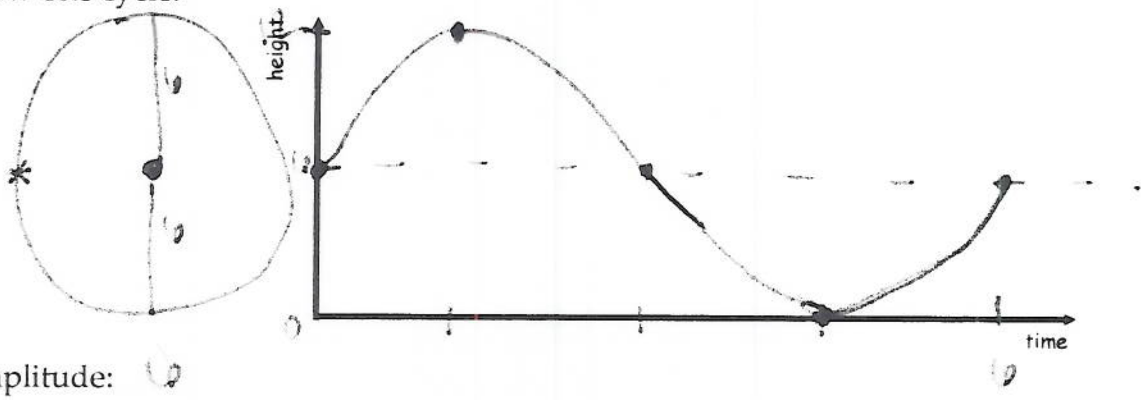
$h(30) = -4\cos\left(\frac{2\pi}{3.5}(30)\right) + 5$

$= 8.6$ Feet

2. A wheel of a bicycle is turning clockwise at 10 rpm [revolutions per minute]. The diameter is 12 feet. At t = 0, a point on the left side of the tire is 6 feet off the ground.

Write an equation to describe the height (h) of the point at any time t, measured in seconds.

Draw one cycle:



Amplitude: 6

Period: $\frac{60}{10} = 6$ sec. $\quad b = \frac{2\pi}{6} = \frac{\pi}{3}$

Midline: 6

Equation: $h(t) = 6\sin\left(\frac{\pi}{3}t\right) + 6$

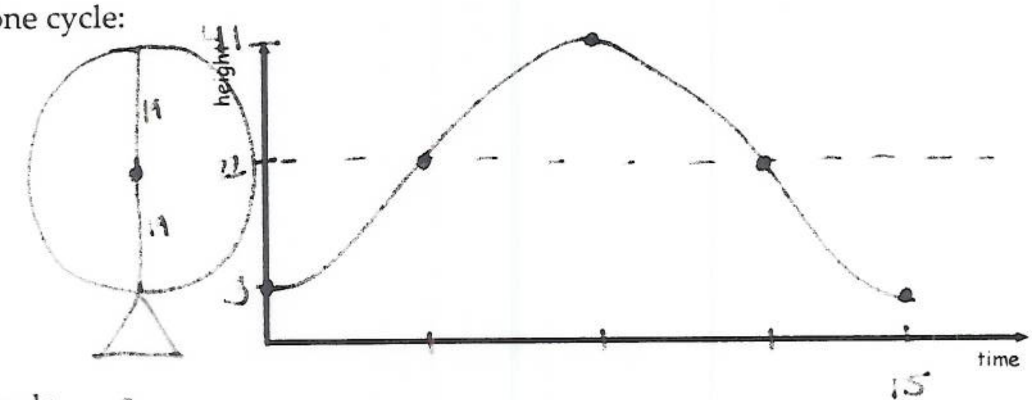
When will the point first reach a height of 12 feet?

$t = 1.5$ sec.

3. As you ride a Ferris wheel, you are experiencing periodic motion. Consider the center of the Ferris wheel to be the equilibrium point. A particular wheel has a diameter of 38 feet, and the seats of the Ferris wheel clear the ground by 3 feet. The wheel makes 4 revolutions each minute.

Write an equation to describe the height (h) of the seat that starts at the bottom of the wheel at time t = 0, measured in seconds.

Draw one cycle:



Amplitude: 19

Period: $\frac{60}{4} = 15$ $\qquad b = \frac{2\pi}{15} =$

Midline: 22

Equation: $-19\cos\left(\frac{2\pi}{15}t\right) + 22$

Find the height of the seat after 22 seconds.

$h(22) = 40.6$ ft.

How long after the ride starts will the seat be 27 feet above the ground?

$t = 4.59$ sec.

4. [2008 Specimen] A spring is suspended from the ceiling. It is pulled down and released, and then oscillates up and down. Its length, l centimeters, is modeled by the function $l = 33 + 5\cos((720t)°)$, where t is time in seconds after release.

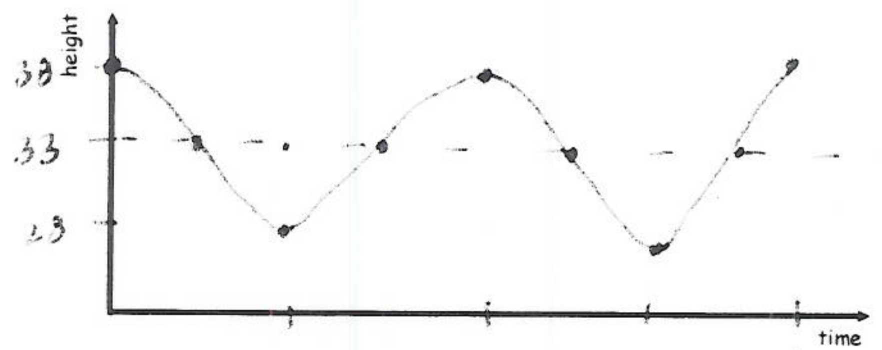
Amplitude: 5

$(2\pi = 360°)$ Period: $\frac{360}{720} = 0.5$

Midline: 33

Draw two cycles:

$l(0) = 38$



Without using a calculator:

a) Find the length of the spring after 1 second. [2]
b) Find the minimum length of the spring. [3]
c) Find the first time at which the length is 33 cm. [3]
d) What is the period of the motion? [2]

a) $l(1) = 38$ cm

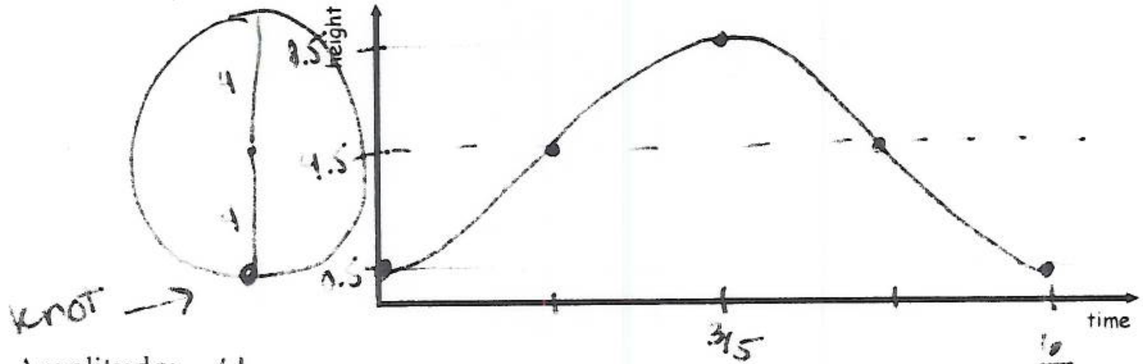
b) 28 cm

c) $t = 0.125$ sec

d) period $= 0.5$ sec.

5. A cowboy spins a lasso in a vertical circle. The diameter of the loop is 8 feet, and the loop spins 50 times each minute. If the lowest point on the rope is 6 inches (0.5 feet…) above the ground, write an equation to describe the height of the knot after t seconds.

Draw one cycle:



Amplitude: 4

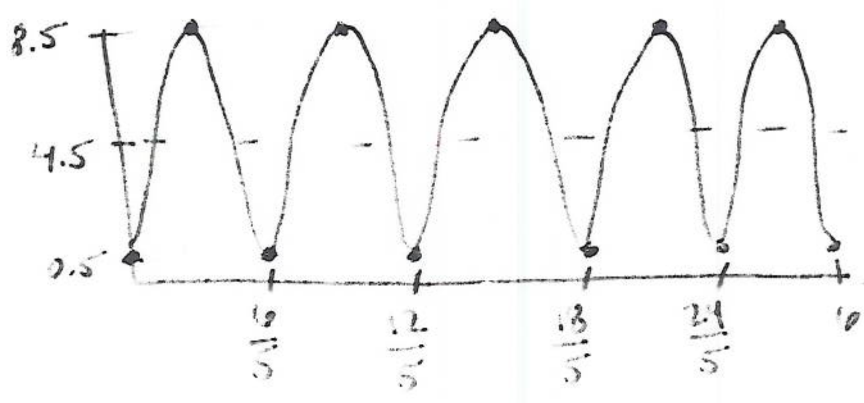
Period: $\frac{60}{50} = \frac{6}{5}$ $\qquad b = \frac{2\pi}{6/5} = \frac{5\pi}{3}$

Equation: $-4\cos\left(\frac{5\pi}{3}x\right) + 0.5$

Draw a graph to show the motion of the lasso in 6 seconds:

$\frac{6}{5}$ sec = 1 cycle

6 sec = 5 cycles



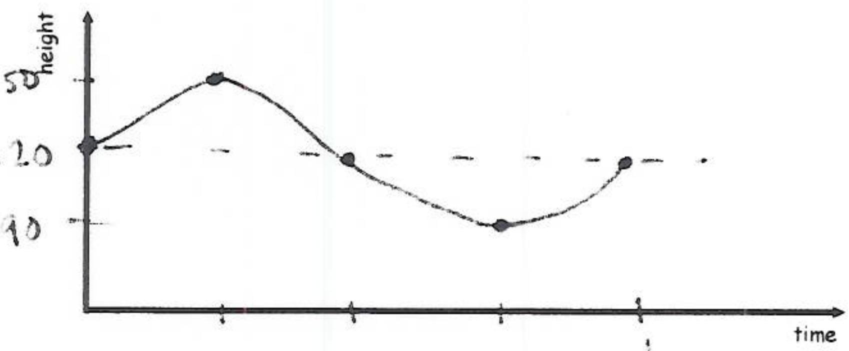
6. The function $P = 120 + 30\sin 2\pi t$ models the blood pressure (in millimeters of mercury) for a person who has high blood pressure; t represents seconds.

Draw one cycle:

$A = 30$

$M = 120$

$Pd = \frac{2\pi}{2\pi} = 1$



What are the expected maximum and minimum blood pressures for this patient?

max = 150 mL

min = 90 mL

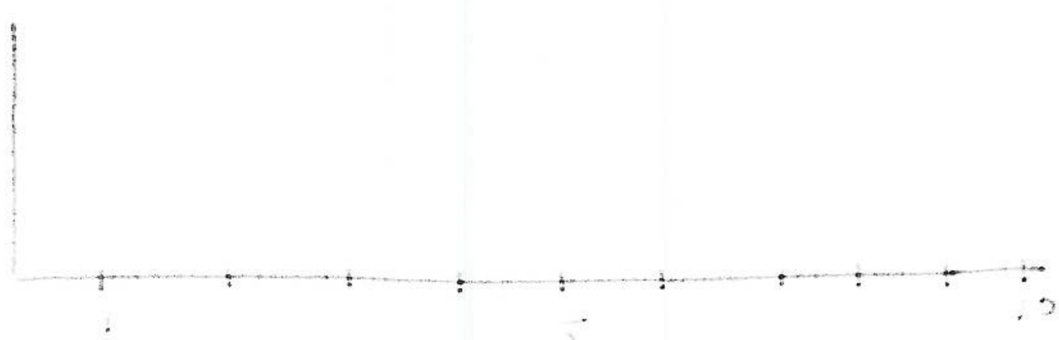
How many heartbeats are there each minute?

cycle

1 beat/sec

60 beats/min

Graph this function to model a 10 second time interval:



7. At a certain latitude in the northern hemisphere, the number, *d*, of hours of daylight in each day of the year is modeled by $d = A + B\sin kt$, where *t* is the number of days after the spring equinox.

a) Assuming that the number of hours of daylight follows an annual cycle of 365 days, find the value of k, giving your answer correct to 3 decimal places.

$$365 = \frac{2\pi}{k}$$

$$k = 0.0172$$

b) The shortest and longest days have 6 and 18 hours of daylight, respectively. Find the values of A and B.

$$A = \frac{6+18}{2} = 12$$

$$B = \frac{18-6}{2} = 6$$

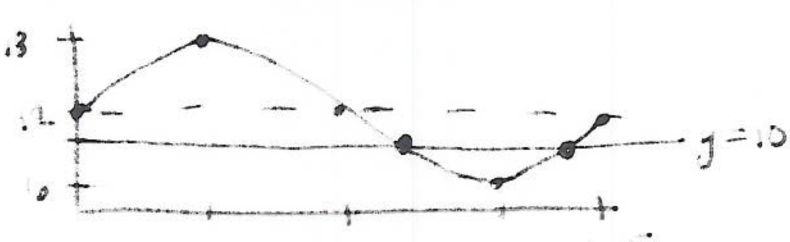
c) Write an equation for d. Use this equation to find, in hours and minutes, the amount of daylight on New Year's Day, which is 80 days before the spring equinox.

$$d = 12 + 6\sin(0.0172t)$$

$t = -80$

$d(-80) = 6.113$ or 6 hrs. and 7 min

d) A town in this latitude holds a fair twice a year on those days having exactly 10 hours of daylight. Find, in relation to the spring equinox, which two days these are. Sketch the graph you used.



$Y_1 = d$

$Y_2 = 10$

window:

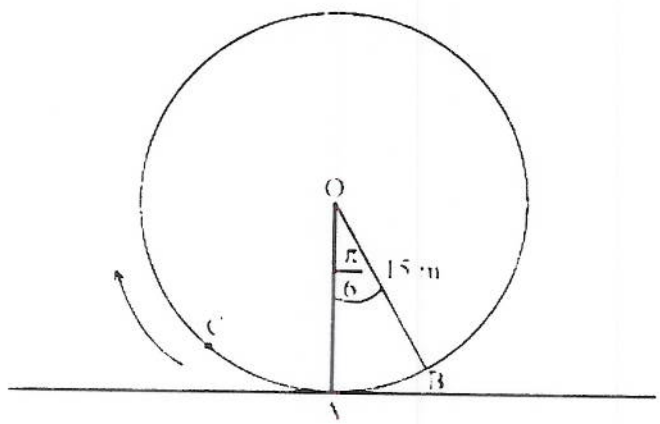
$X_{max} = 365$

$Y_{max} = 20$

$t = 202$ and 345

Use Formulas

8. [November 2007] A Ferris wheel with center O and a radius of 15 metres is represented in the diagram below. Initially, seat A is at ground level. The next seat is B, where $\angle AOB = \dfrac{\pi}{6}$.



(a) Find the length of the arc AB L = 7.85m [2 marks]

(b) Find the area of the sector AOB A = 118 m² [2 marks]

(c) The wheel turns clockwise through an angle of $\dfrac{2\pi}{3}$. Find the height of A above the ground 22.5 m [3 marks]

The height h metres, of seat C above the ground after t minutes, can be modelled by the function

$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right).$$

(d) (i) Find the height of seat C when $t = \dfrac{\pi}{4}$ 25.6 m

(ii) Find the initial height of seat C

$t=0$ 4.39 m

(iii) Find the time at which seat C first reaches its highest point [5 marks]

1.18 min