CHAPTER 7

Section 7−1

2. Answers will vary.

4. In the calculator window one can never see the entire domain of most functions. Moreover, the resolution can never be enough to distinguish small differences in values. The calculator screen can never prove that an equation is an identity. However, if the graphs of the left and the right sides functions differ visibly, then this shows that the equation is not an identity.

6. Verify: $\cos\theta \csc\theta = \cot\theta$

$\cos\theta \csc\theta = \cos\theta \cdot \dfrac{1}{\sin\theta}$	Reciprocal Identity
$= \dfrac{\cos\theta}{\sin\theta}$	Algebra
$= \cot\theta$	Quotient Identity

8. Verify: $\tan\theta \csc\theta \cos\theta = 1$

$\tan\theta \csc\theta \cos\theta = \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\sin\theta} \cdot \cos\theta$	Quotient and Reciprocal Identities
$= 1$	Algebra

10. Verify: $\cot(-x)\tan(x) = -1$

$\cot(-x)\tan(x) = \dfrac{\cos(-x)}{\sin(-x)} \cdot \dfrac{\sin x}{\cos x}$	Quotient Identity
$= \dfrac{\cos x}{-\sin x} \cdot \dfrac{\sin x}{\cos x}$	Identities for Negatives
$= -1$	Algebra

12. Verify: $\tan\alpha = \dfrac{\cos\alpha \sec\alpha}{\cot\alpha}$

$\dfrac{\cos\alpha \sec\alpha}{\cot\alpha} = \dfrac{\cos\alpha \cdot \frac{1}{\cos\alpha}}{\frac{\cos\alpha}{\sin\alpha}}$	Quotient and Reciprocal Identities
$= \dfrac{1}{\frac{\cos\alpha}{\sin\alpha}}$	Algebra
$= \dfrac{\sin\alpha}{\cos\alpha}$	Algebra
$= \tan\alpha$	Quotient Identity

14. Verify: $\tan u + 1 = \sec u(\sin u + \cos u)$

$\sec u(\sin u + \cos u) = \dfrac{1}{\cos u}(\sin u + \cos u)$	Reciprocal Identity
$= \dfrac{\sin u}{\cos u} + \dfrac{\cos u}{\cos u}$	Algebra
$= \tan u + 1$	Quotient Identity

16. Verify: $\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \cot x - \tan x$

$\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \dfrac{\cos^2 x}{\sin x \cos x} - \dfrac{\sin^2 x}{\sin x \cos x}$ Algebra

$= \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}$ Algebra

$= \cot x - \tan x$ Quotient Identity

18. Verify: $\dfrac{\cos^2 t}{\sin t} + \sin t = \csc t$

$\dfrac{\cos^2 t}{\sin t} + \sin t = \dfrac{\cos^2 t + \sin^2 t}{\sin t}$ Algebra

$= \dfrac{1}{\sin t}$ Pythagorean Identity

$= \csc t$ Reciprocal Identity

20. Verify: $\dfrac{\sin u}{1 - \cos^2 u} = \csc u$

$\dfrac{\sin u}{1 - \cos^2 u} = \dfrac{\sin u}{\sin^2 u}$ Pythagorean Identity

$= \dfrac{1}{\sin u}$ Algebra

$= \csc u$ Reciprocal Identity

22. Verify: $(1 - \sin t)(1 + \sin t) = \cos^2 t$

$(1 - \sin t)(1 + \sin t) = 1 - \sin t + \sin t - \sin^2 t$ Algebra

$= 1 - \sin^2 t$ Algebra

$= \cos^2 t$ Pythagorean Identity

24. Verify: $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$(\sin x + \cos x)^2 = \sin^2 x + 2 \sin x \cos x + \cos^2 x$ Algebra

$= \sin^2 x + \cos^2 x + 2 \sin x \cos x$ Algebra

$= 1 + 2 \sin x \cos x$ Pythagorean Identity

26. Verify: $(\csc t - 1)(\csc t + 1) = \cot^2 t$

$(\csc t - 1)(\csc t + 1) = \csc^2 t - \csc t + \csc t - 1$ Algebra

$= \csc^2 t - 1$ Algebra

$= 1 + \cot^2 t - 1$ Pythagorean Identity

$= \cot^2 t$ Algebra

28. Verify: $\sec^2 u - \tan^2 u = 1$

$\sec^2 u - \tan^2 u = 1 + \tan^2 u - \tan^2 u$ Pythagorean Identity

$= 1$ Algebra

30. Verify: $\sin m(\csc m - \sin m) = \cos^2 m$

$\sin m(\csc m - \sin m) = \sin m\left(\dfrac{1}{\sin m} - \sin m\right)$ Reciprocal Identity

$= 1 - \sin^2 m$ Algebra

$= \cos^2 m$ Pythagorean Identity

32. Plug in $x = 0$

34. Plug in $x = 5$

Left side: $\sqrt{0^2 - 10\cdot 0 + 25} = \sqrt{25} = 5$
Right side: $0 - 5 = -5$
The equation is not an identity.

Left side: $|5 - 4| = |1| = 1$
Right side: $\sqrt{5^2 - 16} = \sqrt{9} = 3$
The equation is not an identity.

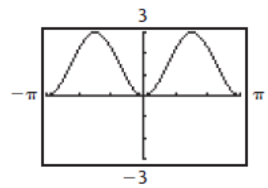
36. Plug in $x = \frac{\pi}{2}$

Left side: $\sin^2 \frac{\pi}{2} - \cos^2 \frac{\pi}{2} = 1^2 - 0^2 = 1$

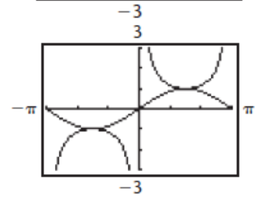
Right side: -1
The equation is not an identity.

38. Plug in $x = 0$
Left side: $\cos(-0) = \cos 0 = 1$
Right side: $-\cos 0 = -1$
The equation is not an identity.

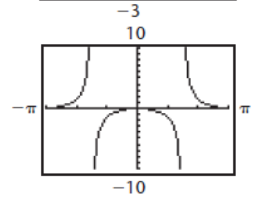
40. The two graphs appear to coincide.
The equation $3 - 3\cos^2 x = 3\sin^2 x$ appears to be an identity.



42. The two graphs do not appear to coincide.
The equation $\sec x \cot x = \sin x$ is not an identity.



44. The two graphs appear to coincide.
The equation $\cos x - \sec x = -\sin x \tan x$ appears to be an identity.



46. Not an identity. If x is negative, the left side is -5.

48. Yes. As long as x is positive, the equation is always true, and the domain of both sides is $x > 0$.

50. Not an identity. $\sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \neq 1$.

52. Yes. $\pi - \pi\cos^2 x = \pi(1 - \cos^2 x) = \pi\sin^2 x$ by the Pythagorean Identity.

54. Verify: $\dfrac{1-\cos^2 y}{(1-\sin y)(1+\sin y)} = \tan^2 y$

$$\frac{1-\cos^2 y}{(1-\sin y)(1+\sin y)} = \frac{\sin^2 y}{1-\sin^2 y}$$ Pythagorean Identity, Algebra

$$= \frac{\sin^2 y}{\cos^2 y}$$ Pythagorean Identity

$$= \left(\frac{\sin y}{\cos y}\right)^2$$ Algebra

$$= \tan^2 y$$ Quotient Identity

56. Verify: $\sin\theta + \cos\theta = \dfrac{\tan\theta + 1}{\sec\theta}$

$$\frac{\tan\theta+1}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}+1}{\frac{1}{\cos\theta}} \cdot \frac{\cos\theta}{\cos\theta}$$ Quotient Identity, Algebra

$$= \sin\theta + \cos\theta$$ Algebra

58. Verify: $1 - \sin y = \dfrac{\cos^2 y}{1+\sin y}$

$$\frac{\cos^2 y}{1+\sin y} = \frac{\cos^2 y}{1+\sin y} \cdot \frac{1-\sin y}{1-\sin y} \qquad \text{Algebra}$$

$$= \frac{(1-\sin^2 y)(1-\sin y)}{(1-\sin^2 y)} \qquad \text{Pythagorean Identity}$$

$$= 1 - \sin y \qquad \text{Algebra}$$

60. Verify: $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

$$\sec^2 x + \csc^2 x = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \qquad \text{Reciprocal Identity}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} \qquad \text{Algebra}$$

$$= \frac{1}{\cos^2 x \sin^2 x} \qquad \text{Pythagorean Identity}$$

$$= \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x} \qquad \text{Algebra}$$

$$= \sec^2 x \csc^2 x \qquad \text{Reciprocal Identity}$$

62. Verify: $\dfrac{1+\sec\theta}{\sin\theta+\tan\theta} = \csc\theta$

$$\frac{1+\sec\theta}{\sin\theta+\tan\theta} = \frac{1+\frac{1}{\cos\theta}}{\sin\theta+\frac{\sin\theta}{\cos\theta}} \cdot \frac{\cos\theta}{\cos\theta} \qquad \text{Quotient and Reciprocal Identities, Algebra}$$

$$= \frac{\cos\theta+1}{\sin\theta\cos\theta+\sin\theta} \qquad \text{Algebra}$$

$$= \frac{\cos\theta+1}{\sin\theta(\cos\theta+1)} \qquad \text{Algebra}$$

$$= \frac{1}{\sin\theta} \qquad \text{Algebra}$$

$$= \csc\theta \qquad \text{Reciprocal Identity}$$

64. Verify: $\ln(\cot x) = \ln(\cos x) - \ln(\sin x)$

$$\ln(\cos x) - \ln(\sin x) = \ln\left(\frac{\cos x}{\sin x}\right) \qquad \text{Algebra}$$

$$= \ln(\cot x) \qquad \text{Quotient Identity}$$

66. Verify: $\dfrac{1-\csc y}{1+\csc y} = \dfrac{\sin y - 1}{\sin y + 1}$

$$\frac{1-\csc y}{1+\csc y} = \frac{1-\frac{1}{\sin y}}{1+\frac{1}{\sin y}} \cdot \frac{\sin y}{\sin y} \qquad \text{Reciprocal Identity, Algebra}$$

$$= \frac{\sin y - 1}{\sin y + 1} \qquad \text{Algebra}$$

68. Verify: $\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = 1$

$$\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 \qquad \text{Algebra}$$

$$= 1^2 \qquad \text{Pythagorean Identity}$$

$$= 1 \qquad \text{Algebra}$$

70. Verify: $\csc n - \dfrac{\sin n}{1+\cos n} = \cot n$

$\csc n - \dfrac{\sin n}{1+\cos n} = \dfrac{1}{\sin n} - \dfrac{\sin n}{1+\cos n}$	Reciprocal Identity
$= \dfrac{1+\cos n - \sin^2 n}{\sin n(1+\cos n)}$	Algebra
$= \dfrac{1-\sin^2 n + \cos n}{\sin n(1+\cos n)}$	Algebra
$= \dfrac{\cos^2 n + \cos n}{\sin n(1+\cos n)}$	Pythagorean Identity
$= \dfrac{\cos n(\cos n + 1)}{\sin n(1+\cos n)}$	Algebra
$= \dfrac{\cos n}{\sin n}$	Algebra
$= \cot n$	Quotient Identity

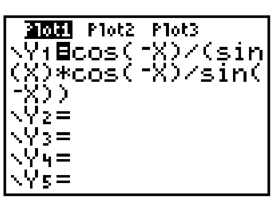
72. Verify: $\dfrac{\sin^2 t + 4\sin t + 3}{\cos^2 t} = \dfrac{3+\sin t}{1-\sin t}$

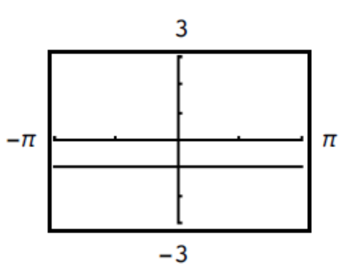
$\dfrac{\sin^2 t + 4\sin t + 3}{\cos^2 t} = \dfrac{(\sin t + 1)(\sin t + 3)}{1-\sin^2 t}$	Algebra, Pythagorean Identity
$= \dfrac{(1+\sin t)(3+\sin t)}{(1+\sin t)(1-\sin t)}$	Algebra
$= \dfrac{3+\sin t}{1-\sin t}$	Algebra

74. Verify: $\dfrac{\cos^3 u + \sin^3 u}{\cos u + \sin u} = 1 - \sin u \cos u$

$\dfrac{\cos^3 u + \sin^3 u}{\cos u + \sin u} = \dfrac{(\cos u + \sin u)(\cos^2 u - \cos u \sin u + \sin^2 u)}{(\cos u + \sin u)}$	Algebra
$= \cos^2 u + \sin^2 u - \cos u \sin u$	Algebra
$= 1 - \sin u \cos u$	Pythagorean Identity

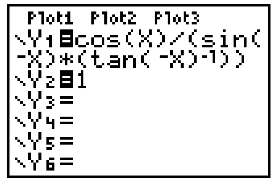
76. As the graph shows, $\dfrac{\cos(-x)}{\sin x \cot(-x)} = 1$ is not an identity since the left hand side is -1 for all x for which it is defined.

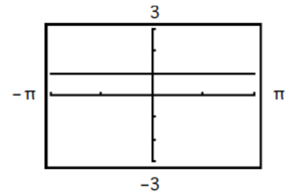




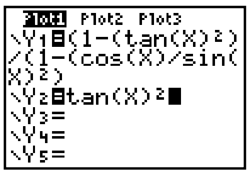
78. As the graph shows, $\dfrac{\cos x}{\sin(-x)\cot(-x)} = 1$ appears to be an identity.

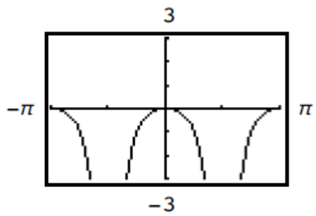
$$\begin{aligned}\frac{\cos x}{\sin(-x)\cot(-x)} &= \frac{\cos x}{-\sin x \cdot \frac{\cos(-x)}{\sin(-x)}} \\ &= \frac{\cos x}{-\sin x \cdot \frac{\cos x}{-\sin x}} \\ &= 1\end{aligned}$$



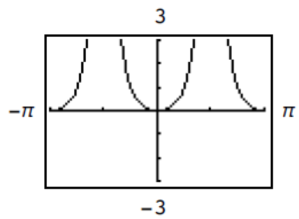


80. As the graphs show, $\dfrac{1-\tan^2 x}{1-\cot^2 x} = \tan^2 x$ does not appear to be an identity.

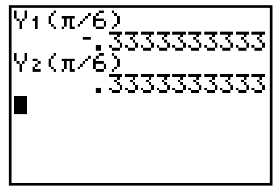




Y_1 = LHS

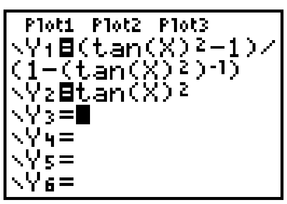


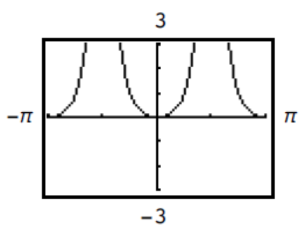
Y_2 = RHS



The last screen shows a value of x for which both sides are defined but not equal.

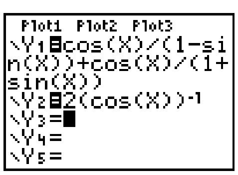
82. As the graph shows, $\dfrac{\tan^2 x - 1}{1-\cot^2 x} = \tan^2 x$ appears to be an identity.

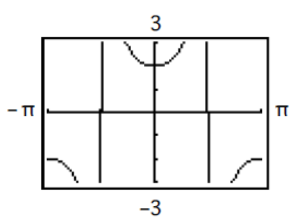




$\dfrac{\tan^2 x - 1}{1-\cot^2 x}$	$= \dfrac{\frac{\sin^2 x}{\cos^2 x} - 1}{1 - \frac{\cos^2 x}{\sin^2 x}}$	Quotient Identity
	$= \dfrac{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}{\frac{\sin^2 x - \cos^2 x}{\sin^2 x}}$	Algebra
	$= \dfrac{1}{\cos^2 x} \cdot \dfrac{\sin^2 x}{1}$	Pythagorean Identity
	$= \tan^2 x$	Algebra, Quotient Identity

84. As the graph shows, $\dfrac{\cos x}{1-\sin x} + \dfrac{\cos x}{1+\sin x} = 2\sec x$ appears to be an identity.





$\dfrac{\cos x}{1-\sin x} + \dfrac{\cos x}{1+\sin x}$	
$= \dfrac{\cos x(1+\sin x) + \cos x(1-\sin x)}{(1-\sin x)(1+\sin x)}$	Algebra
$= \dfrac{\cos x + \cos x \sin x + \cos x - \cos x \sin x}{1-\sin^2 x}$	Algebra
$= \dfrac{2\cos x}{\cos^2 x}$	Algebra, Pythagorean Identity
$= \dfrac{2}{\cos x}$	Algebra
$= 2\sec x$	Reciprocal Identity

86. Verify: $\dfrac{3\cos^2 z+5\sin z-5}{\cos^2 z} = \dfrac{3\sin z-2}{1+\sin z}$

$$\frac{3\cos^2 z+5\sin z-5}{\cos^2 z} = \frac{3(1-\sin^2 z)-5(1-\sin z)}{1-\sin^2 z}$$ Pythagorean Identity, Algebra

$$= \frac{(1-\sin z)[3(1+\sin z)-5]}{(1-\sin z)(1+\sin z)}$$ Algebra

$$= \frac{3(1+\sin z)-5}{1+\sin z}$$ Algebra

$$= \frac{3+3\sin z-5}{1+\sin z}$$ Algebra

$$= \frac{3\sin z-2}{1+\sin z}$$ Algebra

88. Verify: $\dfrac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y} = \dfrac{\tan x+\tan y}{1-\tan x\tan y}$

$$\frac{\tan x+\tan y}{1-\tan x\tan y} = \frac{\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}}{1-\frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}}\cdot\frac{\cos x\cos y}{\cos x\cos y}$$ Quotient Identity, Algebra

$$= \frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}$$ Algebra

90. Verify: $\dfrac{\cot\alpha+\cot\beta}{\cot\alpha\cot\beta-1} = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\frac{1}{\cot\alpha}+\frac{1}{\cot\beta}}{1-\frac{1}{\cot\alpha}\cdot\frac{1}{\cot\beta}}\cdot\frac{\cot\alpha\cot\beta}{\cot\alpha\cot\beta}$$ Reciprocal Identity, Algebra

$$= \frac{\cot\beta+\cot\alpha}{\cot\alpha\cot\beta-1}$$ Algebra

$$= \frac{\cot\alpha+\cot\beta}{\cot\alpha\cot\beta-1}$$ Algebra

92. (A) $\tan x = \dfrac{\sin x}{\cos x}$ (B) $\sin^2 x + \cos^2 x = 1$ (C) $\dfrac{1}{\cos x} = \sec x$

94. $\sqrt{1-\sin^2 x} = \cos x$; $\sqrt{1-\sin^2 x} \ge 0 \Rightarrow \cos x \ge 0 \Rightarrow x$ must be in quadrants I or IV.

96. $\sqrt{1-\cos^2 x} = |\sin x|$. Both sides $\ge 0 \Rightarrow x$ in all quadrants.

98. $\dfrac{\sin x}{\sqrt{1-\sin^2 x}} = -\tan x$. $\sqrt{1-\sin^2 x} > 0 \Rightarrow$ identity when $\sin x$ and $\tan x$ have opposite signs $\Rightarrow x$ in quadrants II or III.

100. $\sqrt{a^2-u^2} = \sqrt{a^2-a^2\cos^2 x} = \sqrt{a^2(1-\cos^2 x)}$

$= a\sqrt{\sin^2 x}$ and since for $0 < x < \pi$, $\sqrt{\sin^2 x} = \sin x$

$= a\sin x$

102. $\sqrt{a^2+u^2} = \sqrt{a^2+a^2\cot^2 x} = \sqrt{a^2(1+\cot^2 x)}$

$= a\sqrt{\csc^2 x}$ and since for $0 < x < \frac{\pi}{2}$, $\sqrt{\csc^2 x} = \csc x$

$= a \csc x$

Section 7−2

2. Since $\cos(x - y) = \cos x \cos y + \sin x \sin y$, $\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$;

since $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$, the right side reduces to $\sin x$.

4. It makes no difference how the angles are measured as long as it is done consistently.

6. Directly using the sum identity involves $\tan\frac{\pi}{2}$, which is undefined.

However, $\tan\left(\frac{\pi}{2}+x\right) = \dfrac{\sin\left(\frac{\pi}{2}+x\right)}{\cos\left(\frac{\pi}{2}+x\right)} = \dfrac{\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x}{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x} = \dfrac{\cos x}{-\sin x} = -\cot x$.

8. Plug in $x = 2$, $y = 1$.
Left side: $(2 - 1)^3 = 1^3 = 1$
Right side: $2^3 - 1^3 = 8 - 1 = 7$
The equation is not an identity.

10. Plug in $x = 2$, $y = \frac{\pi}{6}$.

Left side: $2\tan\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

Right side: $\tan\left(2\cdot\frac{\pi}{6}\right) = \sqrt{3}$

The equation is not an identity.

12. Plug in $x = \frac{\pi}{6}$, $y = \frac{\pi}{6}$.

Left side: $\tan\left(\frac{\pi}{6}+\frac{\pi}{6}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Right side: $\tan\frac{\pi}{6} + \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

The equation is not an identity.

14. Plug in $x = \frac{\pi}{2}$, $y = \frac{\pi}{6}$.

Left side: $\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Right side: $\sin\frac{\pi}{2} - \sin\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$

The equation is not an identity.

16. Plug in $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$.

Left side: $\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = \sin\frac{\pi}{2} = 1$

Right side: $\sin\frac{\pi}{4} + \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

The equation is not an identity.

18. $\cos(x + \pi) = \cos x \cos\pi - \sin x \sin\pi = -\cos x$
Not an identity.

20. $\cot(x + \pi) = \dfrac{\cos(x+\pi)}{\sin(x+\pi)} = \dfrac{\cos x\cos\pi - \sin x\sin\pi}{\sin x\cos\pi + \cos x\sin\pi}$

$= \dfrac{-\cos x}{-\sin x} = \cot x$ Yes

22. $\sec(2\pi - x) = \dfrac{1}{\cos(2\pi - x)} = \dfrac{1}{\cos 2\pi\cos(-x) + \sin 2\pi\sin x} = \dfrac{1}{\cos x} = \sec x$ Yes

24. $\cos\left(x-\frac{\pi}{2}\right) = \cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2} = \sin x$ Not an identity.

26. Verify: $\tan\left(\frac{\pi}{2}-x\right) = \cot x$

$\tan\left(\frac{\pi}{2}-x\right) = \frac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}$	Quotient Identity
$= \frac{\cos x}{\sin x}$	Cofunction Identity
$= \cot x$	Quotient Identity

28. Verify: $\sec\left(\frac{\pi}{2}-x\right) = \csc x$

$\sec\left(\frac{\pi}{2}-x\right) = \frac{1}{\cos\left(\frac{\pi}{2}-x\right)}$	Reciprocal Identity
$= \frac{1}{\sin x}$	Cofunction Identity
$= \csc x$	Reciprocal Identity

30. Verify: $\sin(\pi - x) = \sin x$

$\sin(\pi - x) = \sin \pi \cos x - \cos \pi \sin x$	Difference Identity
$= 0 \cos x - (-1) \sin x$	Known Values
$= \sin x$	Algebra

32.

$\tan(x + y) = \tan[x - (-y)]$	Algebra
$= \frac{\tan x - \tan(-y)}{1 + \tan x \tan(-y)}$	Difference Identity
$= \frac{\tan x + \tan y}{1 - \tan x \tan y}$	Identities for Negatives

34.

$\sin(x - 45°) = \sin x \cos 45° - \sin 45° \cos x$	Difference Identity
$= \sin x \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cos x$	Known Values
$= \frac{1}{\sqrt{2}} (\sin x - \cos x)$	Algebra

36.

$\cos(x + 180°) = \cos x \cos 180° - \sin x \sin 180°$	Sum Identity
$= \cos x(-1) - \sin x(0)$	Known Values
$= -\cos x$	Algebra

38.

$\tan\left(\frac{\pi}{4}-x\right) = \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x}$	Difference Identity
$= \frac{1 - \tan x}{1 + \tan x}$	Known Values

40. $\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \sin 30° \cos 45°$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}} \text{ or } \frac{\sqrt{3}+1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

42. $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

44. $\sin 22° \cos 38° + \cos 22° \sin 38° = \sin(22° + 38°) = \sin 60° = \frac{\sqrt{3}}{2}$

46. $\frac{\tan 110° - \tan 50°}{1+\tan 110° \tan 50°} = \tan(110° - 50°) = \tan 60° = \sqrt{3}$

48. $\sin x = \frac{2}{3}$ (QII), $\cos y = -\frac{1}{4}$ (QIII).

Angle x: $a = -\sqrt{3^2 - 2^2} = -\sqrt{5} \Rightarrow \cos x = -\frac{\sqrt{5}}{3}$

Angle y: $b = -\sqrt{4^2 - (-1)^2} = -\sqrt{15} \Rightarrow \sin y = -\frac{\sqrt{15}}{4}$

$$\sin(x - y) = \sin x \cos y - \sin y \cos x = \frac{2}{3}\cdot\left(-\frac{1}{4}\right) - \frac{-\sqrt{15}}{4}\cdot\frac{-\sqrt{5}}{3} = \frac{-2-5\sqrt{3}}{12}$$

$$\tan x = -\frac{2}{\sqrt{5}},\ \tan y = \sqrt{15}$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{-\frac{2}{\sqrt{5}} + \sqrt{15}}{1 - \frac{-2}{\sqrt{5}}\cdot\sqrt{15}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{-2+5\sqrt{3}}{\sqrt{5}+2\sqrt{15}}$$

50. $\cos x = -\frac{1}{3}$ (QII), $\tan y = \frac{1}{2}$ (QIII)

Angle x: $b = \sqrt{3^2 - (-1)^2} = \sqrt{8} \Rightarrow \sin x = \frac{\sqrt{8}}{3}$, $\tan x = -\sqrt{8}$

Angle y: $c = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5} \Rightarrow \cos y = \frac{-2}{\sqrt{5}}$, $\sin y = \frac{-1}{\sqrt{5}}$

$$\sin(x - y) = \sin x \cos y - \sin y \cos x = \frac{\sqrt{8}}{3}\cdot\frac{-2}{\sqrt{5}} - \frac{-1}{\sqrt{5}}\cdot\frac{-1}{3} = \frac{-4\sqrt{2}-1}{3\sqrt{5}}$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{-\sqrt{8}+\frac{1}{2}}{1-(-\sqrt{8})\left(\frac{1}{2}\right)}\cdot\frac{2}{2} = \frac{1-4\sqrt{2}}{2+2\sqrt{2}}$$

52. Verify: $\sin 2x = \sin(x + x)$ — Algebra

$\sin(x + x) = \sin x \cos x + \cos x \sin x$ — Sum Identity

$= 2 \sin x \cos x$ — Algebra

54. Verify: $\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

$\cot(x - y) = \frac{\cos(x-y)}{\sin(x-y)}$ — Quotient Identity

$$= \frac{\cos x \cos y + \sin x \sin y}{\sin x \cos y - \sin y \cos x} \cdot \frac{\frac{1}{\sin x \sin y}}{\frac{1}{\sin x \sin y}}$$ Difference Identities, Algebra

$$= \frac{\cot x \cot y + 1}{\cot y - \cot x}$$ Algebra, Quotient Identity

56. Verify: $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$

$\cot 2x = \cot(x + x)$ Algebra

$$= \frac{\cos(x+x)}{\sin(x+x)}$$ Quotient Identity

$$= \frac{\cos x \cos x - \sin x \sin x}{\sin x \cos x + \sin x \cos x}$$ Sum Identity

$$= \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x} \cdot \frac{\frac{1}{\sin^2 x}}{\frac{1}{\sin^2 x}}$$ Algebra

$$= \frac{\cot^2 x - 1}{2\cot x}$$ Algebra, Quotient Identity

58. Verify: $\dfrac{\sin(u+v)}{\sin(u-v)} = \dfrac{\tan u + \tan v}{\tan u - \tan v}$

$$\frac{\sin(u+v)}{\sin(u-v)} = \frac{\sin u \cos v + \sin v \cos u}{\sin u \cos v - \sin v \cos u} \cdot \frac{\frac{1}{\cos u \cos v}}{\frac{1}{\cos u \cos v}}$$ Sum and Difference Identities, Algebra

$$= \frac{\tan u + \tan v}{\tan u - \tan v}$$ Algebra, Quotient Identity

60. Verify: $\tan x - \tan y = \dfrac{\sin(x-y)}{\cos x \cos y}$

$$\frac{\sin(x-y)}{\cos x \cos y} = \frac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}$$ Difference Identity

$$= \frac{\sin x \cos y}{\cos x \cos y} - \frac{\sin y \cos x}{\cos x \cos y}$$ Algebra

$= \tan x - \tan y$ Algebra, Quotient Identity

62. Verify: $\tan(x + y) = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \cdot \frac{\frac{1}{\tan x \tan y}}{\frac{1}{\tan x \tan y}}$$ Sum Identity, Algebra

$$= \frac{\frac{1}{\tan y} + \frac{1}{\tan x}}{\frac{1}{\tan x \tan y} - 1}$$ Algebra

$$= \frac{\cot y + \cot x}{\cot x \cot y - 1}$$ Reciprocal Identity

$$= \frac{\cot x + \cot y}{\cot x \cot y - 1}$$ Algebra

64. Verify: $\dfrac{\sin(x+h) - \sin x}{h} = \sin x\left(\dfrac{\cos h - 1}{h}\right) + \cos x\left(\dfrac{\sin h}{h}\right)$

$$\frac{\sin(x+h) - \sin x}{h} = \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$ Sum Identity

$$= \frac{\sin x(\cos h - 1) + \sin h \cos x}{h} \qquad \text{Algebra}$$

$$= \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) \qquad \text{Algebra}$$

66. $x = \frac{\pi}{3}, y = \frac{4\pi}{3}$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

Left Side: $\cos\left(\frac{\pi}{3} + \frac{4\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$

Right Side: $\cos\frac{\pi}{3}\cos\frac{4\pi}{3} - \sin\frac{\pi}{3}\sin\frac{4\pi}{3} = \frac{1}{2}\left(-\frac{1}{2}\right) - \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

Left Side: $\sin\left(\frac{\pi}{3} + \frac{4\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Right Side: $\sin\frac{\pi}{3}\cos\frac{4\pi}{3} + \cos\frac{\pi}{3}\sin\frac{4\pi}{3} = \frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right) + \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$

68. $x = \frac{5\pi}{4}, y = -\frac{3\pi}{4}$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

Left Side: $\cos\left[\frac{5\pi}{4} + \left(-\frac{3\pi}{4}\right)\right] = \cos\frac{\pi}{2} = 0$

Right Side: $\cos\frac{5\pi}{4}\cos\left(-\frac{3\pi}{4}\right) - \sin\frac{5\pi}{4}\sin\left(-\frac{3\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{2} - \frac{1}{2} = 0$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

Left Side: $\sin\left[\frac{5\pi}{4} + \left(-\frac{3\pi}{4}\right)\right] = \sin\frac{\pi}{2} = 1$

Right Side: $\sin\frac{5\pi}{4}\cos\left(-\frac{3\pi}{4}\right) + \cos\frac{5\pi}{4}\sin\left(-\frac{3\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{2} + \frac{1}{2} = 1$

70. $x = 3.042$, $y = 2.384$

$\sin(x - y) = \sin(3.042 - 2.384) = \sin(0.658) \approx 0.6115$

$\sin(x - y) = \sin x \cos y - \sin y \cos x = \sin 3.042 \cos 2.384 - \sin 2.384 \cos 3.042 \approx 0.6115$

$\tan(x + y) = \tan(3.042 + 2.384) = \tan 5.426 \approx -1.155$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\tan 3.042 + \tan 2.384}{1 - \tan 3.042 \tan 2.384} \approx -1.155$

72. $x = 128.3°$, $y = 25.62°$

$\sin(x - y) = \sin(128.3° - 25.62°) = \sin(102.68°) \approx 0.9756$

$\sin x \cos y - \sin y \cos x = \sin 128.3° \cos 25.62° - \sin 25.62° \cos 128.3° \approx 0.9756$

$\tan(x + y) = \tan(128.3° + 25.62°) = \tan(153.92°) \approx -0.4895$

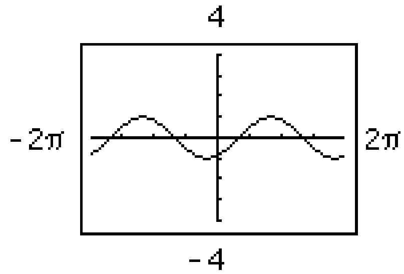
$\frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\tan 128.3° + \tan 25.62°}{1 - \tan 128.3° \tan 25.62°} \approx -0.4895$

74. Evaluate each side for a particular set of values of x and y for which each side is defined. If the left side is not equal to the right side, then the equation is not an identity. For example, for $x = 2$ and $y = 1$, both sides are defined, but are not equal.

76. $y = \sin\left(x - \frac{\pi}{3}\right) = \sin x \cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos x = \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$

$y_1 = \sin(x - \pi/3)$

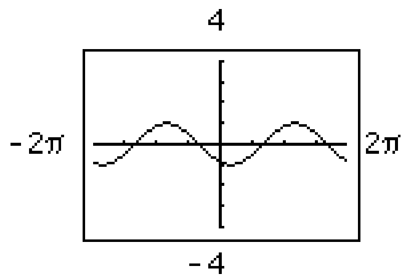
$y_2 = \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$



78. $y = \cos\left(x + \frac{5\pi}{6}\right) = \cos x \cos\left(\frac{5\pi}{6}\right) - \sin x \sin \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

$y_1 = \cos(x + 5\pi/6)$

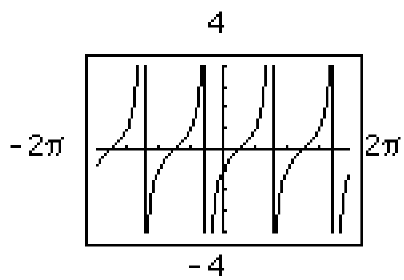
$y_2 = -\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$



80. $y = \tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - \tan\frac{\pi}{4}}{1 + \tan x \tan\frac{\pi}{4}} = \frac{\tan x - 1}{1 + \tan x}$

$y_1 = \tan(x - \pi/4)$

$y_2 = \frac{\tan x - 1}{1 + \tan x}$



82. $\cos\left[\sin^{-1}\left(-\frac{3}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right] = \cos\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]\cos\left[\cos^{-1}\left(\frac{4}{5}\right)\right] - \sin\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]\sin\left[\cos^{-1}\left(\frac{4}{5}\right)\right]$

$= \left(\frac{4}{5}\right)\left(\frac{4}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{16}{25} + \frac{9}{25} = 1$

84. $\cos\left[\arccos\left(-\frac{\sqrt{3}}{2}\right) - \arcsin\left(-\frac{1}{2}\right)\right] = \cos\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)\cos\left(\arcsin\left(-\frac{1}{2}\right)\right) + \sin\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)\sin\left(\arcsin\left(-\frac{1}{2}\right)\right)$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{3}{4} - \frac{1}{4} = -1$

86. Angle x: $b = x$, $c = 1$, $a = \sqrt{1 - x^2}$

Angle y: $a = y$, $c = 1$, $b = \sqrt{1 - y^2}$

$\cos(\sin^{-1} x - \cos^{-1} y) = \cos(\sin^{-1} x)\cos(\cos^{-1} y) + \sin(\sin^{-1} x)\sin(\cos^{-1} y)$

$= \sqrt{1 - x^2} \cdot y + x \cdot \sqrt{1 - y^2} = y\sqrt{1 - x^2} + x\sqrt{1 - y^2}$

88. Verify: $\sin(x + y + z) = \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z - \sin x \sin y \sin z$

$\sin(x + y + z) = \sin((x + y) + z)$	Algebra
$= \sin(x + y)\cos z + \sin z \cos(x + y)$	Sum Identity
$= \cos z[\sin x \cos y + \sin y \cos x] + \sin z[\cos x \cos y - \sin x \sin y]$	Sum Identity
$= \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z - \sin x \sin y \sin z$	Algebra

90. $\dfrac{\sin(x-y)}{\cos(x-y)} = \dfrac{\sin x \cos y - \cos x \sin y}{\cos x \cos y + \sin x \sin y}$

92. $\dfrac{\dfrac{\sin x \cos y - \cos x \sin y}{\cos x \cos y}}{\dfrac{\cos x \cos y + \sin x \sin y}{\cos x \cos y}} = \dfrac{\dfrac{\sin x \cos y}{\cos x \cos y} - \dfrac{\cos x \sin y}{\cos x \cos y}}{\dfrac{\cos x \cos y}{\cos x \cos y} + \dfrac{\sin x \sin y}{\cos x \cos y}}$

94. $\dfrac{\dfrac{\sin x}{\cos x} - \dfrac{\sin y}{\cos y}}{1 + \dfrac{\sin x \sin y}{\cos x \cos y}} = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

96. $y = 3x + 1$ and $y = \dfrac{1}{2}x - 1$

$$\tan(\theta_2 - \theta_1) = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{3 - \frac{1}{2}}{1 + 3\left(\frac{1}{2}\right)} \cdot \frac{2}{2} = \frac{6-1}{2+3} = 1$$

$$\theta_2 - \theta_1 = 45°$$

98. $\alpha = 43°$, $M = 0.25$ inch, $N = 0.11$ inch

$$\tan\beta = \tan\alpha - \frac{N}{M}\sec\alpha = \tan 43° - \frac{0.11}{0.25}\sec 43° \approx 0.33089$$

$$\beta \approx 18°$$

Section 7−3

2. $\sin(x-y) = \sin x \cos y - \cos x \sin y$. After replacing y with x, this reduces to $\sin(x-x) = \sin x \cos x - \cos x \sin x$, or $\sin 0 = 0$, or $0 = 0$.

4. $\cos(x-y) = \cos x \cos y + \sin x \sin y$. After replacing y with x, this reduces to $\cos(x-x) = \cos x \cos x + \sin x \sin x$, or $\cos 0 = \cos^2 x + \sin^2 x$, or $\cos 0 = 1$, or $1 = 1$.

6. Since $\sin\dfrac{x}{2}$ and $\cos\dfrac{x}{2}$ are functions, they can take on only one value for any x. The choice is made according to the quadrant in which $\dfrac{x}{2}$ lies.

8. Verify: $\sin 2x = 2\sin x \cos x$ for $x = 45°$

$\sin 2x = \sin(2 \cdot 45°) = \sin 90° = 1$

$2\sin x \cos x = 2\sin 45° \cos 45°$

$= 2 \cdot \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{2}}{2} = 1$

10. Verify: $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ for $x = \dfrac{\pi}{6}$

$$\tan 2x = \tan\left(2 \cdot \frac{\pi}{6}\right) = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\frac{2\tan x}{1-\tan^2 x} = \frac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} \cdot \frac{3\sqrt{3}}{3\sqrt{3}}$$

$$= \frac{6}{3\sqrt{3}-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$$

12. Verify: $\cos\dfrac{x}{2} = \pm\sqrt{\dfrac{1+\cos x}{2}}$, $x = \dfrac{\pi}{2}$

$\dfrac{x}{2} = \dfrac{\pi}{4}$, Quad I: sign of $\cos\dfrac{x}{2}$ is +.

$$\cos\frac{x}{2} = \cos\frac{\frac{\pi}{2}}{2} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+\cos\frac{\pi}{2}}{2}}$$

$$= \sqrt{\frac{1+0}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

14. $\tan 75° = \tan\dfrac{150°}{2} = \sqrt{\dfrac{1-\cos 150°}{1+\cos 150°}}$

$$= \sqrt{\frac{1-\frac{-\sqrt{3}}{2}}{1+\frac{-\sqrt{3}}{2}}}$$

16. $\tan 15° = \tan\dfrac{30°}{2} = \sqrt{\dfrac{1-\cos 30°}{1+\cos 30°}}$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} \cdot \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$$
$$= \frac{1}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$= 2 + \sqrt{3}$$

$$= \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} \cdot \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$$
$$= \frac{2-\sqrt{3}}{\sqrt{4-3}} = 2 - \sqrt{3}$$

18. $\cos\frac{\pi}{12} = \cos\frac{\pi/6}{2} = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}}$
$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$
$$= \sqrt{\frac{2+\sqrt{3}}{4}}$$
$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

20. $\sin -\frac{7\pi}{8} = \sin\frac{-7\pi/4}{2} = -\sqrt{\frac{1-\cos\left(-\frac{7\pi}{4}\right)}{2}}$
$$= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$
$$= -\sqrt{\frac{2-\sqrt{2}}{4}}$$
$$= \frac{-\sqrt{2-\sqrt{2}}}{2}$$

22. Verify: $\sin 2x = (\tan x)(1 + \cos 2x)$

$(\tan x)(1+\cos 2x) = \frac{\sin x}{\cos x}(1 + 2\cos^2 x - 1)$	Quotient Identity, Double-angle Identity
$= \frac{\sin x}{\cos x}(2\cos^2 x)$	Algebra
$= 2\sin x\cos x$	Algebra
$= \sin 2x$	Double-angle Identity

24. Verify: $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$\frac{1}{2}(\cos 2x + 1) = \frac{1}{2}(2\cos^2 x - 1 + 1)$	Double-angle Identity
$= \frac{1}{2}(2\cos^2 x)$	Algebra
$= \cos^2 x$	Algebra

26. Verify: $1 + \sin 2t = (\sin t + \cos t)^2$

$1 + \sin 2t = 1 + 2\sin t\cos t$	Double-angle Identity
$= \sin^2 t + \cos^2 t + 2\sin t\cos t$	Pythagorean Identity
$= \sin^2 t + 2\sin t\cos t + \cos^2 t$	Algebra
$= (\sin t + \cos t)^2$	Algebra

28. Verify: $\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$

$\cos^2\frac{x}{2} = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$	Half-angle Identity
$= \frac{1+\cos x}{2}$	Algebra

30. Verify: $\cot 2x = \frac{\cot x - \tan x}{2}$

$$\cot 2x = \frac{1}{\tan 2x} = \frac{1}{\frac{2\tan x}{1-\tan^2 x}} = \frac{1-\tan^2 x}{2\tan x}$$

32. Verify: $\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta}$

$$\cot\frac{\pi}{2} = \frac{1}{\tan\frac{\theta}{2}} = \frac{1}{\frac{\sin\theta}{1+\cos\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{2\tan x} - \frac{\tan x}{2} = \frac{\cot x - \tan x}{2}$$

34. Verify: $\dfrac{\cos 2u}{1-\sin 2u} = \dfrac{1+\tan u}{1-\tan u}$

$\dfrac{\cos 2u}{1-\sin 2u} = \dfrac{\cos^2 u - \sin^2 u}{1 - 2\sin u \cos u}$	Double-angle Identity
$= \dfrac{(\cos u - \sin u)(\cos u + \sin u)}{\cos^2 u - 2\sin u \cos u + \sin^2 u}$	Algebra, Pythagorean Identity
$= \dfrac{(\cos u - \sin u)(\cos u + \sin u)}{(\cos u - \sin u)^2}$	Algebra
$= \dfrac{\cos u + \sin u}{\cos u - \sin u} \cdot \dfrac{\frac{1}{\cos u}}{\frac{1}{\cos u}}$	Algebra
$= \dfrac{1+\tan u}{1-\tan u}$	Algebra, Quotient Identity

36. Verify: $\sec 2x = \dfrac{\sec^2 x}{2-\sec^2 x}$

$\sec 2x = \dfrac{1}{\cos 2x}$	Reciprocal Identity
$= \dfrac{1}{2\cos^2 x - 1} \cdot \dfrac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}}$	Double-angle Identity, Algebra
$= \dfrac{\frac{1}{\cos^2 x}}{2 - \frac{1}{\cos^2 x}}$	Algebra
$= \dfrac{\sec^2 x}{2-\sec^2 x}$	Reciprocal Identity

38. Verify: $\cos 2\alpha = \dfrac{\cot\alpha - \tan\alpha}{\cot\alpha + \tan\alpha}$

$\dfrac{\cot\alpha - \tan\alpha}{\cot\alpha + \tan\alpha} = \dfrac{\frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha}}{\frac{\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{\cos\alpha}} \cdot \dfrac{\sin\alpha\cos\alpha}{\sin\alpha\cos\alpha}$	Quotient Identity, Algebra
$= \dfrac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha + \sin^2\alpha}$	Algebra
$= \dfrac{\cos 2\alpha}{1}$	Double-angle Identity, Pythagorean Identity
$= \cos 2\,\alpha$	Algebra

40. Plug in $x = \dfrac{\pi}{6}$.

Left side: $\cos\left(2 \cdot \dfrac{\pi}{6}\right) = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

42. Plug in $x = \dfrac{\pi}{3}$.

Left side: $\tan\dfrac{\pi/3}{2} = \tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$

Right side: $2\cos\frac{\pi}{6} = \sqrt{3}$

The equation is not an identity.

Right side: $\frac{1}{2}\tan\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

The equation is not an identity.

44. Plug in $x = \frac{7\pi}{3}$.

Left side: $\sin\frac{7\pi/3}{2} = \sin\frac{7\pi}{6} = -\frac{1}{2}$

Right side: $\sqrt{\frac{1-\cos\frac{7\pi}{3}}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \frac{1}{2}$

The equation is not an identity.

46. Plug in $x = \frac{\pi}{6}$.

Left side: $\tan\left(2\cdot\frac{\pi}{6}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Right side: $\frac{2\cot\frac{\pi}{6}}{1-\cot^2\frac{\pi}{6}} = \frac{2(\sqrt{3})}{1-(\sqrt{3})^2} = -\sqrt{3}$

The equation is not an identity.

48. Plug in $x = \frac{\pi}{2}$.

Left side: $\sin\frac{\pi/2}{2} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Right side: $\frac{1}{2}\sin\frac{\pi}{2} = \frac{1}{2}\cdot 1 = \frac{1}{2}$

The equation is not an identity.

50. $\csc 2x = \frac{1}{\sin 2x} = \frac{1}{2\sin x\cos x} = \frac{1}{2}\csc x\sec x$

Not an identity.

52. Plug in $x = \frac{\pi}{4}$: $\tan 4\left(\frac{\pi}{4}\right) = \tan\pi = 0$

$$4\tan\frac{\pi}{4} = 4$$

Not an identity.

54. Plug in $x = \frac{\pi}{6}$: $\tan 2\left(\frac{\pi}{6}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

$$\frac{2}{\tan\frac{\pi}{6}-\cot\frac{\pi}{6}} = \frac{2}{\frac{1}{\sqrt{3}}-\sqrt{3}} = \frac{2}{\frac{1-3}{\sqrt{3}}} = -\sqrt{3}$$

Not an identity.

56. $\cos x = -\frac{4}{5},\ \frac{\pi}{2} < x < \pi \qquad a = -4,\ r = 5,\ b = 3$

$\sin 2x = 2\sin x\cos x = 2\cdot\frac{3}{5}\cdot\left(-\frac{4}{5}\right) = -\frac{24}{25}$

$\cos 2x = 1 - 2\sin^2 x = 1 - 2\left(\frac{3}{5}\right)^2 = 1 - \frac{18}{25} = \frac{7}{25}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2} = \frac{-\frac{6}{4}}{1-\frac{9}{16}}\cdot\frac{16}{16} = \frac{-24}{16-9} = -\frac{24}{7}$

58. $\cot x = -\frac{5}{12},\ -\frac{\pi}{2} < x < 0 \qquad a = 5,\ b = -12,\ r = 13$

$\sin 2x = 2\sin x\cos x = 2\cdot\frac{-12}{13}\cdot\frac{5}{13} = -\frac{120}{169}$

$$\cos 2x = 1 - 2\sin^2 x = 1 - 2\left(-\frac{12}{13}\right)^2 = 1 - \frac{288}{169} = \frac{-119}{169}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2} = \frac{-\frac{24}{5}}{1-\frac{144}{25}}\cdot\frac{25}{25} = \frac{-120}{25-144} = \frac{120}{119}$$

60. $\cos x = -\frac{1}{4}$, $\pi < x < \frac{3\pi}{2}$ $a=-1, r=4, b=-\sqrt{15}$ $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ (QII)

$$\sin\frac{x}{2} = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\frac{-1}{4}}{2}} = \sqrt{\frac{4+1}{8}} = \frac{\sqrt{5}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{4}$$

$$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1+\frac{-1}{4}}{2}} = -\sqrt{\frac{4-1}{8}} = \frac{-\sqrt{3}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{6}}{4}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{1-\frac{-1}{4}}{\frac{-\sqrt{15}}{4}} = \frac{4+1}{-\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{-5\sqrt{15}}{15} = -\frac{\sqrt{15}}{3}$$

62. $\tan x = \frac{3}{4}$, $-\pi < x < -\frac{\pi}{2}$ $a=-4, b=-3, r=5$, $-\frac{\pi}{2} < \frac{x}{2} < -\frac{\pi}{4}$ (QIV)

$$\sin\frac{x}{2} = -\sqrt{\frac{1-\cos x}{2}} = -\sqrt{\frac{1-\frac{-4}{5}}{2}} = -\sqrt{\frac{5+4}{10}} = -\frac{3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$$

$$\cos\frac{x}{2} = +\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+\frac{-4}{5}}{2}} = \sqrt{\frac{5-4}{10}} = \frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{1-\frac{-4}{5}}{-\frac{3}{5}} = \frac{5+4}{-3} = \frac{9}{-3} = -3$$

64. Find the exact values of $\sin\theta$ and $\cos\theta$, given $\sec 2\theta = -\frac{5}{4}$, $0° < \theta < 90°$.

(A) $0° < \theta < 90° \Rightarrow 0° < 2\theta < 180°$ and since $\sec 2\theta = \frac{-5}{4} < 0$, 2θ is in QII.

(B) $\sec 2\theta = \frac{-5}{4} \Rightarrow \cos 2\theta = \frac{-4}{5}$

$$\sin 2\theta = \sqrt{1-\cos^2 2\theta} = \sqrt{1-\left(\frac{-4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

(C) $\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$

$\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$

(D) & (E) θ is a quadrant I angle, so

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\frac{-4}{5}}{2}} = \sqrt{\frac{5+4}{10}} = \frac{3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\frac{-4}{5}}{2}} = \sqrt{\frac{5-4}{10}} = \frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

66. $x = 72.358°$

(A) $\tan 2x = \tan(2\cdot 72.358°) = \tan 144.716° \approx -0.70762$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\tan 72.358°}{1-\tan^2 72.358°} \approx -0.70762$$

68. $x = 4$

(A) $\tan 2x = \tan(2\cdot 4) = \tan 8 \approx -6.7997$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\tan 4}{1-\tan^2 4} \approx -6.7997$$

(B) $\cos \frac{x}{2} = \cos \frac{72.358°}{2} = \cos 36.179° \approx 0.80718$

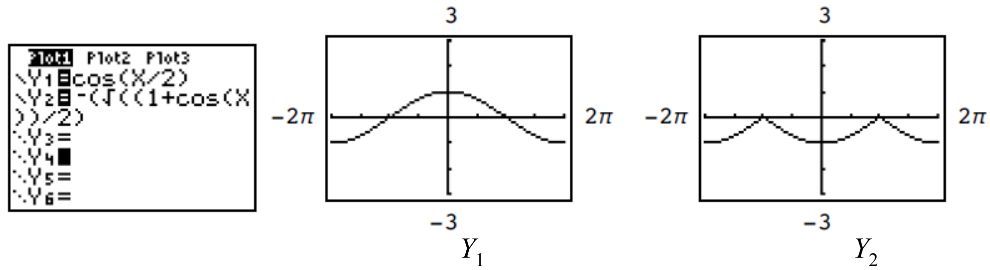
$\cos \frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+\cos 72.358°}{2}} \approx 0.80718$

(B) $\cos \frac{x}{2} = \cos \frac{4}{2} = \cos 2 \approx -0.41615$

$\cos \frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1+\cos 4}{2}} \approx -0.41615$

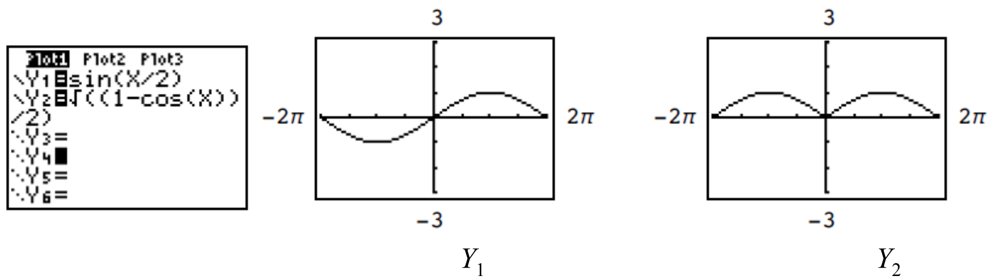
(Quadrant III)

70.



Y_1 and Y_2 are identities on the intervals $[-2\pi, -\pi]$ and $[\pi, 2\pi]$.

72.



Y_1 and Y_2 are identities on the interval $[0, 2\pi]$.

74. Verify: $\sin 3x = 3 \sin x - 4 \sin^3 x$

$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \sin x \cos 2x$	Sum Identity
$= 2 \sin x \cos x \cos x + \sin x(1 - 2 \sin^2 x)$	Double-angle Identity
$= 2 \sin x \cos^2 x + \sin x - 2 \sin^3 x$	Algebra
$= 2 \sin x(1 - \sin^2 x) + \sin x - 2 \sin^3 x$	Pythagorean Identity
$= 2 \sin x - 2 \sin^3 x + \sin x - 2 \sin^3 x$	Algebra
$= 3 \sin x - 4 \sin^3 x$	Algebra

76. Verify: $\sin 4x = (\cos x)(4 \sin x - 8 \sin^3 x)$

$\sin 4x = \sin(2x + 2x)$	Algebra
$= \sin 2x \cos 2x + \sin 2x \cos 2x$	Sum Identity
$= 2 \sin 2x \cos 2x$	Algebra
$= 2(2 \sin x \cos x)(1 - 2 \sin^2 x)$	Double-angle Identity
$= \cos x(4 \sin x)(1 - 2 \sin^2 x)$	Algebra
$= (\cos x)(4 \sin x - 8 \sin^3 x)$	Algebra

78. Use $\sin 2\theta = 2 \sin \theta \cos \theta$

$$\sin\left[2\cos^{-1}\frac{3}{5}\right] = 2 \sin\left(\cos^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}\frac{3}{5}\right) = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$$

80. Use $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$\tan\left[2\tan^{-1}\left(-\frac{3}{4}\right)\right] = \frac{2\tan\left(\tan^{-1}\left(-\frac{3}{4}\right)\right)}{1-\tan^2\left(\tan^{-1}\left(-\frac{3}{4}\right)\right)} = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}\cdot\frac{16}{16} = \frac{-24}{16-9} = -\frac{24}{7}$$

82. Use $\sin\frac{\theta}{2} = -\sqrt{\frac{1-\cos\theta}{2}}$, QIV

$$\sin\left[\frac{1}{2}\tan^{-1}\left(-\frac{4}{3}\right)\right] = -\sqrt{\frac{1-\cos\left(\tan^{-1}\left(-\frac{4}{3}\right)\right)}{2}} = -\sqrt{\frac{1-\frac{3}{5}}{2}} = -\sqrt{\frac{5-3}{10}} = -\frac{\sqrt{2}}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{-2\sqrt{5}}{10} = -\frac{\sqrt{5}}{5}$$

84. $\cos(2x) = 2\cos^2 x - 1$

$1 + \cos 2x = 2\cos^2 x$

$\frac{1+\cos 2x}{2} = \cos^2 x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

86. $\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\tan x\cot^2 x}{(1-\tan^2 x)\cot^2 x}$

$= \frac{2\tan x\cot x\cot x}{\cot^2 x - \tan^2 x\cot^2 x} = \frac{2\cot x}{\cot^2 x - 1}$

88. If $0 < x < \pi$, $\sin x > 0$, then $0 < \frac{x}{2} < \frac{\pi}{2}$, $\tan\frac{x}{2} > 0$. This applies if x is in Quadrants I or II, with $\frac{x}{2}$ in Quadrant I.

If $\pi < x < 2\pi$, $\sin x < 0$, then $\frac{\pi}{2} < \frac{x}{2} < \pi$, $\tan\frac{x}{2} < 0$. This applies if x is in Quadrants III or IV, with $\frac{x}{2}$ in Quadrant II. (If $x = \pi$, $\frac{x}{2} = \frac{\pi}{2}$ and $\tan\frac{x}{2}$ is undefined.)

The truth of the statement that $\tan\frac{x}{2}$ and $\sin x$ always have the same sign follows for all other values of x since $\sin(x + 2k\pi) = \sin x$ and $\tan\left(\frac{x+2k\pi}{2}\right) = \tan(x + k\pi) = \tan x$.

90. $\left|\tan\frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{1+\cos x}}$ (A) Half-angle identity

$= \sqrt{\frac{1-\cos x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}}$ Multiplied by 1

$= \sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}}$ Algebra

$= \sqrt{\frac{(1-\cos x)^2}{\sin^2 x}}$ (B) Pythagorean identity

$= \frac{\sqrt{(1-\cos x)^2}}{\sqrt{\sin^2 x}}$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$= \frac{1-\cos x}{|\sin x|}$ (C) $\sqrt{\sin^2 x} = |\sin x|$ because $\sqrt{a^2} = |a|$ for any real number a;

$\sqrt{(1-\cos x)^2} = 1-\cos x$ because $1 - \cos x$ is never negative.

$= \frac{1-\cos x}{\sin x}$ (D) Since $\tan\frac{x}{2}$ and $\sin x$ always have the same sign, and since $1 - \cos x$ is never negative, $\tan\frac{x}{2}$ and $(1 - \cos x)/\sin x$ always have the same sign for any x.

92. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, $\tan\theta = \frac{2}{x}$, $\tan 2\theta = \frac{6}{x}$

$$\frac{6}{x} = \frac{2\cdot\frac{2}{x}}{1-\left(\frac{2}{x}\right)^2}\cdot\frac{x^2}{x^2}$$

$$\frac{6}{x} = \frac{4x}{x^2-4}$$

$$6x^2 - 24 = 4x^2$$

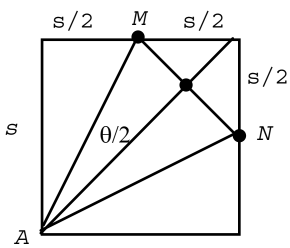
$$x^2 = 12$$

$$x = 2\sqrt{3} \approx 3.464 \text{ ft.}$$

$$\tan\theta = \frac{2}{x} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = 30.000°$$

94.



$$AM = \sqrt{s^2+\left(\frac{s}{2}\right)^2} = \sqrt{\frac{5s^2}{4}} = \frac{\sqrt{5}\cdot s}{2} \qquad MN = \sqrt{\left(\frac{s}{2}\right)^2+\left(\frac{s}{2}\right)^2} = \sqrt{\frac{s^2}{2}} = \frac{\sqrt{2}\cdot s}{2}$$

$$\sin\frac{\theta}{2} = \frac{\frac{MN}{2}}{AM} = \frac{\frac{\sqrt{2}\cdot s}{4}}{\frac{\sqrt{5}\cdot s}{2}} = \frac{\sqrt{2}}{2\sqrt{5}} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\frac{1-\cos\theta}{2} = \frac{2}{20} = \frac{1}{10}$$

$$1-\cos\theta = \frac{2}{10} = \frac{1}{5}$$

$$\cos\theta = \frac{4}{5}$$

Section 7−4

2. A sum-product identity expresses a sum of two trigonometric functions as a product of two other trigonometric functions.

4. The identity for $\cos(x - y)$.

6. The double-angle identities for sine and cosine.

8. $\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

$\cos 7A \cos 5A = \frac{1}{2}[\cos(7A + 5A) + \cos(7A - 5A)] = \frac{1}{2}(\cos 12A + \cos 2A) = \frac{1}{2}\cos 12A + \frac{1}{2}\cos 2A$

10. $\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$

$\cos 2\theta \sin 3\theta = \frac{1}{2}[\sin(2\theta + 3\theta) - \sin(2\theta - 3\theta)] = \frac{1}{2}[\sin 5\theta - \sin(-\theta)] = \frac{1}{2}[\sin 5\theta + \sin\theta]$

$= \frac{1}{2}\sin 5\theta + \frac{1}{2}\sin\theta$

12. $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos 7\theta + \cos 5\theta = 2\cos\frac{7\theta+5\theta}{2}\cos\frac{7\theta-5\theta}{2} = 2\cos 6\theta\cos\theta$

14. $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

$\sin u - \sin 5u = 2\cos\dfrac{u+5u}{2}\sin\dfrac{u-5u}{2} = 2\cos 3u\sin(-2u) = -2\cos 3u\sin 2u$

16. $\cos x\sin y = \dfrac{1}{2}[\sin(x+y) - \sin(x-y)]$

$\cos 75° \sin 15° = \dfrac{1}{2}[\sin(75° + 15°) - \sin(75° - 15°)] = \dfrac{1}{2}[\sin 90° - \sin 60°]$

$= \dfrac{1}{2}\left(1 - \dfrac{\sqrt{3}}{2}\right) = \dfrac{1}{2}\left(\dfrac{2-\sqrt{3}}{2}\right) = \dfrac{2-\sqrt{3}}{4}$

18. $\sin x\sin y = \dfrac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\sin 112.5° \sin 22.5° = \dfrac{1}{2}[\cos(112.5° - 22.5°) - \cos(112.5° + 22.5°)] = \dfrac{1}{2}[\cos 90° - \cos 135°]$

$= \dfrac{1}{2}\left[0 - \left(-\dfrac{\sqrt{2}}{2}\right)\right] = \dfrac{\sqrt{2}}{4}$

20. $\sin x\cos y = \dfrac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\sin 262.5° \cos 52.5° = \dfrac{1}{2}[\sin(262.5° + 52.5°) + \sin(262.5° - 52.5°)] = \dfrac{1}{2}[\sin 315° + \sin 210°]$

$= \dfrac{1}{2}\left[-\dfrac{\sqrt{2}}{2} + \dfrac{-1}{2}\right] = \dfrac{-(\sqrt{2}+1)}{4}$

22. $\cos x\cos y = \dfrac{1}{2}[\cos(x-y) + \cos(x+y)]$

$\cos\dfrac{3\pi}{8}\cos\dfrac{7\pi}{8} = \dfrac{1}{2}\left[\cos\left(\dfrac{3\pi}{8} - \dfrac{7\pi}{8}\right) + \cos\left(\dfrac{3\pi}{8} + \dfrac{7\pi}{8}\right)\right] = \dfrac{1}{2}\left[\cos\dfrac{-\pi}{2} + \cos\dfrac{5\pi}{4}\right] = \dfrac{1}{2}\left[0 - \dfrac{\sqrt{2}}{2}\right] = -\dfrac{\sqrt{2}}{4}$

24. $\cos x\sin y = \dfrac{1}{2}[\sin(x+y) - \sin(x-y)]$

$\cos\dfrac{7\pi}{12}\sin\dfrac{5\pi}{12} = \dfrac{1}{2}\left[\sin\left(\dfrac{7\pi}{12} + \dfrac{5\pi}{12}\right) - \sin\left(\dfrac{7\pi}{12} - \dfrac{5\pi}{12}\right)\right] = \dfrac{1}{2}\left[\sin\pi - \sin\dfrac{\pi}{6}\right] = \dfrac{1}{2}\left[0 - \dfrac{1}{2}\right] = -\dfrac{1}{4}$

26. $\sin x\sin y = \dfrac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\sin\dfrac{17\pi}{24}\sin\dfrac{\pi}{24} = \dfrac{1}{2}\left[\cos\left(\dfrac{17\pi}{24} - \dfrac{\pi}{24}\right) - \cos\left(\dfrac{17\pi}{24} + \dfrac{\pi}{24}\right)\right] = \dfrac{1}{2}\left[\cos\dfrac{2\pi}{3} - \cos\dfrac{3\pi}{4}\right] = \dfrac{1}{2}\left[-\dfrac{1}{2} - \left(-\dfrac{\sqrt{2}}{2}\right)\right] = \dfrac{-1+\sqrt{2}}{4}$

28. $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

$\cos 105° + \cos 15° = 2\cos\dfrac{105°+15°}{2}\cos\dfrac{105°-15°}{2} = 2\cos 60° \cos 45° = 2\cdot\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{2}$

30. $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin 165° + \sin 105° = 2\sin\frac{165°+105°}{2}\cos\frac{165°-105°}{2} = 2\sin 135°\cos 30° = 2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

32. $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$\cos\frac{13\pi}{12} - \cos\frac{5\pi}{12} = -2\sin\frac{\frac{13\pi}{12}+\frac{5\pi}{12}}{2}\sin\frac{\frac{13\pi}{12}-\frac{5\pi}{12}}{2} = -2\sin\frac{3\pi}{4}\sin\frac{\pi}{3} = -2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{6}}{2}$

34. $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$\sin\frac{11\pi}{12} - \sin\frac{7\pi}{12} = 2\cos\frac{\frac{11\pi}{12}+\frac{7\pi}{12}}{2}\sin\frac{\frac{11\pi}{12}-\frac{7\pi}{12}}{2} = 2\cos\frac{3\pi}{4}\sin\frac{\pi}{6} = 2\cdot-\frac{\sqrt{2}}{2}\cdot\frac{1}{2} = -\frac{\sqrt{2}}{2}$

36. Verify: $\sin x\sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$$\begin{aligned}\frac{1}{2}[\cos(x-y) - \cos(x+y)] &= \frac{1}{2}[\cos x\cos y + \sin x\sin y - (\cos x\cos y - \sin x\sin y)]\\ &= \frac{1}{2}[2\sin x\sin y]\\ &= \sin x\sin y\end{aligned}$$

38. Start with the product-sum identity

$\cos u\cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$

Let $x = u + v$

$y = u - v$

Solving this system gives $u = \frac{x+y}{2}$, $v = \frac{x-y}{2}$. Substituting into the product-sum identity,

$\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{1}{2}[\cos x + \cos y]$ or $\cos x + \cos y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

40. Verify: $\frac{\cos t - \cos 3t}{\sin t + \sin 3t} = \tan t$

$\frac{\cos t - \cos 3t}{\sin t + \sin 3t} = \frac{-2\sin\frac{t+3t}{2}\sin\frac{t-3t}{2}}{2\sin\frac{t+3t}{2}\cos\frac{t-3t}{2}}$	Sum-product Identities
$= \frac{-\sin 2t\sin(-t)}{\sin 2t\cos(-t)}$	Algebra
$= \frac{\sin 2t\sin t}{\sin 2t\cos t}$	Identities for negatives
$= \frac{\sin t}{\cos t}$	Algebra
$= \tan t$	Quotient Identity

42. Verify: $\dfrac{\sin x+\sin y}{\cos x+\cos y}=\tan\dfrac{x+y}{2}$

$$\frac{\sin x+\sin y}{\cos x+\cos y}=\frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$$ Sum-product Identities

$$=\frac{\sin\frac{x+y}{2}}{\cos\frac{x+y}{2}}$$ Algebra

$$=\tan\frac{x+y}{2}$$ Quotient Identity

44. Verify: $\dfrac{\cos x-\cos y}{\sin x+\sin y}=-\tan\dfrac{x-y}{2}$

$$\frac{\cos x-\cos y}{\sin x+\sin y}=\frac{-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}}{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}$$ Sum-product Identities

$$=\frac{-\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}}$$ Algebra

$$=-\tan\frac{x-y}{2}$$ Quotient Identity

46. Verify: $\dfrac{\sin x+\sin y}{\sin x-\sin y}=\dfrac{\tan\left[\frac{1}{2}(x+y)\right]}{\tan\left[\frac{1}{2}(x-y)\right]}$

$$\frac{\sin x+\sin y}{\sin x-\sin y}=\frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}$$ Sum-product Identities

$$=\frac{\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{\cos\frac{x+y}{2}\sin\frac{x-y}{2}}$$ Algebra

$$=\tan\frac{x+y}{2}\cot\frac{x-y}{2}$$ Quotient Identities

$$=\tan\frac{x+y}{2}\,\frac{1}{\tan\frac{x-y}{2}}$$ Reciprocal Identity

$$=\frac{\tan\frac{x+y}{2}}{\tan\frac{x-y}{2}}$$ Algebra

48. Let $x = y = 0$.
Left side: $\cos 0 \sin 0 = 0$
Right side: $\cos 0 - \sin 0 = 1$
The equation is not an identity.

50. Let $x=\dfrac{\pi}{6}$ and $y=\dfrac{\pi}{6}$.

Left side: $\cos\dfrac{\pi}{6}\cos\dfrac{\pi}{6}=\dfrac{3}{4}$

Right side: $\cos\left(\dfrac{\pi}{6}+\dfrac{\pi}{6}\right)=\cos\dfrac{\pi}{3}=\dfrac{1}{2}$

The equation is not an identity.

52. Let $x = 0$ and $y = \frac{\pi}{2}$.

Left side: $\sin 0 + \sin \frac{\pi}{2} = 1$

Right side: $\sin 0 \sin \frac{\pi}{2} = 0$

The equation is not an identity.

54. Let $x = \frac{\pi}{2}$ and $y = 0$.

Left side: $\cos \frac{\pi}{2} - \cos 0 = -1$

Right side: $2 \sin \frac{\pi/2+0}{2} \sin \frac{\pi/2-0}{2} = 2 \sin^2 \frac{\pi}{4} = 1$

The equation is not an identity.

56. Try a product-sum identity:

$2 \sin x \cos 2x = 2\left[\frac{1}{2}(\sin(x+2x)+\sin(x-2x)\right] = \sin 3x + \sin(-x) = \sin 3x - \sin x$

The equation is not an identity.

58. Try a product-sum identity:

$2 \cos 3x \cos 5x = 2\left[\frac{1}{2}(\cos(3x+5x)+\cos(3x-5x)\right] = \cos 8x + \cos(-2x) = \cos 8x + \cos 2x$

The equation is an identity.

60. Try a product-sum identity:

$2 \sin 4x \cos 2x = 2 \cdot \frac{1}{2}[\sin(4x + 2x) + \sin(4x - 2x)] = \sin 6x + \sin 2x$

Not an identity.

62. $x = 50.137°$, $y = 18.044°$

(A) $\cos 50.137° \sin 18.044° \approx 0.19853$

$\frac{1}{2}[\sin(50.137° + 18.044°) - \sin(50.137° - 18.044°)] = \frac{1}{2}[\sin 68.181 - \sin 32.093] \approx 0.19853$

(B) $\cos 50.137° + \cos 18.044° \approx 1.5918$

$2 \cos \frac{50.137°+18.044°}{2} \cos \frac{50.137°-18.044°}{2} = 2 \cos 34.0905 \cos 16.0465 \approx 1.5918$

64. $x = 0.03917$, $y = 0.61052$

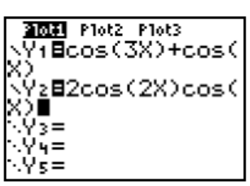
(A) $\cos 0.03917 \sin 0.61052 \approx 0.57285$

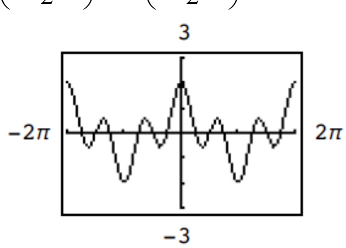
$\frac{1}{2}[\sin(0.03917 + 0.61052) - \sin(0.03917 - 0.61052)] = \frac{1}{2}[\sin 0.64969 - \sin(-0.57135)] \approx 0.57285$

(B) $\cos 0.039817 + \cos 0.61052 \approx 1.8186$

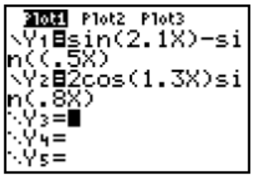
$2 \cos \frac{0.03917+0.61052}{2} \cos \frac{0.03917-0.61052}{2} = 2 \cos 0.324845 \cos(-0.285675) \approx 1.8186$

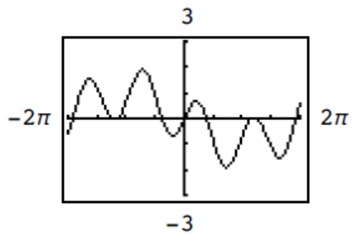
66. $y = \cos 3x + \cos x = 2 \cos\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) = 2 \cos 2x \cos x$



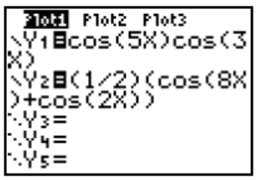


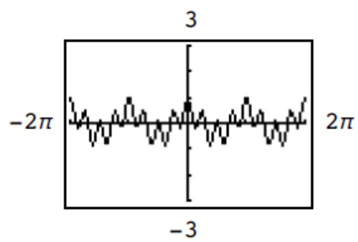
68. $y = \sin 2.1x - \sin 0.5x = 2\cos\dfrac{2.1x+0.5x}{2}\sin\dfrac{2.1x-0.5x}{2} = 2\cos 1.3x \sin 0.8x$



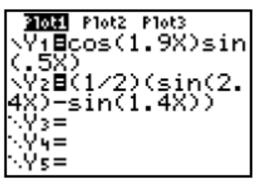


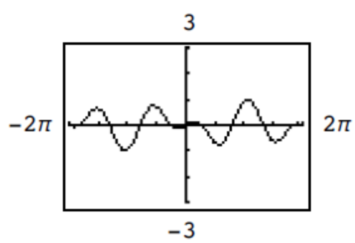
70. $y = \cos 5x \cos 3x = \dfrac{1}{2}[\cos(5x+3x) + \cos(5x-3x)] = \dfrac{1}{2}[\cos 8x + \cos 2x]$





72. $y = \cos 1.9x \sin 0.5x = \dfrac{1}{2}[\sin(1.9x+0.5x) - \sin(1.9x-0.5x)] = \dfrac{1}{2}[\sin 2.4x - \sin 1.4x]$

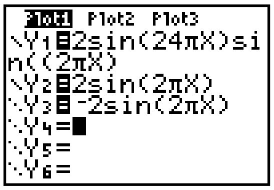


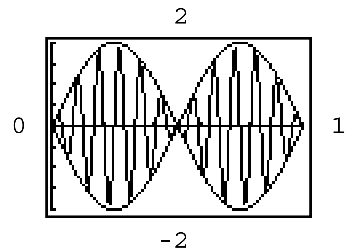


74. Verify: $\sin x \sin y \sin z = \dfrac{1}{4}[\sin(x+y-z) + \sin(y+z-x) + \sin(z+x-y) - \sin(x+y+z)]$

$\sin x \sin y \sin z = \sin x\left\{\dfrac{1}{2}[\cos(y-z) - \cos(y+z)]\right\}$	Product-Sum Identity
$= \dfrac{1}{2}\sin x\cos(y-z) - \dfrac{1}{2}\sin x\cos(y+z)$	Algebra
$= \dfrac{1}{2}\left\{\dfrac{1}{2}[\sin(x+y-z) + \sin(x-\{y-z\})]\right\} - \dfrac{1}{2}\left\{\dfrac{1}{2}[\sin(x+y+z) + \sin(x-\{y+z\})]\right\}$	Product-Sum Identity
$= \dfrac{1}{4}\sin(x+y-z) + \dfrac{1}{4}\sin(x-y+z) - \dfrac{1}{4}\sin(x+y+z) - \dfrac{1}{4}\sin(x-y-z)$	Algebra
$= \dfrac{1}{4}[\sin(x+y-z) - \sin(x-y-z) + \sin(z+x-y) - \sin(x+y+z)]$	Algebra
$= \dfrac{1}{4}[\sin(x+y-z) + \sin\{-(x-y-z)\} + \sin(z+x-y) - \sin(x+y+z)]$	Identity for negatives
$= \dfrac{1}{4}[\sin(x+y-z) + \sin(y+z-x) + \sin(z+x-y) - \sin(x+y+z)]$	Algebra

76. (A)

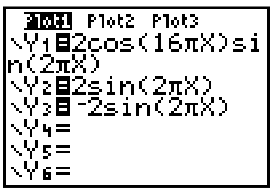


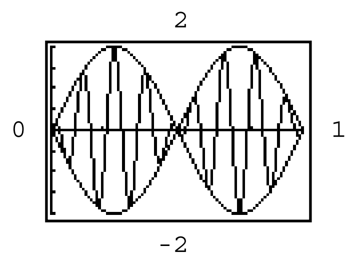


(B) $Y_1 = 2\sin(24\pi x)\ \sin(2\pi x) = 2 \cdot \frac{1}{2}[\cos(24\pi x - 2\pi x) - \cos(24\pi x + 2\pi x)]$

$= \cos(22\pi x) - \cos(26\pi x)$. Graph is the same.

78. (A)





(B) $Y_1 = 2\cos(16\pi x)\ \sin(2\pi x) = 2 \cdot \frac{1}{2}[\sin(16\pi x + 2\pi x) - \sin(16\pi x - 2\pi x)]$

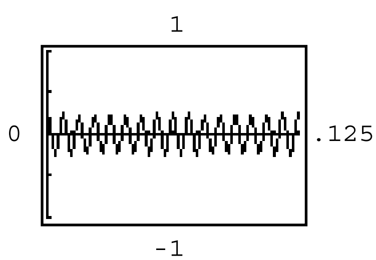
$= \sin(18\pi x) - \sin(14\pi x)$. Graph is the same.

80. (A) $0.25\cos(256\pi t) - 0.25\cos(288\pi t) = 0.25[\cos(256\pi t) - \cos(288\pi t)]$

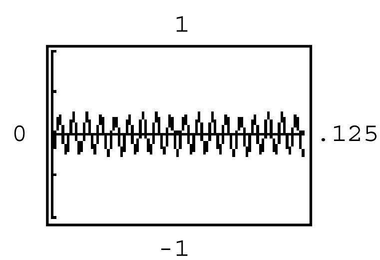
$$= 0.25\left[-2\sin\left(\frac{256\pi t + 288\pi t}{2}\right)\sin\left(\frac{256\pi t - 288\pi t}{2}\right)\right]$$

$$= -\frac{1}{2}\ \sin(272\pi t)\ \sin(-16\pi t) = \frac{1}{2}\ \sin(272\pi t)\ \sin(16\pi t)$$

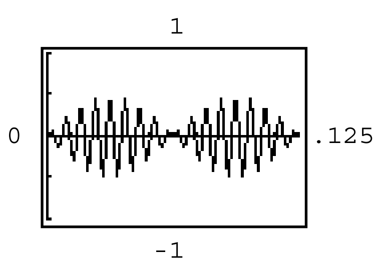
(B) $y = 0.25\cos(256\pi t)$



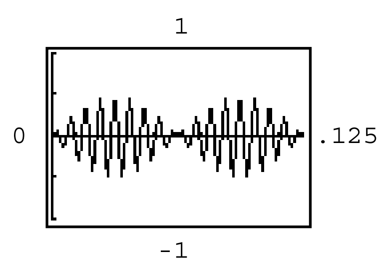
$y = -0.25\cos(288\pi t)$



$y = 0.25\cos(256\pi t) - 0.25\cos(288\pi t)$



$y = 0.5\sin(16\pi t)\sin(272\pi t)$



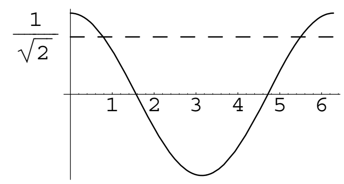
Section 7−5

2. Because the trigonometric functions are periodic, given one solution there are usually an infinite number of solutions formed by adding multiples of π to the first solution.

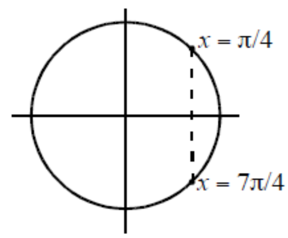
4. Answers will vary.

6. $\cos x = \frac{1}{\sqrt{2}}$

Sketch a graph of $y = \cos x$ and $y = \frac{1}{\sqrt{2}}$ on the interval $[0, 2\pi)$.



Or use a unit circle diagram.



8. We have found all solutions of $\cos x = \frac{1}{\sqrt{2}}$ over one period in problem 6. Since the cosine function is periodic with period 2π, all solutions are given by

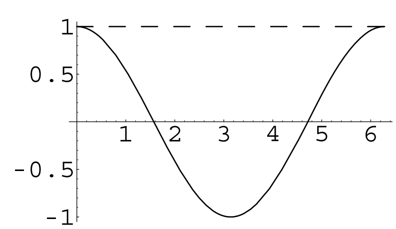
$$\left.\begin{aligned} x &= \frac{\pi}{4} + 2k\pi \\ x &= \frac{7\pi}{4} + 2k\pi \end{aligned}\right\} k \text{ any integer}$$

10. $\cos x - 1 = 0$

$\cos x = 1$

Sketch a graph of $y = \cos x$ and $y = 1$ on the interval $[0, 2\pi)$. $x = 0$

Or use a unit circle diagram (not shown).



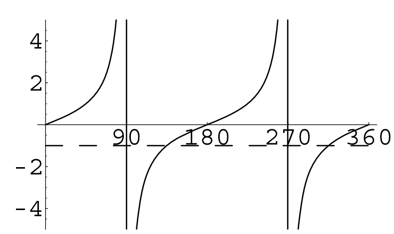
12. We have found all solutions of $\cos x - 1 = 0$ over one period in problem 10. Since the cosine function is periodic with period 2π, all solutions are given by

$x = 2k\pi$, k any integer

14. $\tan\theta + 1 = 0$

$\tan\theta = -1$

Sketch a graph of $y = \tan\theta$ and $y = -1$ on the interval $[0, 360°]$. $\theta = 135°, 315°$

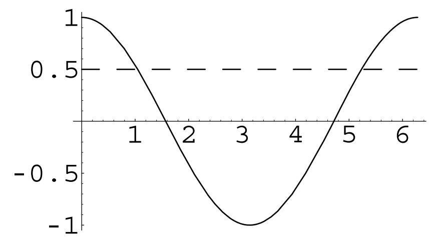


16. We have found all solutions of tan θ + 1 = 0 over two periods in problem 14. Since the tangent function is periodic with period 180˚, all solutions are given by
θ = 135˚ + k180˚, k any integer

18. $2\cos x - 1 = 0$

$$\cos x = \frac{1}{2}$$

Sketch a graph of $y = \cos x$ and $y = \frac{1}{2}$ on the interval $[0, 2\pi)$. $x = \frac{\pi}{3}, \frac{5\pi}{3}$



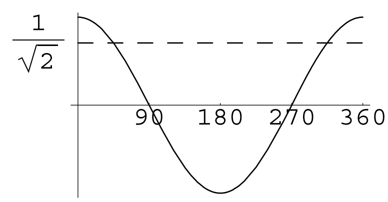
20. We have found all solutions of $2\cos x - 1 = 0$ over one period in problem 18. Since the cosine function is periodic with period 2π, all solutions are given by

$$\left.\begin{array}{l} x = \frac{\pi}{3} + 2k\pi \\ x = \frac{5\pi}{3} + 2k\pi \end{array}\right\} k \text{ any integer}$$

22. $\sqrt{2}\cos\theta - 1 = 0$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

Sketch a graph of $y = \cos\theta$ and $y = \frac{1}{\sqrt{2}}$ on the interval [0, 360˚). θ = 45˚, 315˚



24. We have found all solutions of $\sqrt{2}\cos\theta - 1 = 0$ over one period in problem 22. Since the cosine function is periodic with period 360˚, all solutions are given by

$$\left.\begin{array}{l} \theta = 45° + k360° \\ \theta = 315° + k360° \end{array}\right\} k \text{ any integer}$$

26. We have found all solutions of $\tan x - \sqrt{3} = 0$ over two periods in problem 25. Since the tangent function is periodic with period π, all solutions are given by $x = \frac{\pi}{3} + k\pi$, k any integer.

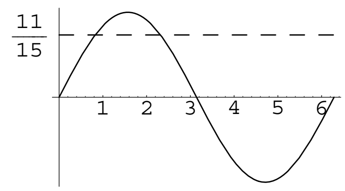
28. $-3\sin x - 4 = 5$
$-3\sin x = 9$
$\sin x = -3$
No solution. sin x has values only between −1 and 1.

30. $5\cos x - 2 = 0,\ 0 \le x < 2\pi$
$5\cos x = 2$
$\cos x = \frac{2}{5}$
$x \approx 1.1593, 5.1239$

32. $4\tan\theta + 15 = 0,\ 0° \le \theta < 180°$
$4\tan\theta = -15$
$\tan\theta = \frac{-15}{4}$
$\theta = \tan^{-1}\left(-\frac{15}{4}\right) = -75.0686°$
$\theta \approx 104.9314°$
For correct domain, θ = 180° − 75.0686°

34. $\frac{1}{5}\cos x + \frac{3}{5} = 0$
$\frac{1}{5}\cos x = -\frac{3}{5}$
$\cos x = -3$
No solution. cos x has values only between −1 and 1.

36. $\frac{11}{3} - 5\sin x = 0$



$-5\sin x = -\frac{11}{3}$

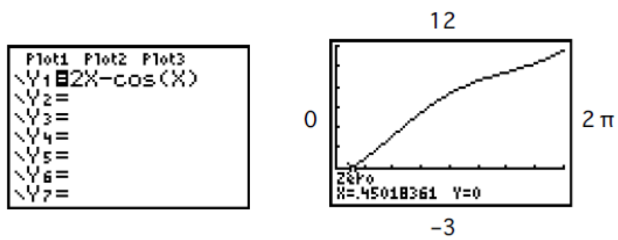
$\sin x = \frac{11}{15}$

$x = \begin{cases} \sin^{-1}\frac{11}{15} \\ \pi - \sin^{-1}\frac{11}{15} \end{cases} = \begin{cases} 0.8232 \\ 2.3184 \end{cases}$ are the solutions over one period $0 \le x < 2\pi$

If x can range over all real numbers,

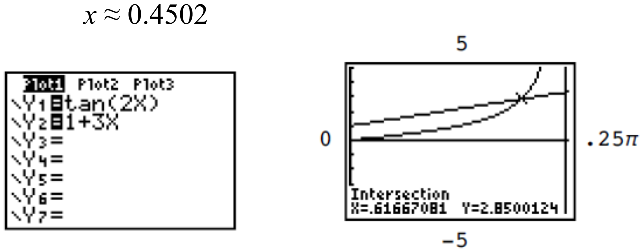
$x = \begin{cases} 0.8232 + 2k\pi \\ 2.3184 + 2k\pi \end{cases}$ k any integer

38.



$x \approx 0.4502$

40.



$x \approx 0.6167$

42. $\cos^2\theta = \frac{1}{2}\cdot\sin 2\theta$, all θ

$\cos^2\theta = \frac{1}{2}\cdot 2\sin\theta\cos\theta$

$\cos^2\theta - \sin\theta\cos\theta = 0$

$\cos\theta\cdot(\cos\theta - \sin\theta) = 0$

So

$\cos\theta = 0$ or $\cos\theta - \sin\theta = 0$

$\theta = 90° + k\cdot 180°$

$\cos\theta = \sin\theta$

$\frac{\cos\theta}{\sin\theta} = 1$

$\tan\theta = 1$

$\theta = 45° + k\cdot 180°$, k any integer

44. $\cos x = \cot x, \quad 0 \le x < 2\pi$

$\cos x = \frac{\cos x}{\sin x}$

$\sin x\cos x - \cos x = 0$

$\cos x(\sin x - 1) = 0$

$\cos x = 0 \qquad \sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin x = 1$

$x = \frac{\pi}{2}$

46. $\tan\left(\frac{x}{2}\right) - 1 = 0, \quad 0 \le x < 2\pi \Leftrightarrow 0 \le \frac{x}{2} < \pi$

$\tan\frac{x}{2} = 1$

$\frac{x}{2} = \frac{\pi}{4}$

$x = \frac{\pi}{2}$

48. $\sin^2\theta + 2\cos\theta = -2, \quad 0° \le \theta < 360°$

$$1 - \cos^2\theta + 2\cos\theta + 2 = 0$$
$$\cos^2\theta - 2\cos\theta - 3 = 0$$
$$(\cos\theta + 1)(\cos\theta - 3) = 0$$
$$\cos\theta + 1 = 0 \quad \text{or} \quad \cos\theta - 3 = 0$$
$$\cos\theta = -1 \qquad \cos\theta = 3, \text{ no solution}$$
$$\theta = 180°$$

50. $\cos 2\theta + \sin^2\theta = 0, 0° \le \theta < 360°$

$$1 - 2\sin^2\theta + \sin^2\theta = 0$$
$$-\sin^2\theta = -1$$
$$\sin^2\theta = 1$$
$$\sin\theta = \pm 1$$
$$\theta = 90°, 270°$$

52. $4\cos^2 2x - 4\cos 2x + 1 = 0, 0 \le x < 2\pi \Leftrightarrow 0 \le 2x < 4\pi$

$$(2\cos 2x - 1)^2 = 0$$
$$\cos 2x = \frac{1}{2}$$
$$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

54. Since $|\sin x| \le 1$ and $|\cos x| \le 1$ for all x,
$\sin x + \cos x \le 2$ for all x.
This equation has no solutions.

56. Since $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ is an identity,
$2\cos^2\theta = 1 + \cos^2\theta$ for all θ.
The solutions of this equation are all θ, $0° \le \theta < 360°$.

58. $4\cos^2\theta = 7\cos\theta + 2, 0° \le \theta \le 180°$

$$4\cos^2\theta - 7\cos\theta - 2 = 0$$
$$(4\cos\theta + 1)(\cos\theta - 2) = 0$$
$$4\cos\theta + 1 = 0 \quad \text{or} \quad \cos\theta - 2 = 0$$
$$4\cos\theta = -1 \qquad \cos\theta = 2, \text{ no solution}$$
$$\cos\theta = \frac{-1}{4}$$
$$\theta = \cos^{-1}(-0.25)$$
$$\theta \approx 104.5°$$

60. $8\sin^2 x + 10\sin x = 3, 0 \le x \le \frac{\pi}{2}$

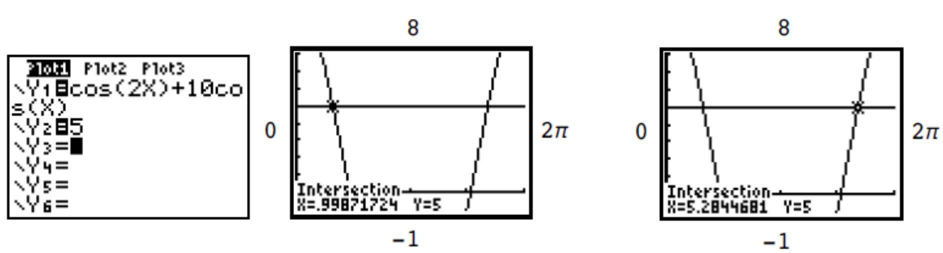
$$8\sin^2 x + 10\sin x - 3 = 0$$
$$(4\sin x - 1)(2\sin x + 3) = 0$$
$$4\sin x - 1 = 0 \quad \text{or} \quad 2\sin x + 3 = 0$$
$$4\sin x = 1 \qquad 2\sin x = -3$$
$$\sin x = \frac{1}{4} \qquad \sin x = \frac{-3}{2}, \text{ no solution}$$
$$x = \sin^{-1}(0.25)$$
$$x \approx 0.2527$$

62. $\cos 2x + 10\cos x = 5, 0 \le x < 2\pi$

$$2\cos^2 x - 1 + 10\cos x - 5 = 0$$
$$2\cos^2 x + 10\cos x - 6 = 0$$
$$\cos x = \frac{-10 \pm \sqrt{10^2 - 4(2)(-6)}}{2(2)} = \frac{-10 \pm 2\sqrt{37}}{4} = \frac{-5 \pm \sqrt{37}}{2}$$
$$\cos x = \frac{-5+\sqrt{37}}{2} = 0.5413812651\ldots \qquad \text{or} \quad \cos x = \frac{-5-\sqrt{37}}{2} = -5.54138265\ldots$$
$$x \approx 0.9987, 5.284\ (2\pi - 0.9987) \qquad \text{no solution}$$

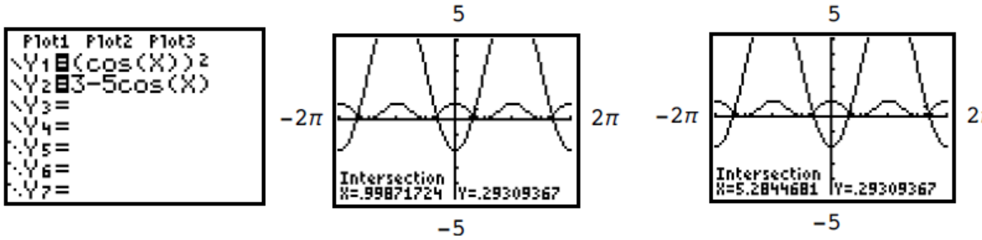
64. $\cos^2 x = 3 - 5\cos x$, all real x
$\cos^2 x + 5\cos x - 3 = 0$ has the same solutions as problem 62,
$x \approx 0.9987 + 2\pi k, 5.284 + 2\pi k$, k any integer $[-0.9987 + 2\pi = 5.284]$

66.



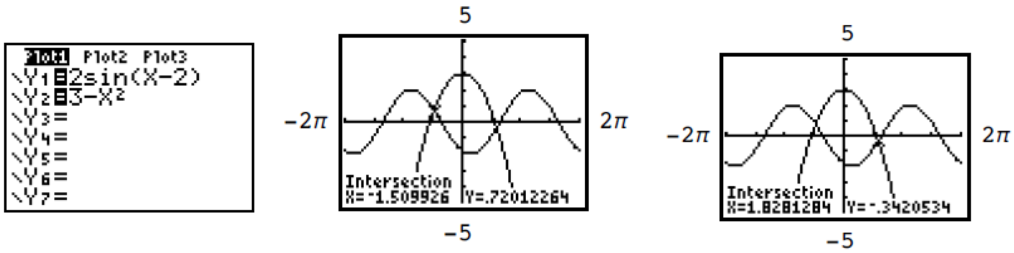
$x \approx 0.9987, 5.2845$

68.



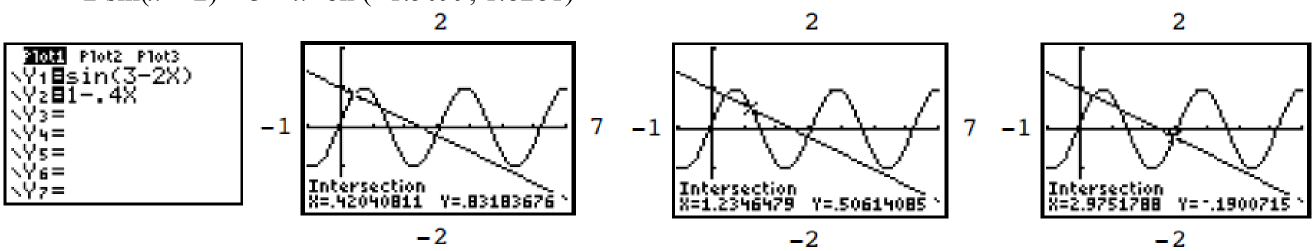
$x \approx 0.9987 + 2\pi k$, $5.2845 + 2\pi k$, k any integer

70.



$2\sin(x-2) < 3 - x^2$ on $(-1.5099, 1.8281)$

72.



$\sin(3-2x) \ge 1 - 0.4x$ on $[0.4204, 1.2346]$, $[2.9752, \infty)$

74.

$$5\sin x + 7 < 0$$
$$5\sin x < -7$$
$$\sin x < -1.4$$

Since $|\sin x| \le 1$, the statement $\sin x < -1.4$ is never true, and the original inequality has no solution.

76. Evaluating $\cos^{-1}(-0.7334)$ gives a unique number, ≈ 2.3941, the value of the inverse cosine function at -0.7334, while solving $\cos x = -0.7334$ involves finding an infinite number of x values whose cosine is -0.7334 by adding $2\pi k$, k any integer, to each solution in one period of $\cos x$.

78.

$$\sin x + \cos x = 1, \quad 0 \le x < 2\pi$$
$$\sin^2 x + 2\sin x\cos x + \cos^2 x = 1$$
$$1 + 2\sin x\cos x = 1$$
$$\sin x\cos x = 0$$
$$\sin x = 0 \quad \text{or} \quad \cos x = 0$$
$$x = 0, \pi \qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$\sin 0 + \cos 0 = 1$ $\qquad$ $\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1$

$\sin\pi + \cos\pi = -1$ (extraneous) $\qquad$ $\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2} = -1$(extraneous)

$x = 0, \frac{\pi}{2}$ are the solutions

80. $\sec x + \tan x = 1$, $0 \le x < 2\pi$

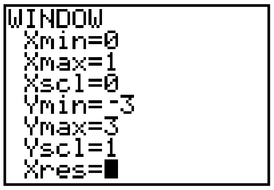
$$\sec x = 1 - \tan x$$
$$\sec^2 x = 1 - 2\tan x + \tan^2 x$$
$$\sec^2 x = \sec^2 x - 2\tan x$$
$$0 = -2\tan x$$
$$\tan x = 0$$
$$x = 0, \pi$$

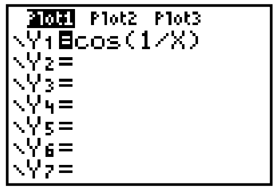
$\sec 0 + \tan 0 = 1$

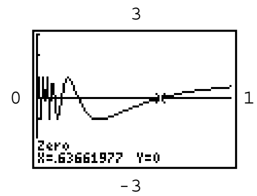
$\sec\pi + \tan\pi = -1$ (extraneous)

$x = 0$ is the solution

82. $g(x) = \cos\left(\frac{1}{x}\right)$ for $x > 0$







(A) 0.6366 is the largest zero. As $x \to \infty$, $1/x \to 0 \Rightarrow \cos\left(\frac{1}{x}\right) \to 1$ so $y = 1$ is a horizontal asymptote for the graph of g.

(B) Infinitely many zeros exist between 0 and b, for any b, however small. The exploration graphs suggest this conclusion, which is reinforced by the following reasoning: Note that for each interval $(0, b]$, however small, as x tends to zero through positive numbers, $1/x$ increases without bound, and as $1/x$ increases without bound, $\cos(1/x)$ will cross the x axis an unlimited number of times. The function g does not have a smallest zero, because, between 0 and b, no matter how small b is, there is always an unlimited number of zeros.

84. Solve

$$-8\cos 2t = 4$$
$$\cos 2t = -\frac{1}{2}$$
$$2t = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \quad \text{(first four solutions)}$$
$$t = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$
t = 1.05 sec, 2.09 sec, 4.19 sec, 5.24 sec

86. $I = 30\sin(120\pi t)$, $I = 25$ amps

$$25 = 30\sin(120\pi t)$$
$$\sin(120\pi t) = \frac{25}{30}$$
$$120\pi t = \sin^{-1}\left(\frac{25}{30}\right)$$
$$t = \frac{1}{120\pi} \cdot \sin^{-1}\left(\frac{5}{6}\right)$$
$t \approx 0.002613$ seconds

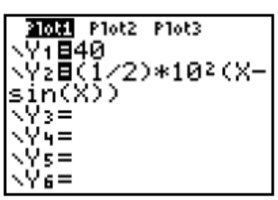
88. $I_L = I_E \cdot \cos^2\theta$, $I_L = 70\%\ I_E$

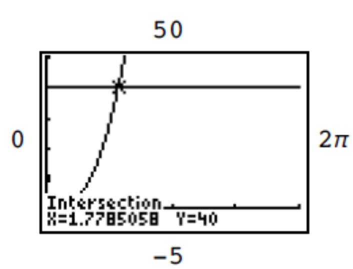
$$0.7I_E = I_E \cdot \cos^2\theta$$
$$0.7 = \cos^2\theta$$
$$\cos\theta = \pm\sqrt{0.7}$$
$$\theta = \cos^{-1}(\pm\sqrt{0.7})$$
$$\theta \approx 33.21°, 146.79°, 213.21°, 326.79°$$
Smallest positive θ: 33.21°

90. $r = \dfrac{3.44 \times 10^7}{1 - 0.206\cos\theta}$

$$3.78 \times 10^7 = \frac{3.44 \times 10^7}{1 - 0.206\cos\theta}$$
$$3.78 \times 10^7 - 7{,}786{,}800\cos\theta = 3.44 \times 10^7$$
$$-7{,}786{,}800\cos\theta = -3{,}400{,}000$$
$$\cos\theta \approx 0.436636359$$
$$\theta = \cos^{-1}(0.436636359)$$
$$\theta \approx 64.1°, 296°$$
Smallest positive θ: 64.1°

92.





$\theta \approx 1.779$ rad

94. (A) $\sin\theta = \frac{a}{R} = \frac{5.4}{R}$ $\quad\cos\theta = \frac{x}{R}$

$R = \frac{5.4}{\sin\theta}$ $\quad x = R\cos\theta$

$R = x + b$

$\frac{5.4}{\sin\theta} = R\cos\theta + 2.4$

$\frac{5.4}{\sin\theta} = \frac{5.4\cos\theta}{\sin\theta} + 2.4$

Graphing each side of the equation and finding the intersection gives $\theta \approx 0.83644866$.

$R = \frac{5.4}{\sin 0.83644866} \approx 7.274999994$

$L = R(2\theta)$

$L \approx 7.274999994(2 \cdot 0.83644866)$

$L \approx 12.1703$ mm to 4 decimal places

(B) Increase a to 5.5 mm:

$L = R(2\theta)$

$\theta = \frac{L}{2R}$

$\theta \approx \frac{12.17032799}{\frac{2(5.5)}{\sin\theta}}$

Graphing each side of the equation and finding the intersection gives $\theta \approx 0.77096792$.

$R \approx \frac{5.5}{\sin 0.770967922} \approx 7.892888691$

$x = R\cos\theta$

$x \approx 7.892888691 \cos 0.77096792 \approx 5.661068098$

$b = R - x$

$b \approx 7.892888691 - 5.661068098$

$b \approx 2.2318$ mm to 4 decimal places

96. $r = 2\sin\theta,\ 0° \le \theta \le 360°$

$r = 2(1 - \sin\theta)$

$2\sin\theta = 2(1 - \sin\theta)$

$4\sin\theta = 2$

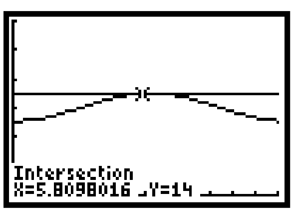
$\sin\theta = \frac{1}{2},\ r = 2\left(\frac{1}{2}\right) = 1$

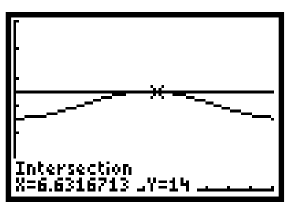
$\theta = 30°, 150° \Rightarrow (1, 30°), (1, 150°)$

98. We are to solve

$1.912\sin(0.511x - 1.608) + 12.13 = 14$

The graphs show the following:





Using $x = 5$ for May 15, we can interpret $x = 5.8098016$ as $0.8098016(31) = 25.1$ days later, June 9 or 10.

Using $x = 6$ for June 15, we can interpret $x = 6.6316713$ as $0.6316713(30) = 18.9$ days later, July 3 or 4.

Chapter 7 Group Activity

(A) $y = M\sin Bt + N\cos Bt$

$y = \frac{\sqrt{M^2+N^2}}{\sqrt{M^2+N^2}}(M\sin Bt + N\cos Bt)$

$y = \sqrt{M^2+N^2}\left(\frac{M}{\sqrt{M^2+N^2}}\sin Bt + \frac{N}{\sqrt{M^2+N^2}}\cos Bt\right)$

Now, if C has (M, N) on its terminal side,

$\cos C = \frac{M}{\sqrt{M^2+N^2}}$ $\quad\sin C = \frac{N}{\sqrt{M^2+N^2}}$

Thus $y = \sqrt{M^2 + N^2}\ (\cos C \sin Bt + \sin C \cos Bt)$

$y = \sqrt{M^2 + N^2}\ \sin(Bt + C)$ as required

(B) $A = \sqrt{M^2 + N^2} = \sqrt{(-4)^2 + 3^2} = 5$

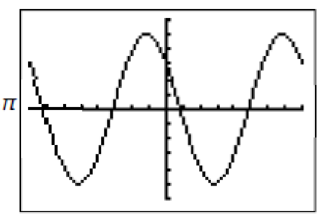
$B = \frac{1}{2}$

$\cos C = \frac{-4}{5}$ $\sin C = \frac{3}{5}$ C is a 2nd quadrant angle: $C = \cos^{-1}\left(-\frac{4}{5}\right) = 2.498$

$y_2 = 5 \sin\left(\frac{1}{2}t + 2.498\right)$

Amplitude = 5, Period = $\frac{2\pi}{1/2} = 4\pi$, Phase shift = $-\frac{C}{B} = 4.996$

(C)



(D) $A = \sqrt{(-3)^2 + (-4)^2} = 5$

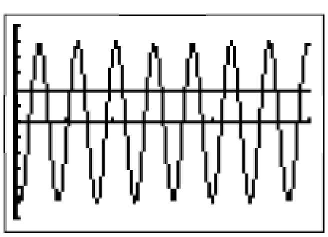
$B = 8$

$\cos C = -\frac{3}{5}$ $\sin C = -\frac{4}{5}$ C is a third–quadrant angle: $C = \pi + \sin^{-1}\left(\frac{4}{5}\right) = 4.069$

$y_2 = 5 \sin(8t + 4.069)$

Amplitude = 5 Period = $\frac{2\pi}{8} = \frac{\pi}{4}$ Phase shift = $-\frac{4.069}{8} = -0.509$

(E) Graphing y_1, y_2 and $y_3 = 2$:



The bottom of the weight will pass $y = 2$ 15 times in the first 6 seconds.

(F) Solve

$5 \sin(8t + 4.069) = 2$

$\sin(8t + 4.069) = 0.4$

$8t + 4.069 = \sin^{-1} 0.4$ or $\frac{\pi}{2} - \sin^{-1} 0.4$

$t = \frac{\sin^{-1} 0.4 - 4.069}{8}$ or $\frac{\frac{\pi}{2} - \sin^{-1} 0.4 - 4.069}{8}$

$t = -0.457$ or -0.364

Adding multiples of period $\frac{\pi}{8}$: $t = 0.328$ is the first positive solution, confirmed by the calculator intersection routine.