CHAPTER 6

Section 6−1

1. A positive angle is produced by a counterclockwise rotation from the initial side to the terminal side, a negative angle by a clockwise rotation.

3. Answers will vary.

5. Answers will vary.

7. Since 1 rotation corresponds to 360°, $\frac{1}{9}$ rotation corresponds to $\frac{1}{9}(360°) = 40°$

9. Since 1 rotation corresponds to 360°, $\frac{3}{4}$ rotation corresponds to $\frac{3}{4}(360°) = 270°$

11. Since 1 rotation corresponds to 360°, $\frac{9}{8}$ rotation corresponds to $\frac{9}{8}(360°) = 405°$

13. $\theta = \frac{s}{r} = \frac{24 \text{ centimeters}}{4 \text{ centimeters}} = 6$ radians

15. $\theta = \frac{s}{r} = \frac{30 \text{ feet}}{12 \text{ feet}} = 2.5$ radians

17. Since 1 rotation corresponds to 2π radians, $\frac{1}{8}$ rotation corresponds to $\frac{1}{8}(2\pi) = \frac{\pi}{4}$ radians

19. As in problem 17, $\frac{3}{4}$ rotation corresponds to $\frac{3}{4}(2\pi) = \frac{3\pi}{2}$ radians

21. As in problem 17, $\frac{13}{12}$ rotation corresponds to $\frac{13}{12}(2\pi) = \frac{13\pi}{6}$ radians

23. Using the relation $\theta_R = \frac{\pi \text{ rad}}{180°}\theta_D$,

we have:
$$\frac{\pi \text{ rad}}{180°}30° = \frac{\pi}{6}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}60° = \frac{\pi}{3}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}90° = \frac{\pi}{2}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}120° = \frac{2\pi}{3}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}150° = \frac{5\pi}{6}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}180° = \pi \text{ rad}$$

25. Using the relation $\theta_R = \frac{\pi \text{ rad}}{180°}\theta_D$,

we have:
$$\frac{\pi \text{ rad}}{180°}(-45°) = -\frac{\pi}{4}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}(-90°) = -\frac{\pi}{2}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}(-135°) = -\frac{3\pi}{4}\text{rad}$$
$$\frac{\pi \text{ rad}}{180°}(-180°) = -\pi \text{ rad}$$

27. Using the relation $\theta_D = \frac{180°}{\pi}\text{rad}\,\theta_R$,

we have:
$$\frac{180°}{\pi}\frac{\pi}{3} = 60° \qquad \frac{180°}{\pi}\frac{4\pi}{3} = 240°$$
$$\frac{180°}{\pi}\frac{2\pi}{3} = 120° \qquad \frac{180°}{\pi}\frac{5\pi}{3} = 300°$$
$$\frac{180°}{\pi}\pi = 180° \qquad \frac{180°}{\pi}2\pi = 360°$$

29. Using the relation $\theta_D = \frac{180°}{\pi}\text{rad}\,\theta_R$,

we have:
$$\frac{180°}{\pi}\left(-\frac{\pi}{2}\right) = -90°$$
$$\frac{180°}{\pi}(-\pi) = -180°$$
$$\frac{180°}{\pi}\left(-\frac{3\pi}{2}\right) = -270°$$
$$\frac{180°}{\pi}(-2\pi) = -360°$$

31. True. "Standard position" means that both angles have the positive *x* axis as their initial sides. If they have the same initial side and the same measure, they must have the same terminal side as well.

33. True. Angles that are complementary have measures that add up to 90°, so if both are positive and add to 90° the measure of each has to be between 0° and 90°. That's the definition of acute.

35. False. The terminal side of the angle −315° (or $-\frac{7\pi}{4}$) is in quadrant I.

37. $5°51'33'' = \left(5+\frac{51}{60}+\frac{33}{3600}\right)^{\circ} = 5.859°$

39. $354°8'29'' = \left(354+\frac{8}{60}+\frac{29}{3600}\right)^{\circ} = 354.141°$

41. $3.042° = 3°(0.042\cdot 60)' = 3°2.52' = 3°2'(0.52\cdot 60)'' = 3°2'31''$

43. $403.223° = 403°(0.223\cdot 60)' = 403°13.38' = 403°13'(0.38\cdot 60)'' = 403°13'23''$

45. Using the relation $\theta_R = \frac{\pi \text{ rad}}{180°}\theta_D$, we have $\frac{\pi \text{ rad}}{180°}64° = 1.117$ rad

47. Using the relation $\theta_R = \frac{\pi \text{ rad}}{180°}\theta_D$, we have $\frac{\pi \text{ rad}}{180°}108.413° = 1.892$ rad

49. First we convert 13°25'14" to decimal degrees:

$$13°25'14'' = \left(13+\frac{25}{60}+\frac{14}{3600}\right)^{\circ} = 13.421°$$

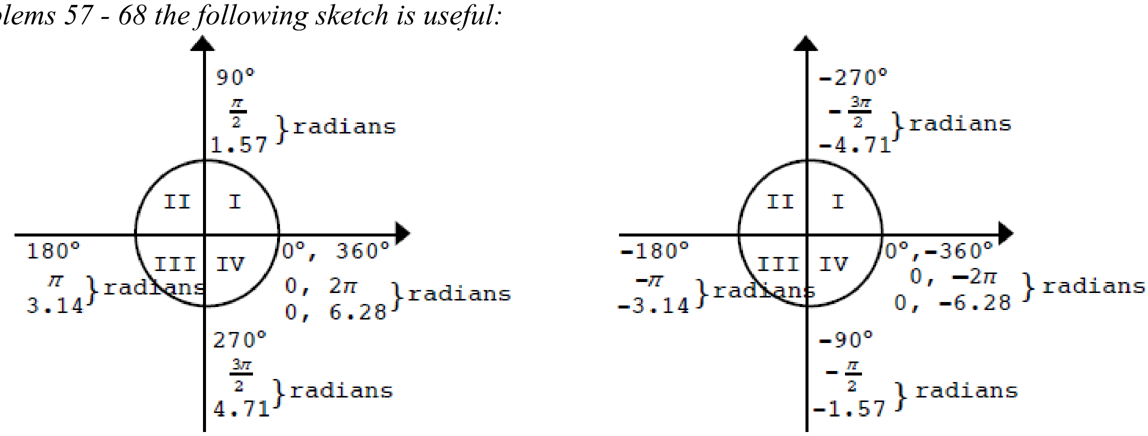
Then we use the relation $\theta_R = \frac{\pi \text{ rad}}{180°}\theta_D$ to find $\theta_R = \frac{\pi \text{ rad}}{180°}(13.421°) = 0.234$ radians.

51. Using the relation $\theta_D = \frac{180°}{\pi \text{ rad}}\theta_R$, we have $\frac{180°}{\pi}(0.93) = 53.29°$

53. Using the relation $\theta_D = \frac{180°}{\pi \text{ rad}}\theta_R$, we have $\frac{180°}{\pi}(1.13) = 64.74°$

55. Using the relation $\theta_D = \frac{180°}{\pi \text{ rad}}\theta_R$, we have $\frac{180°}{\pi}(-2.35) = -134.65°$

For Problems 57 - 68 the following sketch is useful:



57. From the sketch, we find, since 180° < 250° < 270°, 250° is a III quadrant angle.

59. Since 270° < 275° < 360°, 275° is a IV quadrant angle.

61. Since $\frac{\pi}{2} < \frac{3\pi}{4} < \pi$, $\frac{3\pi}{4}$ is a II quadrant angle.

63. $\frac{3\pi}{2}$ is a quadrantal angle.

65. Since −360° < −330° < −270°, −330° is a I quadrant angle.

67. Since −1.57 < −1.5 < 0, −1.5 is a IV quadrant angle.

69. A central angle of radian measure 1 is an angle subtended by an arc of the same length as the radius of the circle.

71. Of all the coterminal angles $210^\circ + n360^\circ$, $210^\circ + 360^\circ = 570^\circ$ is in the correct interval.

73. Of all the coterminal angles $45^\circ + n360^\circ$, $45^\circ - 360^\circ = -315^\circ$ is in the correct interval.

75. Of all the coterminal angles $\frac{\pi}{3} + n2\pi$, $\frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$ and $\frac{\pi}{3} + 4\pi = \frac{13\pi}{3}$ are in the correct interval.

77. Of all the coterminal angles $\pi + n2\pi$, $\pi - 2\pi = -\pi$ and $\pi - 4\pi = -3\pi$ are in the correct interval.

79. We apply $\frac{s}{c} = \frac{\theta^\circ}{360^\circ}$ with $s = 500$ mi, and $\theta^\circ = 7.5^\circ$

Then $\frac{500}{c} = \frac{7.5^\circ}{360^\circ}$ or $\frac{500}{c} = \frac{7.5}{360}$

$$360c \cdot \frac{500}{c} = 360c \cdot \frac{7.5}{360}$$
$$180{,}000 = 7.5c$$
$$c = 24{,}000 \text{ mi}$$

81. The 7.5° angle and θ have a common side. (An extended vertical pole in Alexandria will pass through the center of the earth.) The sun's rays are essentially parallel when they arrive at the earth. Thus, the other two sides of the angles are parallel, since a sun ray to the bottom of the well, when extended, will pass through the center of the earth. From geometry we know that the alternate interior angles made by a line intersecting two parallel lines are equal. Therefore, $\theta = 7.5^\circ$.

83. To calculate angular speed, we need radians and seconds.

200 revolutions × 2π radians = 400π radians

1 minute = 60 sec.

angular speed = $\frac{400\pi \text{ radians}}{60\text{ sec}} = 20.94 \frac{\text{rad}}{\text{sec}}$

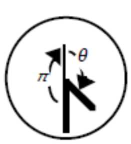
To calculate linear speed, we need the circumference of the wheel.

$c = \pi d = 6\pi$

In 200 revolutions, the linear distance is $200 \times 6\pi = 1200\pi$ feet.

linear speed = $\frac{1200\pi \text{ feet}}{60\text{ sec}}$ = 62.83 ft/sec

85.



From the figure, it should be clear that the minute (larger) hand is displaced π radians from noon, while the hour (smaller) hand is displaced $\theta = \frac{4\frac{1}{2}}{12}$ of a full revolution, or $\frac{4\frac{1}{2}}{12} \cdot 2\pi$ radians from noon.

Hence the larger angle between the hands has measure $\theta + \pi$, or $\frac{4\frac{1}{2}}{12} \cdot 2\pi + \pi = \frac{3}{4}\pi + \pi = \frac{7}{4}\pi$ radians.

87. We use $\theta = \frac{s}{r}$ with $r = \frac{1}{2}(10) = 5$ centimeters and $s = 10 \text{ meters} \times 100 \frac{\text{centimeters}}{\text{meter}} = 1000$ centimeters. Then $\theta = \frac{1000}{5} = 200$ radians.

89. In one year the line sweeps out one full revolution, or 2π radians. In one week the line sweeps out $\frac{1}{52}$ of one full revolution, or $\frac{1}{52} \cdot 2\pi = \frac{\pi}{26}$ radians. $\frac{\pi}{26} = 0.12$ radian to two decimal places.

91. Following example 4, we reason that points on the circumference of each wheel travel the same distance. Then $s_1 = r_1\theta_1 = r_2\theta_2 = s_2$. The radius of the front wheel is $\frac{1}{2}(40)$ or 20 centimeters. The radius of the back wheel is $\frac{1}{2}(60)$ or 30 centimeters. So

$$20\,\theta_1 = 8(30)$$
$$\theta_1 = \frac{240}{20}$$
$$\theta_1 = 12 \text{ radians}$$

93. Front wheel: $d = 40$ cm; $c = \pi d = 40\pi$ cm. One complete revolution is 2π radians. So 2π rad $= 40\pi$ cm or 1 cm $= \frac{1}{20}$ radian. In one hour, the linear distance is 10 km which is 1,000,000 cm.

$$1{,}000{,}000 \text{ cm} = \frac{1}{20} \cdot 1{,}000{,}000 \text{ radian} = 50{,}000 \text{ radians}$$
$$\frac{50{,}000 \text{ radians}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = 13.9 \text{ rad/sec}$$

Back wheel: $d = 60$ cm; $c = 60\pi$ cm

2π rad $= 60\pi$ cm or 1 cm $= \frac{1}{30}$ radian

$$1{,}000{,}000 \text{ cm} = \frac{1}{30} \cdot 1{,}000{,}000 \text{ radian} = 33{,}333.3 \text{ radians}$$
$$\frac{33{,}333.3 \text{ radians}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = 9.26 \text{ rad/sec}$$

95. We use $c \approx s = r\,\theta$ with $r = 9.3 \times 10^7$ mi and $\theta = 9.3 \times 10^{-3}$ rad.
Then $c \approx (9.3 \times 10^7)(9.3 \times 10^{-3}) = 865{,}000$ mi

97. We use $c \approx s = r\,\theta$. $r = 750$ ft. θ must be converted to radians to use the formula, thus, using $\theta_R = \frac{\pi \text{ rad}}{180°} 2.5°$ we have $\theta_R = \frac{\pi}{72}$ rad. Then $c \approx 750 \cdot \frac{\pi}{72} = 33$ ft.

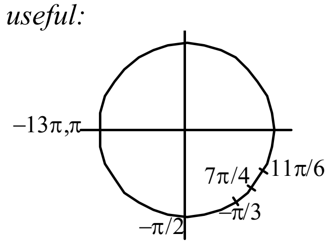
Section 6−2

1. The unit circle is the circle with center the origin and radius 1. In the *uv*-coordinate system it has the equation $u^2 + v^2 = 1$.

3. Answers will vary.

5. Since $\tan x = \frac{b}{a}$, when $a = 0$ the tangent is undefined. This occurs when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

For Problems 7−18, the following sketch is useful:



7. $W(\pi) = (-1, 0)$

9. $W\left(-\frac{\pi}{2}\right) = (0, -1)$

11. $W\left(\frac{7\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

13. $W\left(-\frac{\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

15. $W\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

17. $W(-13\pi) = (-1, 0)$

It's helpful to remember these rules for finding the coordinates of the circular points when x is a multiple of π:

1) If x has a denominator of 6, like $\frac{5\pi}{6}$, the coordinates are $\left(\pm\frac{\sqrt{3}}{2}, \pm\frac{1}{2}\right)$. Locate the quadrant to decide on the appropriate signs.

2) If x has a denominator of 4, like $-\frac{3\pi}{4}$, the coordinates are $\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$. Locate the quadrant to decide on the appropriate signs.

3) If x has a denominator of 3, like $\frac{7\pi}{3}$, the coordinates are $\left(\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$. Locate the quadrant to decide on the appropriate signs.

4) If the denominator is 2, or there is no denominator, like $\frac{3\pi}{2}$ or -6π, x is on one of the coordinate axes and you should be able to find the coordinates by just locating x on the unit circle.

IMPORTANT: These rules are only valid if x is a <u>reduced</u> <u>fraction</u>! For example, $\frac{6\pi}{4}$ has denominator 4 but can be reduced to $\frac{3\pi}{2}$.

19. $W(\pi) = (-1, 0)$ so $\sin \pi = 0$.

21. $W\left(-\frac{\pi}{2}\right) = (0, -1)$ so $\sec\left(-\frac{\pi}{2}\right) = \frac{1}{0}$ is not defined.

23. $W\left(\frac{7\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ so $\tan\frac{7\pi}{4} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$

25. $W\left(-\frac{\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ so $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

27. $W\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ so $\csc\frac{11\pi}{6} = \frac{1}{-1/2} = -2$

29. $W(-13\pi) = (-1, 0)$ so $\cot(-13\pi) = \frac{-1}{0}$ is not defined.

31. Since $\cos x = a$, it is negative in quadrants II and III.

33. Since $\sin x = b$, it is positive in quadrants I and II.

35. Since $\cot x = \frac{a}{b}$, it is negative if a and b have opposite signs. This occurs in quadrants II and IV.

37. -0.6573 **Common Error:** $\cos 2.288 \neq 0.9992$; calculator must be in *radian* mode.

39. -14.60

41. 1.000

43. 0.4226 (calculator in degree mode)

45. -1.573 (calculator in radian mode)

47. $\sin(113°27'13'') = \sin\left(113 + \frac{27}{60} + \frac{13}{3600}\right)^{\circ} = 0.9174$

49. False. The domain of the wrapping function is the set of all real numbers.

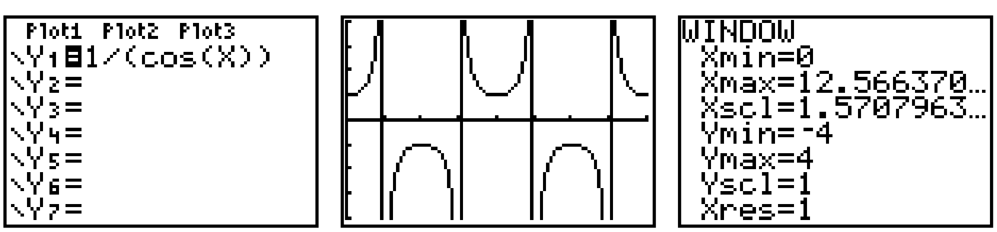
51. False. The wrapping function is not one-to-one. $W(0) = W(2\pi)$ but $0 \neq 2\pi$.

53. False. The reciprocal of $\sin x$ is $\csc x$, unless $\sin x = 0$.

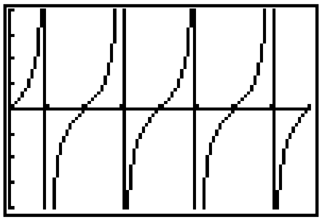
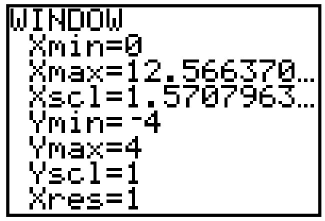
55. False. The secant function is not one-to-one. $\sec 0 = \sec 2\pi$, but $0 \neq 2\pi$.

57. False. The domain of $\sin x$ is all real numbers; the domain of $\csc x$ excludes all integer multiples of π.

59.

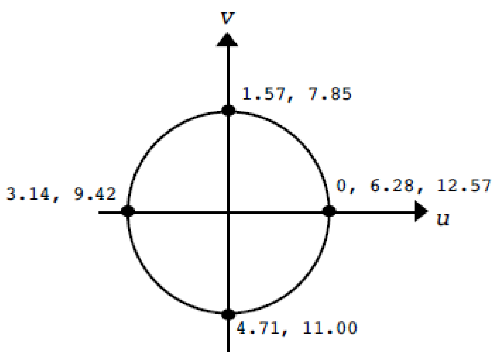


Since there is no secant button, we enter $y = \sec x$ as $y = 1/\cos x$. There are no zeros and 3 turning points. They occur when the graph is at height 1 or −1. Since $\sec x = \dfrac{1}{\cos x}$, $\sec x = \pm 1$ when $\cos x = \pm 1$. This occurs for $x = \pi$, 2π, and 3π, so the turning points are $(\pi, -1)$, $(2\pi, 1)$, and $(3\pi, -1)$.

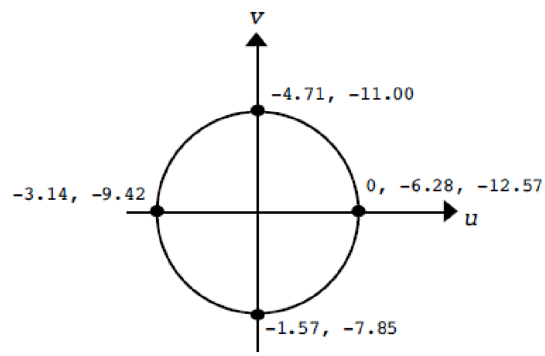
61.  

There are 5 zeros. Since $\tan x = \dfrac{\sin x}{\cos x}$, the zeros occur when $\sin x = 0$. The x values are 0, π, 2π, 3π, and 4π. There are no turning points.

For Problems 63 - 72*, the following sketches are useful.*



Sketch 1: counterclockwise wrapping



Sketch 2: clockwise wrapping

63. Using sketch 1 and $1.57 < 2 < 3.14$, we see that $W(2)$ is in the second quadrant. Hence, if $W(2) = (a, b)$, then $a < 0$ and $b > 0$, that is, a is negative and b is positive.

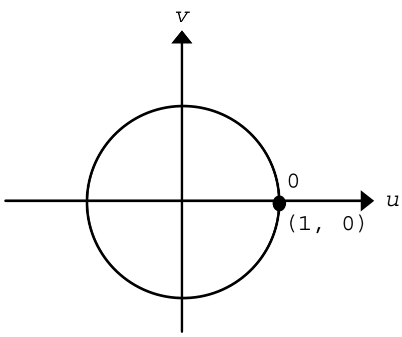
65. Using sketch 1 and $1.57 < 3 < 3.14$, we see that $W(3)$ is in the second quadrant. Hence, if $W(3) = (a, b)$, then $a < 0$ and $b > 0$, that is, a is negative and b is positive.

67. Using sketch 1 and $4.71 < 5 < 6.28$, we see that $W(5)$ is in the fourth quadrant. Hence, if $W(5) = (a, b)$, then $a > 0$ and $b < 0$, that is, a is positive and b is negative.

69. Using sketch 2 and $-3.14 < -2.5 < -1.57$, we see that $W(-2.5)$ is in the third quadrant. Hence, if $W(-2.5) = (a, b)$, then $a < 0$ and $b < 0$, that is, a and b are negative.

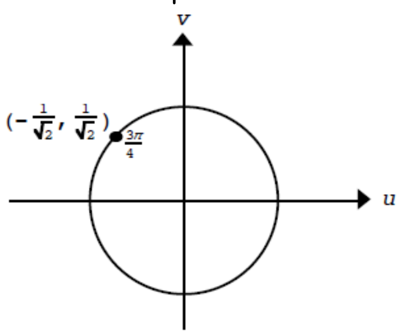
71. Using sketch 2 and $-6.28 < -6.1 < -4.71$, we see that $W(-6.1)$ is in the first quadrant. Hence, if $W(-6.1) = (a, b)$, then $a > 0$ and $b > 0$, that is, a and b are positive.

73.



From the sketch, if $W(x) = (1, 0)$, $0 \le x < 2\pi$, then $x = 0$. Since $W(x) = W(x + 2k\pi)$, k any integer, if there are no restrictions on x, then $x = 0 + 2k\pi$ or $x = 2k\pi$, k any integer.

75.



From the sketch, if $W(x) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $0 \le x < 2\pi$, then $x = \frac{3\pi}{4}$. Since $W(x) = W(x + 2k\pi)$, k any integer, if there are no restrictions on x, then $x = \frac{3\pi}{4} + 2k\pi$, k any integer.

77. $W(x)$ is the coordinates of a point on a unit circle that is $|x|$ units from (1, 0), in a counterclockwise direction if x is positive and in a clockwise direction if x is negative. $W(x + 4\pi)$ has the same coordinates as $W(x)$, since we return to the same point every time we go around the unit circle any integer multiple of 2π units (the circumference of the circle) in either direction.

79. $\sin x < 0$ in quadrants III and IV; $\cot x < 0$ in quadrants II and IV; therefore, both are true in quadrant IV.

81. $\cos x < 0$ in quadrants II and III; $\sec x > 0$ in quadrants I and IV; therefore, it is not possible to have both true for the same value of x.

83. $\cos x = a$ is always defined. There are no values for which it is undefined.

85. $\tan x = \frac{b}{a}$ is undefined if and only if $a = 0$. This occurs at points on the vertical axis. The only values of x between 0 and 2π for which $W(x)$ is on the vertical axis are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

87. $\sec x = \frac{1}{a}$ is undefined if and only if $a = 0$. This occurs at points on the vertical axis. The only values of x between 0 and 2π for which $W(x)$ is on the vertical axis are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

89. Two points are given; to find the equation of the line we first find the slope, then use point-slope form.

$$m = \frac{b-0}{a-0} = \frac{b}{a} \qquad y - 0 = \frac{b}{a}(x - 0)$$

$$y = \frac{b}{a}x$$

$$ay = bx$$

$$bx - ay = 0$$

91. Given $n = 12$, $r = 5$, we use the given formula $A = \frac{1}{2}nr^2 \sin \frac{2\pi}{n}$ to obtain:

$$A = \frac{1}{2}(12)(5)^2 \sin \frac{2\pi}{12} = 150 \sin \frac{\pi}{6} = 150\left(\frac{1}{2}\right) = 75 \text{ square meters}$$

93. Given $n = 3$, $r = 4$, we use the given formula $A = \frac{1}{2}nr^2 \sin \frac{2\pi}{n}$ to obtain:

$$A = \frac{1}{2}(3)(4)^2 \sin \frac{2\pi}{3} = 24 \sin \frac{2\pi}{3} = 24\left(\frac{\sqrt{3}}{2}\right) = 12\sqrt{3} \approx 20.78 \text{ square inches}$$

95. $a_1 = 0.5$

$a_2 = a_1 + \cos a_1 = 0.5 + \cos 0.5 = 1.377583$

$a_3 = a_2 + \cos a_2 = 1.377583 + \cos 1.377583 = 1.569596$

$a_4 = a_3 + \cos a_3 = 1.569596 + \cos 1.569596 = 1.570796$

$a_5 = a_4 + \cos a_4 = 1.570796 + \cos 1.570796 = 1.570796$

$\frac{\pi}{2} = 1.570796$ to six decimal places.

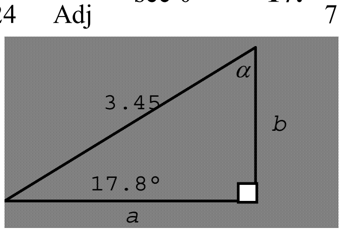
Section 6−3

1. Given two sides, or one acute angle and a side, solving for the remaining three quantities is called solving the right triangle.
3. Yes, a diagonal of the rectangle partitions it into two right triangles.
5. Any number of similar triangles can have a given set of angles, as long as the sides have equal ratios, thus, the length of the sides is not determined.

7. $\sin\theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{7}{25}$ **9.** $\csc\theta = \frac{\text{Hyp}}{\text{Opp}} = \frac{25}{7}$ **11.** $\tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{7}{24}$ **13.** $\frac{24}{25} = \frac{\text{Adj}}{\text{Hyp}} = \cos\theta$

15. $\frac{25}{24} = \frac{\text{Hyp}}{\text{Adj}} = \sec\theta$ **17.** $\frac{24}{7} = \frac{\text{Adj}}{\text{Opp}} = \cot\theta$ **19.** 60.55° **21.** 82.90° **23.** 37.09°

25.



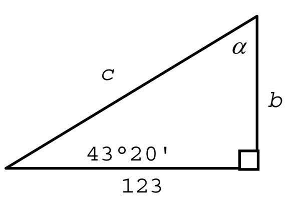
Solve for α: $\alpha = 90° - 17.8° = 72.2°$

Solve for b: $\sin\beta = \frac{b}{c}$ Solve for a: $\cos\beta = \frac{a}{c}$

$\sin 17.8° = \frac{b}{3.45}$ $\cos 17.8° = \frac{a}{3.45}$

$b = 3.45 \sin 17.8° = 1.05$ $a = 3.45 \cos 17.8° = 3.28$

27.



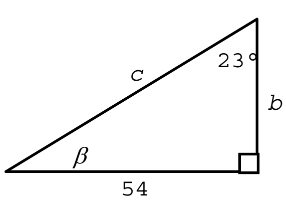
Solve for α: $\alpha = 90° - 43°20' = 46°40'$

Solve for b: $\tan\beta = \frac{b}{a}$ Solve for c: $\sec\beta = \frac{c}{a}$

$\tan 43°20' = \frac{b}{123}$ $\sec 43°20' = \frac{c}{123}$

$b = 123 \tan 43°20' = 116$ $c = 123 \sec 43°20' = 169$

29.



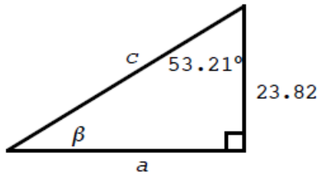
Solve for β: $\beta = 90° - 23°0' = 67°0'$

Solve for b: $\tan\beta = \frac{b}{a}$ Solve for c: $\csc\alpha = \frac{c}{a}$

$\tan 67°0' = \frac{b}{54}$ $\csc 23°0' = \frac{c}{54}$

$b = 54 \tan 67°0' = 127$ $c = 54 \csc 23°0' = 138$

31.



Solve for β: $\beta = 90° - 53.21° = 36.79°$

Solve for a: $\tan\alpha = \dfrac{a}{b}$

$\tan 53.21° = \dfrac{a}{23.82}$

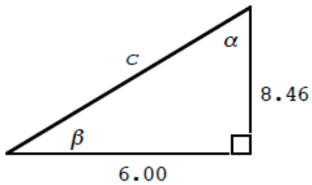
$a = 23.82\tan 53.21° = 31.85$

Solve for c: $\sec\alpha = \dfrac{c}{b}$

$\sec 53.21° = \dfrac{c}{23.82}$

$c = 23.82\sec 53.21° = 39.77$

33.



Solve for β: $\tan\beta = \dfrac{b}{a}$

$\tan\beta = \dfrac{8.46}{6.00}$

$\beta = \tan^{-1}\dfrac{8.46}{6.00} = 54.7°$ or $54°40'$

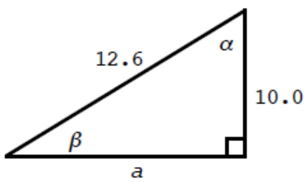
Solve for α: $\alpha = 90° - 54°40' = 35°20'$

Solve for c: $\csc\beta = \dfrac{c}{b}$

$\csc 54°40' = \dfrac{c}{8.46}$

$c = 8.46\csc 54°40' = 10.4$

35.



Solve for β: $\sin\beta = \dfrac{b}{c}$

$\sin\beta = \dfrac{10.0}{12.6}$

$\beta = \sin^{-1}\dfrac{10.0}{12.6} = 52.5°$ or $52°30'$

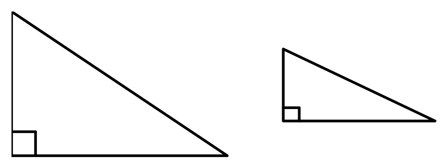
Solve for α: $\alpha = 90° - 52°30' = 37°30'$

Solve for a: $\cot\beta = \dfrac{a}{b}$

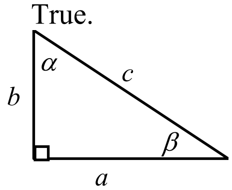
$\cot 52°30' = \dfrac{a}{10.0}$

$a = 10.0\cot 52°30' = 7.67$

37. False. Knowing only the three angles can't tell us anything about the lengths of the sides. The two triangles at the right have the exact same angles but totally different side lengths.



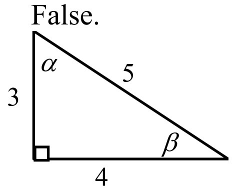
39. True.



According to the diagram,

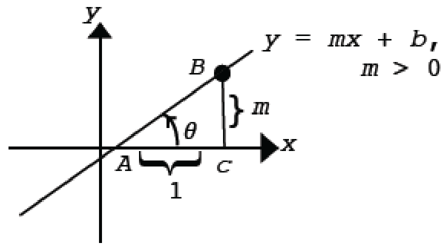
$\sin\alpha = \dfrac{\text{Opp}}{\text{Hyp}} = \dfrac{a}{c}$ and $\cos\beta = \dfrac{\text{Adj}}{\text{Hyp}} = \dfrac{a}{c}$.

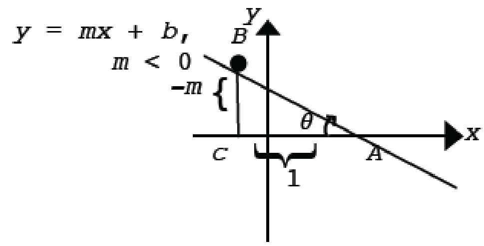
41. False.



According to the diagram,

$\sec\alpha = \dfrac{\text{Hyp}}{\text{Adj}} = \dfrac{5}{3}$ and $\cos\beta = \dfrac{\text{Adj}}{\text{Hyp}} = \dfrac{4}{5}$.





43. See the figure. A line of positive slope m will form a right triangle ABC as shown. $\tan\theta = \frac{m}{1}$.

Here $\tan\theta = \frac{1}{2}$

$$\theta = \tan^{-1}\frac{1}{2} = 26.6^\circ$$

45. See the figure. A line of positive slope m will form a right triangle ABC as shown. $\tan\theta = \frac{m}{1}$.

Here $\tan\theta = 5$

$$\theta = \tan^{-1} 5 = 78.7^\circ$$

47. See the figure. A line of negative slope m will form a right triangle ABC as shown. $\tan\theta = \frac{-m}{1}$.

Here $\tan\theta = -\frac{-2}{1} = 2$

$$\theta = \tan^{-1} 2 = 63.4^\circ$$

49. See the figure. An angle θ can be formed by either of two lines, one with positive slope m and the other with negative slope m.
Thus $m = \pm\tan\theta$
Here $m = \pm\tan 20^\circ = \pm 0.36$

51. See the figure. An angle θ can be formed by either of two lines, one with positive slope m and the other with negative slope m.
Thus $m = \pm\tan\theta$
Here $m = \pm\tan 80^\circ = \pm 5.67$

53. See the figure. An angle θ can be formed by either of two lines, one with positive slope m and the other with negative slope m.
Thus $m = \pm\tan\theta$
Here $m = \pm\tan\left(\frac{\pi}{30}\right) = \pm 0.11$

55. (A) In triangle OAD, $\cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{OA}{1} = OA$

(B) In triangle OED, angle $EOD = 90^\circ - \theta$, angle $OED = 90^\circ - (90^\circ - \theta) = \theta$.

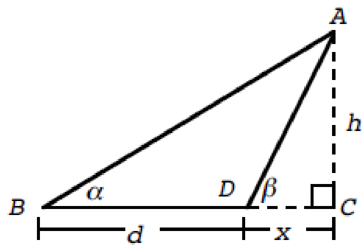
Thus $\cot OED = \frac{\text{Adj}}{\text{Opp}} = \frac{DE}{1} = DE = \cot\theta$

(C) In triangle ODC, $\sec\theta = \frac{\text{Hyp}}{\text{Adj}} = \frac{OC}{1} = OC$

57. (A) As θ approaches 90°, $OA = \cos\theta$ approaches 0.
(B) As θ approaches 90°, $DE = \cot\theta$ approaches 0.
(C) As θ approaches 90°, $OC = \sec\theta$ increases without bound.

59. (A) As θ approaches 0°, $AD = \sin\theta$ approaches 0.
(B) As θ approaches 0°, $CD = \tan\theta$ approaches 0.
(C) As θ approaches 0°, $OE = \csc\theta$ increases without bound.

61.



Label as shown at the left.

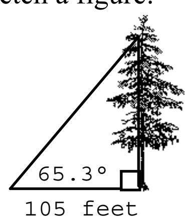
In right triangle ADC, $\cot\beta = \dfrac{x}{h}$

In right triangle ABC, $\cot\alpha = \dfrac{d+x}{h}$

Hence $x = h\cot\beta$ and $d + x = h\cot\alpha$

$$d = h\cot\alpha - x$$
$$d = h\cot\alpha - h\cot\beta$$
$$d = h(\cot\alpha - \cot\beta)$$
$$h = \frac{d}{\cot\alpha - \cot\beta}$$

63. Sketch a figure:



Let h = height of tree
From the figure, it should be clear that

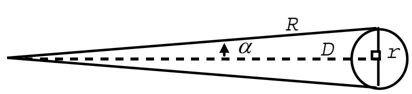
$$\tan 65.3° = \frac{h}{105}$$
$$h = 105\tan 65.3° = 228 \text{ feet}$$

65. Sketch a figure:

Let h = how far train climbs.

$$\sin 1°23' = \frac{h}{5280}$$
$$h = 5280\sin 1°23' = 127.5 \text{ feet}$$

67. Sketch a figure:



Note $\alpha = \dfrac{1}{2}(32') = 16'$

diameter $= d = 2r$

$$\tan\alpha = \frac{r}{D}$$
$$\tan 16' = \frac{r}{239{,}000}$$
$$r = 239{,}000\tan 16'$$
$$d = 2(239{,}000)\tan 16'$$
$$= 2225 \text{ miles}$$

Alternatively, we can write

$$\sin\alpha = \frac{r}{R}$$
$$\sin 16' = \frac{r}{239{,}000}$$
$$r = 239{,}000\sin 16'$$
$$d = 2(239{,}000)\sin 16' = 2225 \text{ miles}$$

Although it is not clear whether 239,000 miles is to be interpreted as D, R, or $D - r$, at this accuracy it does not matter.

69. Sketch a figure:

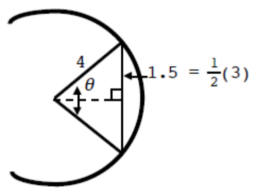
We will find $\dfrac{1}{2}\theta$ and double it.

$$\sin\frac{1}{2}\theta = \frac{1.5}{4}$$

71. We use $g = \dfrac{v}{t\sin\theta}$

with $v = 4.1$, $t = 3.0$, $\theta = 8.0°$

$$g = \frac{4.1}{3.0\sin 8.0°}$$

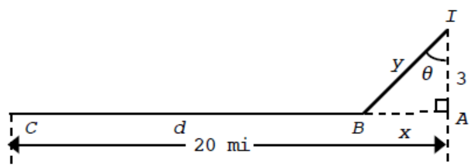


$$\frac{1}{2}\theta = \sin^{-1}\frac{1.5}{4}$$

$$\theta = 2\sin^{-1}\frac{1.5}{4} = 44°$$

$g = 9.8$ meters/second^2

73.



(A) We note that the cable consists of water section *IB*, and shore section *CB* = 20 mi − *AB*.
Let $y = IB$ and $d = CB = 20 - x$
In right triangle *ABI*

$$\sec\theta = \frac{y}{3\text{ mi}} \quad \text{and } \tan\theta = \frac{x}{3\text{ mi}}$$

$$y = 3\sec\theta\text{ mi} \qquad x = 3\tan\theta\text{ mi}$$

Thus the cost of the cable $=\left(\begin{array}{c}\text{Cost of Water}\\ \text{Section Per Mile}\end{array}\right)\left(\begin{array}{c}\text{Number of}\\ \text{Water miles} = y\end{array}\right) + \left(\begin{array}{c}\text{Cost of Shore}\\ \text{Section Per Mile}\end{array}\right)\left(\begin{array}{c}\text{Number of}\\ \text{Shore miles} = d\end{array}\right)$

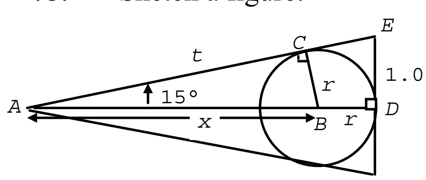
$C(\theta) = (25{,}000\ \frac{\text{dollars}}{\text{mi}})(3\sec\theta\text{ mi}) + (15{,}000\ \frac{\text{dollars}}{\text{mi}})(20 - 3\tan\theta\text{ mi})$

$C(\theta) = 75{,}000\sec\theta + 300{,}000 - 45{,}000\tan\theta$

(B)

θ	$C(\theta)$
10°	\$368,222
20°	\$363,435
30°	\$360,622
40°	\$360,146
50°	\$363,050

75. Sketch a figure:



In triangle *ABC*, we have $\frac{r}{x} = \sin 15°$ In triangle *ADE*, we have $\frac{1.0}{r+x} = \tan 15°$

Eliminating *x*, we see that

$$\frac{r+x}{1.0} = \frac{1}{\tan 15°} = \cot 15° \text{ (reciprocal identity)}$$

$$x = \cot 15° - r$$

Substituting, we have

$$\frac{r}{\cot 15° - r} = \sin 15°$$

$$r = \sin 15°(\cot 15° - r)$$

$$r = \sin 15° \cot 15° - \sin 15° r$$

$$r(1 + \sin 15°) = \sin 15° \cot 15°$$

$$r = \frac{\sin 15° \cot 15°}{1 + \sin 15°}$$

$$r = 0.77 \text{ meters}$$

Section 6–4

1. An equation is an identity if it holds true for all replacements of the variable or variables by real numbers for which both sides are defined.

3. To evaluate $\sin^2 x$ and $(\sin x)^2$, find $\sin\frac{\pi}{6}$ and square the result. To evaluate $\sin x^2$, find $\left(\frac{\pi}{6}\right)^2$ and then sine of the result. Calculator in radian mode.

5. Since one can only see a portion of the graph of a function, it is not possible to tell from a graph whether or not the function is periodic.

7. sine: 2π; cotangent: π; cosecant: 2π

9. (A) Since the range of the cosine function is $[-1, 1]$, the largest and smallest y values on its graph are, respectively, 1 and -1. The largest deviation of the function from the x axis is therefore 1 unit.
(B) Since the range of the tangent function is all real numbers, the graph deviates indefinitely far from the x axis.
(C) Since the range of the cosecant function is all real numbers $y \geq 1$ or $y \leq -1$, the graph deviates indefinitely far from the x axis.

11. (A) $-2\pi, -\pi, 0, \pi, 2\pi$ (B) $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ (C) No x intercepts

13. (A) Defined for all real x (B) $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ (C) $-2\pi, -\pi, 0, \pi, 2\pi$

15. (A) There are no vertical asymptotes (B) $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ (C) $-2\pi, -\pi, 0, \pi, 2\pi$

17. (A) A shift of $\pi/2$ to the left will transform the cosecant graph into the secant graph. [The answer is not unique––see part (B).]
(B) The graph of $y = -\csc(x - \pi/2)$ is a $\pi/2$ shift to the right and a reflection in the x axis of the graph of $y = \csc x$. The result is the graph of $y = \sec x$. The graph of $y = -\csc\left(x + \frac{\pi}{2}\right)$ is a $\pi/2$ shift to the left and a reflection in the x axis of the graph of $y = \csc x$. The result is not the graph of $y = \sec x$.

19. Let $f(x) = \frac{\tan x}{x}$

$$f(-x) = \frac{\tan(-x)}{(-x)} = \frac{-\tan x}{-x} = \frac{\tan x}{x} = f(x)$$

$y = \frac{\tan x}{x}$ is even.

21. Let $f(x) = \frac{\csc x}{x}$

$$f(-x) = \frac{\csc(-x)}{-x} = \frac{\frac{1}{\sin(-x)}}{-x} = \frac{\frac{1}{-\sin x}}{-x} = \frac{-\csc x}{-x} = \frac{\csc x}{x} = f(x)$$

$y = \frac{\csc x}{x}$ is even.

23. Let $f(x) = \sin x \cos x$

$$f(-x) = \sin(-x)\cos(-x) = -\sin x \cos x = -f(x)$$

$y = \sin x \cos x$ is odd.

25. Let $f(x) = \sin x - \cos x$

$$f(-x) = \sin(-x) - \cos(-x) = -\sin x - \cos x$$

Since $f(x) \neq f(-x)$ and $-f(x) \neq f(-x)$,

27. Let $f(x) = x^2 + \sin^2 x$

$$f(-x) = (-x)^2 + \sin^2(-x) = x^2 + (-\sin x)^2 = x^2 + \sin^2 x$$

29. Let $f(x) = x^2 \sin x$

$$f(-x) = (-x)^2 \sin(-x) = x^2(-\sin x) = -x^2 \sin x$$

$y = \sin x - \cos x$ is neither even nor odd.

$= f(x)$
$y = x^2 + \sin^2 x$ is even.

$= -f(x)$
$y = x^2 \sin x$ is odd.

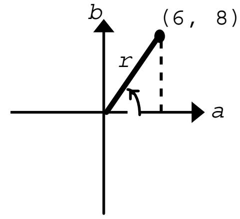
31. $(a, b) = (6, 8)$

$$r = \sqrt{a^2+b^2} = \sqrt{6^2+8^2} = \sqrt{100} = 10$$

$$\sin\theta = \frac{b}{r} = \frac{8}{10} = \frac{4}{5} \qquad \csc\theta = \frac{r}{b} = \frac{10}{8} = \frac{5}{4}$$

$$\cos\theta = \frac{a}{r} = \frac{6}{10} = \frac{3}{5} \qquad \sec\theta = \frac{r}{a} = \frac{10}{6} = \frac{5}{3}$$

$$\tan\theta = \frac{b}{a} = \frac{8}{6} = \frac{4}{3} \qquad \cot\theta = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$$



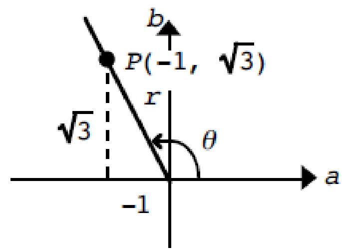
33. $(a, b) = (-1, \sqrt{3})$

$$r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+(\sqrt{3})^2} = \sqrt{4} = 2$$

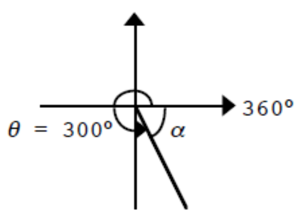
$$\sin\theta = \frac{b}{r} = \frac{\sqrt{3}}{2} \qquad \csc\theta = \frac{r}{b} = \frac{2}{\sqrt{3}}$$

$$\cos\theta = \frac{a}{r} = \frac{-1}{2} = -\frac{1}{2} \qquad \sec\theta = \frac{r}{a} = \frac{2}{-1} = -2$$

$$\tan\theta = \frac{b}{a} = \frac{\sqrt{3}}{-1} = -\sqrt{3} \qquad \cot\theta = \frac{a}{b} = \frac{-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

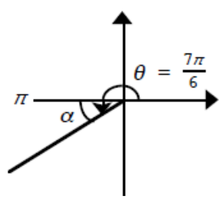


35.



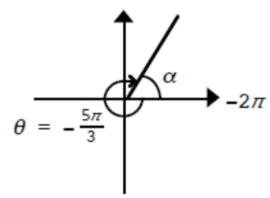
$\alpha = 360° - 300° = 60°$

37.



$\alpha = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$

39.

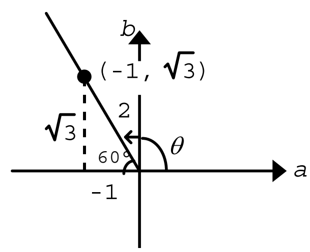


$\alpha = -\frac{5\pi}{3} + 2\pi = \frac{\pi}{3}$

41. $\cos\theta = \frac{a}{r} = \frac{-1}{2}$ because $r > 0$.

a is negative in the II and III quadrants. The smallest positive θ is associated with a 60° reference triangle in the II quadrant as shown in the figure.

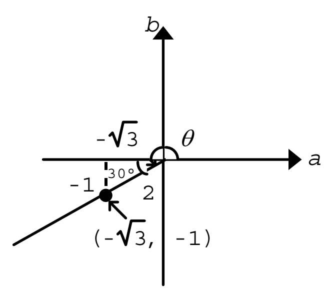
$\theta = 120°$ or $\frac{2\pi}{3}$ radians



43. $\sin\theta = \frac{b}{r} = \frac{-1}{2}$ because $r > 0$.

b is negative in the III and IV quadrants. The smallest positive θ is associated with a 30° reference triangle in the III quadrant as drawn.

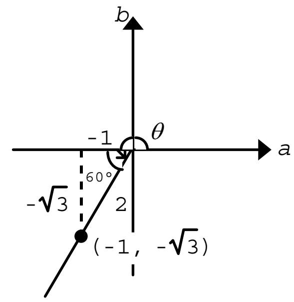
$\theta = 210°$ or $\frac{7\pi}{6}$ radians



45. $\csc\theta = \frac{r}{b} = \frac{2}{-\sqrt{3}}$ because $r > 0$.

b is negative in the III and IV quadrants. The smallest positive θ is associated with a 60° reference triangle in the III quadrant as drawn.

$\theta = 240°$ or $\frac{4\pi}{3}$ radians

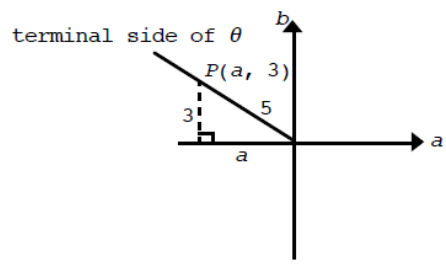


47. Since $\sin\theta = \frac{3}{5} > 0$ and $\cos\theta < 0$, θ is a II quadrant angle. We sketch a reference triangle.

Since $\sin\theta = \frac{b}{r} = \frac{3}{5}$, we know that $b = 3$ and $r = 5$. Use the Pythagorean theorem to find a.

$a^2 + 3^2 = 5^2$
$a^2 = 16$
$a = -4$

(a must be negative because θ is a II quadrant angle)
Using $(a, b) = (-4, 3)$ and $r = 5$, we have



$\cos\theta = \frac{a}{r} = \frac{-4}{5} = -\frac{4}{5}$ $\qquad$ $\tan\theta = \frac{b}{a} = \frac{3}{-4} = -\frac{3}{4}$

$\sec\theta = \frac{r}{a} = \frac{5}{-4} = -\frac{5}{4}$ $\qquad$ $\cot\theta = \frac{a}{b} = \frac{-4}{3} = -\frac{4}{3}$

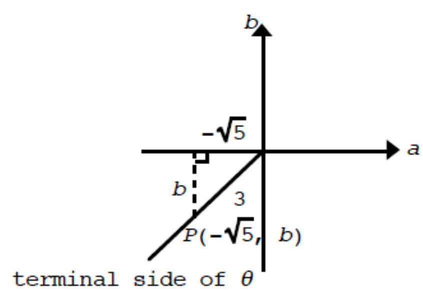
$\csc\theta = \frac{r}{b} = \frac{5}{3}$

49. Since $\cos\theta = -\frac{\sqrt{5}}{3} < 0$ and $\cot\theta > 0$, θ is a III quadrant angle. We sketch a reference triangle.

Since $\cos\theta = \frac{a}{r} = -\frac{\sqrt{5}}{3}$ we know that $a = -\sqrt{5}$ and $r = 3$. Use the Pythagorean theorem to find b.

$(-\sqrt{5})^2 + b^2 = 3^2$
$b^2 = 4$
$b = -2$

(b must be negative because θ is a III quadrant angle)
Using $(a, b) = (-\sqrt{5}, -2)$ and $r = 3$, we have



$\sin\theta = \frac{b}{r} = -\frac{2}{3}$ $\qquad$ $\sec\theta = \frac{r}{a} = -\frac{3}{\sqrt{5}}$

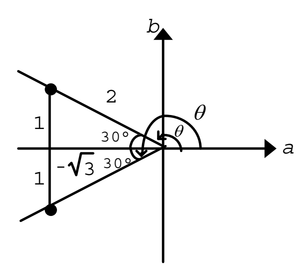
$\tan\theta = \frac{b}{a} = \frac{-2}{-\sqrt{5}} = \frac{2}{\sqrt{5}}$ $\qquad$ $\cot\theta = \frac{a}{b} = \frac{-\sqrt{5}}{-2} = \frac{\sqrt{5}}{2}$

$\csc\theta = \frac{r}{b} = \frac{3}{-2} = -\frac{3}{2}$

51. In these situations $P(a, b)$ is restricted so that $a = 0$. In this case, functions for which a is in the denominator are not defined. These functions are tangent and secant.

53. $\cos\theta = \frac{a}{r} = -\frac{\sqrt{3}}{2} = \frac{-\sqrt{3}}{2}$

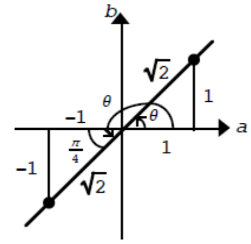
Thus $(a, b) = (-\sqrt{3}, 1)$ or $(-\sqrt{3}, -1)$ θ is associated with a 30° reference triangle in the II quadrant or the III quadrant as drawn:
$\theta = 150°$ or $210°$



55. $\tan\theta = \frac{b}{a} = \frac{1}{1} = \frac{-1}{-1}$

Thus $(a, b) = (1, 1)$ or $(-1, -1)$. θ is associated with a $\frac{\pi}{4}$ reference triangle in the I quadrant or the III quadrant as drawn:

$\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$



57. False. $f(x) = x + 1$ is neither even nor odd.

59. False. $\sin x$ and $\cos x$ have period 2π; however, $\frac{\sin x}{\cos x} = \tan x$ has period π.

61. True. If $f(-x) = -f(x)$ and $g(-x) = -g(x)$, then $(fg)(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = (fg)(x)$ for all x in the domain of $fg(x)$.

63. If $f(x + p) = f(x)$ for all x, then

$$\begin{aligned} a(x+p)+b &= ax+b \\ ap+b &= b \\ a &= 0 \end{aligned}$$

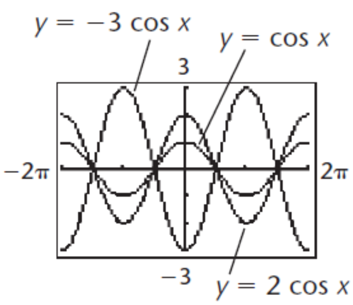
Hence $f(x) = b$, b any real number.

65. If $f(-x) = f(x)$ for all x, then

$$\begin{aligned} a(-x)+b &= ax+b \\ 0 &= 2ax \end{aligned}$$

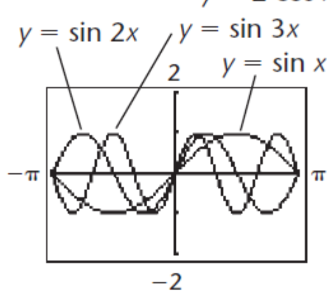
Hence a must be 0, $f(x) = b$, b any real number

67. (A)



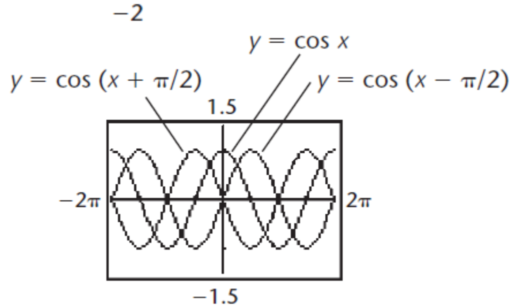
(B) The x intercepts do not change.
(C) The deviation of $y = \cos x$ from the x axis is 1 unit; the deviation of $y = 2\cos x$ from the x axis is 2 units; the deviation of $y = -3\cos x$ from the x axis is 3 units.
(D) The deviation of the graph from the x axis is changed by changing A. The deviation appears to be $|A|$.

69. (A)



(B) 1 period of $y = \sin x$ appears. 2 periods of $y = \sin 2x$ appear. 3 periods of $y = \sin 3x$ appear.
(C) n periods of $y = \sin nx$ would appear.

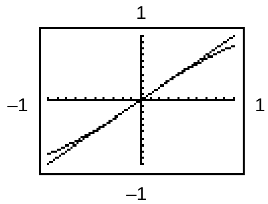
71. (A)



(B) The graph of $y = \cos x$ is shifted $|C|$ units to the right if $C < 0$ and $|C|$ units to the left if $C > 0$.

73. For each case, the number is not in the domain of the function and an error message of some type will appear.

75. Here are graphs of $f(x) = \sin x$ and $g(x) = x$, $-1 \le x \le 1$.



(A) The graphs become more indistinguishable the closer x is to the origin.

(B)

x	−0.3	−0.2	−0.1	0.0	0.1	0.2	0.3
$\sin x$	−0.296	−0.199	−0.100	0.000	0.100	0.199	0.296

77. (A) Since $\theta_R = \frac{s}{r}$ and r = radius of circle = 4, we have $\theta_R = \frac{7}{4}$ or 1.75 radians

(B) Since $\sin\theta = \frac{b}{r}$ and $\cos\theta = \frac{a}{r}$, we can write

$$a = r\cos\theta = 4\cos\frac{7}{4} = -0.713$$

$$b = r\sin\theta = 4\sin\frac{7}{4} = 3.936$$

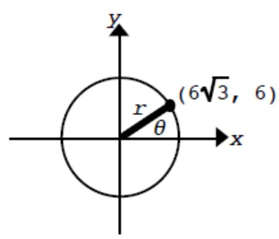
$(a, b) = (-0.713, 3.936)$

79. We know that $s = r\,\theta$. $(a, b) = (6\sqrt{3}, 6)$.
From the Pythagorean theorem,

$$r = \sqrt{a^2 + b^2} = \sqrt{(6\sqrt{3})^2 + 6^2} = 12$$

Since $\tan\theta = \frac{b}{a} = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}}$, $\theta = \frac{\pi}{6}$.

Hence $s = r\,\theta = 12\left(\frac{\pi}{6}\right) = 2\pi$ units.



81. $I = k\cos\theta$
For $\theta = 0°$, $I = k\cos 0° = k$
For $\theta = 30°$, $I = k\cos 30° = 0.866k$
For $\theta = 60°$, $I = k\cos 60° = 0.5k$

83. From the figure, the following relations are clear:

$$a^2 + b^2 = r^2 \quad r = 1 \quad \frac{a}{1} = \cos\theta \quad \frac{b}{1} = \sin\theta$$

Using the Pythagorean theorem in the right triangle whose hypotenuse is the rod connecting the piston to the wheel, we have

$$(y - b)^2 + a^2 = 4^2$$
$$(y - b)^2 = 4^2 - a^2$$
$$y - b = \sqrt{4^2 - a^2}$$
$$y = b + \sqrt{4^2 - a^2}$$

Since $a = \cos\theta$ and $b = \sin\theta$ and $\theta = 6\pi t$, we have $y = \sin 6\pi t + \sqrt{16 - (\cos 6\pi t)^2}$

85. $A = n\tan\left(\frac{180°}{n}\right)$

(A) For $n = 8$, $A = 8\tan\frac{180°}{8} = 3.31371$

For $n = 100$, $A = 100\tan\frac{180°}{100} = 3.14263$

For $n = 1000$, $A = 1000\tan\frac{180°}{1000} = 3.14160$

For $n = 10{,}000$, $A = 10{,}000\tan\frac{180°}{10{,}000} = 3.14159$

(B) as $n \to \infty$, A seems to approach $\pi(=3.1415926\ldots)$, the area of the circle.

87. (A) Using the formula given:
For $\theta = 88.7°$, $m = \tan\theta = \tan 88.7° = 44.07$
For $\theta = 162.3°$, $m = \tan\theta = \tan 162.3° = -0.32$

(B) Using the formula for inclination, the slope m is given by $m = \tan 137° = -0.93$. We now use the point-slope form of the equation of a line.

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 5 &= -0.93[x - (-4)] \\ y - 5 &= -0.93x - 3.72 \\ y &= -0.93x + 1.28 \end{aligned}$$

Section 6−5

1. Motion with the same frequency and amplitude indefinitely is simple harmonic motion.

3. The graph of $y = A\cos(Bx + C)$ is the graph of $y = A\cos Bx$ shifted horizontally by an amount $-C/B$.

5. Amplitude $= |A| = |3| = 3$ Period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{1} = 2\pi$

7. Amplitude $= |A| = \left|-\dfrac{1}{2}\right| = \dfrac{1}{2}$ Period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{1} = 2\pi$

9. Amplitude is not defined for the cotangent function. Period $= \dfrac{\pi}{B} = \dfrac{\pi}{4}$

11. Amplitude is not defined for the tangent function. Period $= \dfrac{\pi}{B} = \dfrac{\pi}{8\pi} = \dfrac{1}{8}$

13. Amplitude is not defined for the cosecant function. Period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{1/2} = 4\pi$

15. Amplitude $= |A| = |1| = 1$. Period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{\pi} = 2$

The basic sine function $\sin t$ has zeros when $t = k\pi$, k an integer. We examine $\pi x = k\pi$, $-2 \le x \le 2$ and find $x = k$ falls in this interval when $x = -2, -1, 0, 1, 2$.

17. Amplitude is not defined for the cotangent function. Period $= \dfrac{\pi}{1/2} = 2\pi$.

The basic cotangent function $\cot t$ has zeros when $t = \dfrac{\pi}{2} + k\pi$, k an integer.

We examine $\dfrac{x}{2} = \dfrac{\pi}{2} + k\pi$, $0 < x < 4\pi$, and find $x = \pi + 2k\pi$ falls in this interval when $x = \pi$ and $x = 3\pi$.

19. Amplitude $= |A| = |3| = 3$. Period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{2} = \pi$

The basic cosine function $\cos t$ has turning points when $t = k\pi$, k an integer.

We examine $2x = k\pi$, $-\pi \le x \le \pi$, and find that $x = \dfrac{k\pi}{2}$ falls in this interval when $x = -\pi, -\dfrac{\pi}{2}, 0, \dfrac{\pi}{2}, \pi$.

The turning points are therefore $(-\pi, 3)$, $\left(-\dfrac{\pi}{2}, -3\right)$, $(0, 3)$, $\left(\dfrac{\pi}{2}, -3\right)$, $(\pi, 3)$.

21. Amplitude is not defined for the secant function. Period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{\pi} = 2$.

The basic secant function $\sec t$ has turning points when $t = k\pi$, k an integer.
We examine $\pi x = k\pi$, $-1 \le x \le 3$, and find that $x = k$ falls in this interval when $x = -1, 0, 1, 2, 3$.
The turning points are therefore $(-1, -2)$, $(0, 2)$, $(1, -2)$, $(2, 2)$, $(3, -2)$.

23. $A = 3$ $P = \dfrac{\pi}{2} = \dfrac{2\pi}{4}$.

Hence $B = 4$, $y = 3\sin 4x$, $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{2}$.

25. $|A| = 10$ $P = 2 = \dfrac{2\pi}{\pi}$.

Hence $B = \pi$, $A = -10$, since the graph has the form of the standard sine curve turned upside down. $y = -10\sin \pi x$ $-1 \le x \le 2$.

27. $A = 5 \qquad P = 8\pi = 2\pi \cdot 4 = 2\pi \div \frac{1}{4}$.

Hence $B = \frac{1}{4}$, $y = 5 \cos \frac{1}{4}x$

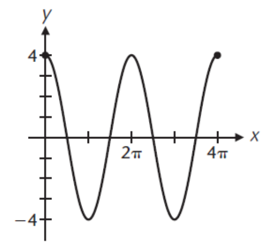
$-4\pi \le x \le 8\pi$.

29. $|A| = 0.5, \quad P = 8 = 2\pi \cdot \frac{4}{\pi} = 2\pi \div \frac{\pi}{4}$.

Hence $B = \frac{\pi}{4}$, $A = -0.5$, since the graph has the form of the standard cosine curve turned upside down. $y = -0.5 \cos \frac{\pi x}{4} \quad -4 \le x \le 8$

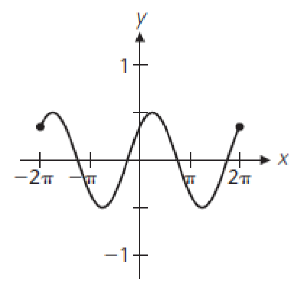
31. $y = 4 \cos x$

Amplitude = $|A| = 4$ Period = 2π Phase Shift = 0



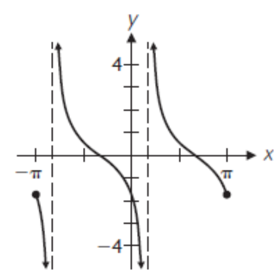
33. $y = \frac{1}{2} \sin\left(x + \frac{\pi}{4}\right)$ completes one cycle as $x + \frac{\pi}{4}$ varies from $x + \frac{\pi}{4} = 0$ to $x + \frac{\pi}{4} = 2\pi$, that is as x varies from $-\frac{\pi}{4}$ to $-\frac{\pi}{4} + 2\pi$. Amplitude: $|A| = \frac{1}{2}$ Period: 2π Phase shift: $-\frac{\pi}{4}$

Divide the interval $\left[-\frac{\pi}{4}, -\frac{\pi}{4} + 2\pi\right]$ into four equal parts and sketch one cycle of $y = \frac{1}{2} \sin\left(x + \frac{\pi}{4}\right)$. Then extend the graph to cover $[-2\pi, 2\pi]$, deleting the small portion beyond $x = 2\pi$.



35. $y = \cot\left(x - \frac{\pi}{6}\right)$ completes one period as $x - \frac{\pi}{6}$ varies from $x - \frac{\pi}{6} = 0$ to $x - \frac{\pi}{6} = \pi$, that is as x varies from $\frac{\pi}{6}$ to $\frac{\pi}{6} + \pi$. Period: π Phase shift: $\frac{\pi}{6}$

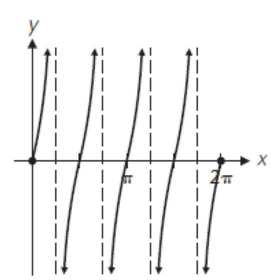
Sketch one cycle of $y = \cot\left(x - \frac{\pi}{6}\right)$, the graph of $y = \cot x$ shifted $\frac{\pi}{6}$ units to the right, on the interval $\left(\frac{\pi}{6}, \frac{\pi}{6} + \pi\right)$. Then extend the graph to cover $[-\pi, \pi]$, deleting the portion beyond π.



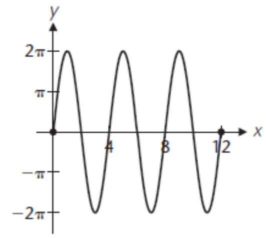
37. $y = 3 \tan 2x$ completes one period as $2x$ varies from $2x = -\frac{\pi}{2}$ to $2x = \frac{\pi}{2}$ that is, as x varies from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$.

Period: $\frac{\pi}{2}$ Phase shift: 0

Sketch one cycle of $y = 3 \tan 2x$, the graph of $y = \tan x$ stretched vertically by a factor of 3 and shrunk horizontally by a factor of 2, on the interval $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$. Then extend the graph to cover $[0, 2\pi]$.



39. $y = 2\pi \sin \frac{\pi x}{2}$ completes one cycle as $\frac{\pi x}{2}$ varies from $\frac{\pi x}{2} = 0$ to $\frac{\pi x}{2} = 2\pi$ that is, as x varies from 0 to 4.
Amplitude = $|A| = 2\pi$ Period: 4 Phase shift: 0
Divide the interval [0, 4] into four equal parts and sketch one cycle of $y = 2\pi \sin \frac{\pi x}{2}$, then extend the graph to cover [0, 12].



41. $y = -3 \sin\left[2\pi\left(x+\frac{1}{2}\right)\right]$ completes one cycle as $2\pi\left(x+\frac{1}{2}\right)$ varies from

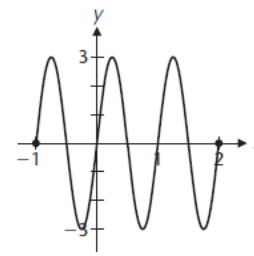
$$2\pi\left(x+\frac{1}{2}\right) = 0 \quad \text{to} \quad 2\pi\left(x+\frac{1}{2}\right) = 2\pi$$

$$x + \frac{1}{2} = 0 \qquad\qquad x + \frac{1}{2} = 1$$

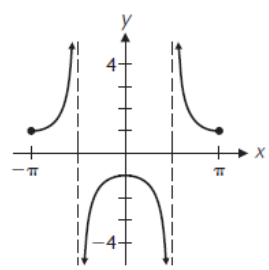
$$x = -\frac{1}{2} \qquad\qquad x = \frac{1}{2}$$

Amplitude = $|A| = |-3| = 3$ Period: $\frac{1}{2} - \left(-\frac{1}{2}\right) = 1$ Phase shift: $-\frac{1}{2}$

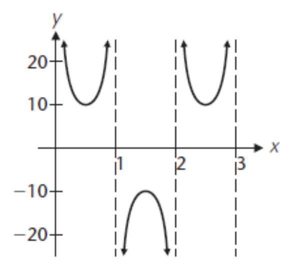
Divide the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ into four equal parts and sketch one cycle of $y = -3 \sin\left[2\pi\left(x+\frac{1}{2}\right)\right]$ -- an upside-down sine curve. Then extend the graph to [−1, 2].



43. $y = \sec(x + \pi)$ has period 2π and completes one period as $x + \pi$ varies from $x + \pi = 0$ to $x + \pi = 2\pi$, that is, as x varies from $-\pi$ to π.
Phase shift: $-\pi$
The required graph is this one period, the graph of $y = \sec x$ shifted π units to the left.



45. $y = 10 \csc \pi x$ completes one period as πx varies from $\pi x = 0$ to $\pi x = 2\pi$, that is, as x varies from 0 to 2. Period: 2 Phase shift: 0
Sketch one period of $y = 10 \csc \pi x$, the graph of $y = \csc x$ stretched vertically by a factor of 10 and shrunk horizontally by a factor of $\frac{1}{\pi}$, on this interval. Then extend the graph to [0, 3].

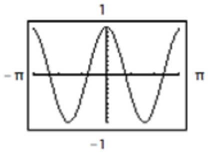


47. True. If $x = 0$, $y = A \sin B(0) = 0$. Thus the point (0, 0) is on the graph.

49. False. The function $y = \sin\left(x - \frac{\pi}{4}\right)$ is neither even nor odd.

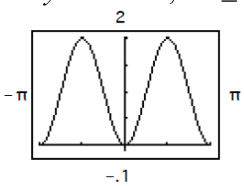
51. True. Every function of form $A \sin(Bx + C)$ or $A \cos(Bx + C)$ has period $\frac{2\pi}{B}$.

53. Here is a graph of $y = \cos^2 x - \sin^2 x$, $-\pi \le x \le \pi$.



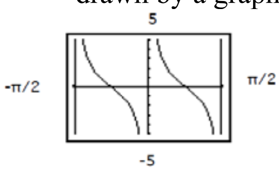
The graph has amplitude 1 and period π. It appears to be the graph of $y = A \cos Bx$ with $A = 1$, and $B = 2\pi \div P = 2\pi \div \pi = 2$, that is, $y = \cos 2x$.

55. Here is a graph of $y = 2 \sin^2 x$, $-\pi \le x \le \pi$.



The graph has amplitude $\frac{2-0}{2} = 1$ and period π. It appears to be the graph of $y = \cos 2x$ turned upside down and shifted up one unit, that is, $y = -\cos 2x + 1$ or $y = 1 - \cos 2x$.

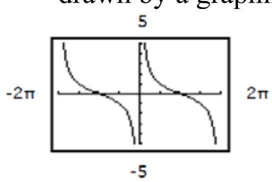
57. Here is a graph of $y = \cot x - \tan x$ drawn by a graphing calculator.



The graph appears to have the form $y = A \cot Bx$. Since the period is $\frac{\pi}{2}$, set $\frac{\pi}{2} = \frac{\pi}{B}$ to obtain $B = 2$. The graph of $y = A \cot 2x$ shown appears to pass through $\left(\frac{\pi}{8}, 2\right)$, thus $2 = A \cot 2\left(\frac{\pi}{8}\right) = A \cot \frac{\pi}{4} = A$

The equation of the graph can be written $y = 2 \cot 2x$.

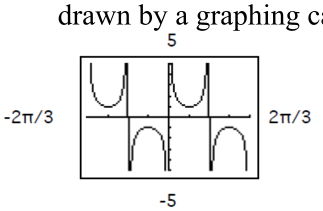
59. Here is a graph of $y = \csc x + \cot x$ drawn by a graphing calculator.



The graph appears to have the form $y = A \cot Bx$. Since the period is 2π, set $2\pi = \frac{\pi}{B}$ to obtain $B = \frac{1}{2}$. The graph of $y = A \cot \frac{1}{2}x$ shown appears to pass through $\left(\frac{\pi}{2}, 1\right)$, thus $1 = A \cot \frac{1}{2}\left(\frac{\pi}{2}\right) = A \cot \frac{\pi}{4} = A$

The equation of the graph can be written $y = \cot \frac{1}{2}x$.

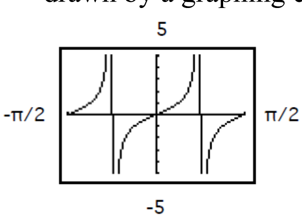
61. Here is a graph of $y = \sin 3x + \cos 3x \cot 3x$ drawn by a graphing calculator.



The graph appears to have the form $y = A \csc Bx$. Since the period is $\frac{2\pi}{3}$, set $\frac{2\pi}{3} = \frac{2\pi}{B}$ to obtain $B = 3$. The graph of $y = A \csc 3x$ shown appears to pass through $\left(\frac{\pi}{6}, 1\right)$, thus $1 = A \csc 3\left(\frac{\pi}{6}\right) = A \csc \frac{\pi}{2} = A$

The equation of the graph can be written $y = \csc 3x$.

63. Here is a graph of $y = \frac{\sin 4x}{1 + \cos 4x}$ drawn by a graphing calculator.



The graph appears to have the form $y = A \tan Bx$. Since the period is $\frac{\pi}{2}$, set $\frac{\pi}{B} = \frac{2\pi}{3}$ to obtain $B = 2$. The graph of $y = A \tan 2x$ shown appears to pass through $\left(\frac{\pi}{8}, 1\right)$, thus $1 = A \tan 2\left(\frac{\pi}{8}\right) = A \tan \frac{\pi}{4} = A$

The equation of the graph can be written $y = \tan 2x$.

65. $|A| = 4$. Hence $A = 4$ or -4. The graph completes one full cycle as x varies over the interval $[-1, 3]$. Since $-\frac{C}{B}$ is required between 0 and 2, we cannot simply set $-\frac{C}{B} = -1$. We must (mentally) extend the curve so that the phase shift is positive. Then the (extended) graph is that of an upside down sine curve that completes one full cycle as x varies over the interval $[1, 5]$. Hence

$$A = -4$$

$$-\frac{C}{B} = 1 \qquad -\frac{C}{B} + \frac{2\pi}{B} = 5$$

$$C = -B \qquad \frac{2\pi}{B} = 4 \qquad B = \frac{2\pi}{4} = \frac{\pi}{2} \qquad C = -\frac{\pi}{2}$$

The equation is then $y = A\sin(Bx + C)$

$$y = -4\sin\left(\frac{\pi}{2}x - \frac{\pi}{2}\right)$$

67. $|A| = \frac{1}{2}$. Hence $A = \frac{1}{2}$ or $-\frac{1}{2}$. The graph completes one full cycle of the cosine function as x varies over the (mentally extended) intervals $[-\pi, 7\pi]$ or $[3\pi, 11\pi]$. Since the phase shift is required between 0 and 4π, we must set $-\frac{C}{B} = 3\pi$. Then the (extended) graph has the form of a standard cosine curve. Hence

$$A = \frac{1}{2} \qquad -\frac{C}{B} + \frac{2\pi}{B} = 11\pi$$

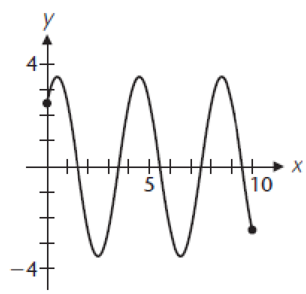
$$-\frac{C}{B} = 3\pi \qquad \frac{2\pi}{B} = 8\pi \quad B = \frac{2\pi}{8\pi} = \frac{1}{4} \quad C = -3\pi B = -\frac{3\pi}{4}$$

$$C = -3\pi B$$

The equation is then $y = A\cos(Bx + C)$

$$y = \frac{1}{2}\cos\left(\frac{1}{4}x - \frac{3\pi}{4}\right)$$

69. $y = 3.5\sin\left[\frac{\pi}{2}(t + 0.5)\right]$



$A = 3.5$

Solve $\frac{\pi}{2}(t + 0.5) = 0 \qquad \frac{\pi}{2}(t + 0.5) = 2\pi$

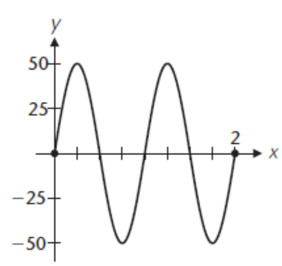
$$t + 0.5 = 0 \qquad t + 0.5 = 4$$

$$t = -0.5 \qquad t = -0.5 + 4 = 3.5$$

Phase shift $\quad$ Period $P = 4$

The graph completes one full cycle as t varies over the interval $[-0.5, 3.5]$.

71. $y = 50\cos[2\pi(t - 0.25)]$



$A = 50$

Solve $2\pi(t - 0.25) = 0 \quad 2\pi(t - 0.25) = 2\pi$

$$t - 0.25 = 0 \qquad t - 0.25 = 1$$

$$t = 0.25 \qquad t = 0.25 + 1 = 1.25$$

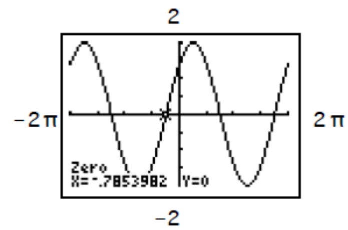
Phase shift $\quad$ Period $P = 1$

The graph completes one full cycle as t varies over the interval $[0.25, 1.25]$.

73. Here is a graph of $y = \sqrt{2}\sin x + \sqrt{2}\cos x$, $-2\pi \le x \le 2\pi$.
It appears that this is a sine curve shifted to the left, with $A = 2$ and, since $P = \frac{2\pi}{B}$ and P appears to be 2π, $B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$.
The x intercept closest to the origin, to three decimal places, is -0.785.
To find C, substitute $B = 1$ and $x = -0.785$ into the phase-shift formula $x = -\frac{C}{B}$ and solve for C:

$$\begin{aligned} x &= -\frac{C}{B} \\ -0.785 &= -\frac{C}{1} \\ C &= 0.785 \end{aligned}$$

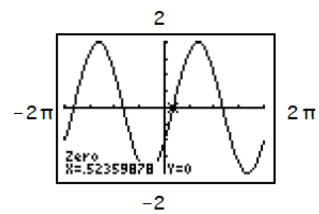
The equation is thus $y = 2\sin(x + 0.785)$.



75. Here is a graph of $y = \sqrt{3}\sin x - \cos x$, $-2\pi \le x \le 2\pi$.
It appears that this is a sine curve shifted to the right, with $A = 2$ and, since $P = \frac{2\pi}{B}$ and P appears to be 2π, $B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$.
The x intercept closest to the origin, to three decimal places, is 0.524. To find C, substitute $B = 1$ and $x = 0.524$ into the phase-shift formula $x = -\frac{C}{B}$ and solve for C:

$$\begin{aligned} x &= -\frac{C}{B} \\ 0.524 &= -\frac{C}{1} \\ C &= -0.524 \end{aligned}$$

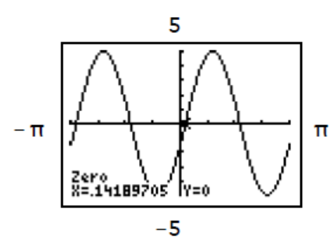
The equation is thus $y = 2\sin(x - 0.524)$.



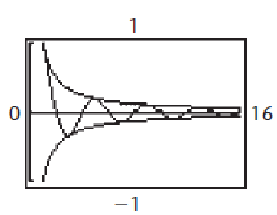
77. Here is a graph of $y = 4.8\sin 2x - 1.4\cos 2x$, $-\pi \le x \le \pi$.
It appears that this is a sine curve shifted to the right, with $A = 5$ and, since $P = \frac{2\pi}{B}$ and P appears to be π, $B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$.
The x intercept closest to the origin, to three decimal places, is 0.142. To find C, substitute $B = 2$ and $x = 0.142$ into the phase-shift formula $x = -\frac{C}{B}$ and solve for C:

$$\begin{aligned} x &= -\frac{C}{B} \\ 0.142 &= -\frac{C}{2} \\ C &= -0.284 \end{aligned}$$

The equation is thus $y = 5\sin(2x - 0.284)$.

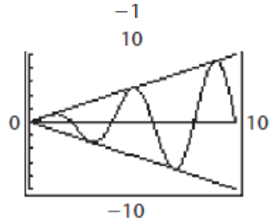


79.



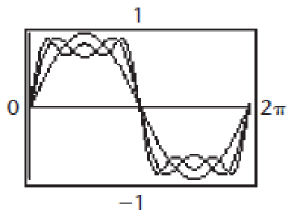
The amplitude is decreasing with time. This is often referred to as a *damped sine wave*. Examples are the vertical motion of a car after going over a bump (which is damped by the suspension system) and the slowing down of a pendulum that is released away from the vertical line of suspension (air resistance and friction).

81.



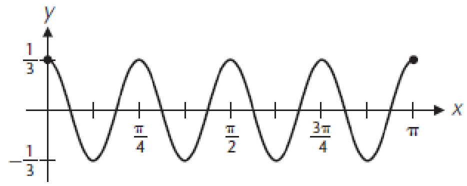
The amplitude is increasing with time. In physical and electrical systems this is referred to as *resonance*. Some examples are the swinging of a bridge during high winds and the movement of tall buildings during an earthquake. Some bridges and buildings are destroyed when the resonance reaches the elastic limits of the structure.

83.



85. $A = \frac{1}{3}$

$P = \frac{2\pi}{8} = \frac{\pi}{4}$



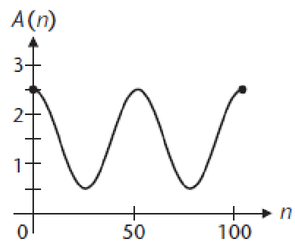
87. When $t = 0$ $y = -8$. Hence $-8 = A \cos B(0)$, that is, $A = -8$. Since the period is 0.5 seconds, $\frac{2\pi}{B} = 0.5$,

$B = \frac{2\pi}{0.5} = 4\pi$. Hence the equation is $y = -8 \cos 4\pi t$.

89. The graph is the same as the graph of $y = \cos \frac{n\pi}{26}$, shifted 1.5 units up.

$A = 1$

$P = 2\pi \div \frac{\pi}{26} = 52$



The graph shows the seasonal changes of sulfur dioxide pollutant in the atmosphere; more is produced during winter months because of increased heating.

91. $I = 15 \cos\left(120\pi t + \frac{\pi}{2}\right)$

$A = 15$

Solve $120\pi t + \frac{\pi}{2} = 0$ $\qquad$ $120\pi t + \frac{\pi}{2} = 2\pi$

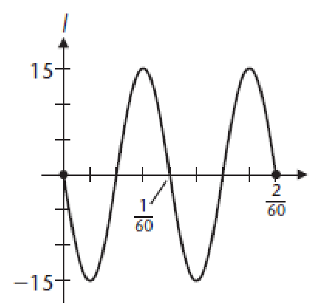
$120\pi t = -\frac{\pi}{2}$ $\qquad$ $120\pi t = -\frac{\pi}{2} + 2\pi$

$t = -\frac{\pi}{2} \div 120\pi$ $\qquad$ $t = \left(-\frac{\pi}{2} + 2\pi\right) \div 120\pi$

$t = -\frac{1}{240}$ $\qquad$ $t = -\frac{1}{240} + \frac{1}{60} = \frac{3}{240}$ or $\frac{1}{80}$

Phase Shift $= -\frac{1}{240}$ $\qquad$ Period $P = \frac{1}{60}$

The graph completes one full cycle as t varies over the interval $\left[-\frac{1}{240}, \frac{3}{240}\right]$.



93. If the disk rotates through an angle θ in t seconds, we see

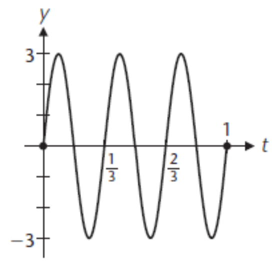
$$\theta = 3\,\frac{\text{revolutions}}{\text{second}} \cdot 2\pi\,\frac{\text{radians}}{\text{revolution}} \cdot t \text{ seconds} = 6\pi t \text{ radians}$$

Then $\frac{y}{R} = \sin\theta = \sin 6\pi t$

$y = R\sin 6\pi t$

Since the disk has radius 3, $y = 3\sin 6\pi t$.

This function has $A = 3$, $P = \frac{2\pi}{6\pi} = \frac{1}{3}$.



95. (A) In triangle MNP, we can write

$\cos\theta = \frac{20}{c}$

$c\cos\theta = 20$

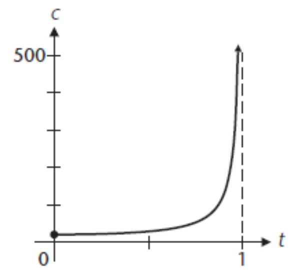
$c = \frac{20}{\cos\theta}$

$c = 20\sec\theta$

Since $\theta = \frac{\pi t}{2}$, the equation for c in terms of t is $c = 20\sec\frac{\pi t}{2}$.

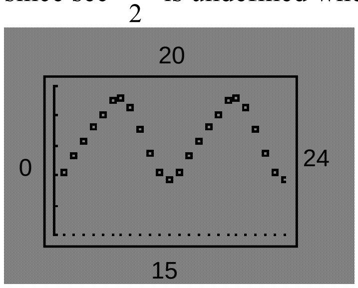
The equation is valid for $0 \le t < 1$, since $\sec\frac{\pi t}{2}$ is undefined when $t = 1$.

(B) The graph has an asymptote at $t = 1$. Sketch the portion of $y = \sec\frac{\pi t}{2}$ on [0, 1).



(C) The length of the light beam starts at 20 ft and increases slowly at first, then increases rapidly without end.

97. (A)



(B) From the table, Max y = 19:35 and Min y = 16:51.

Hence $A = \frac{\text{Max } y - \text{Min } y}{2} = \frac{19\frac{35}{60} - 16\frac{51}{60}}{2} = 1.37$

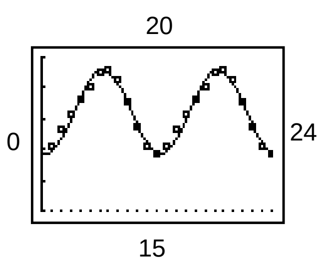
$B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{12} = \frac{\pi}{6}$

$C = -1.75$ from the graph

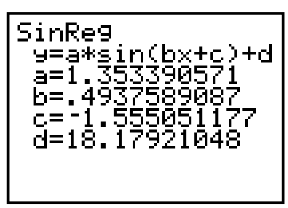
$k = \text{Min } y + A = 16\frac{51}{60} + 1.37 = 18.22$

Thus, $y = 18.22 + 1.37\sin(\frac{\pi x}{6} - 1.75)$

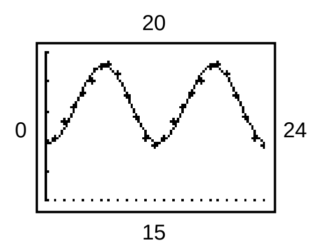
(C)



(D)



Compute the regression equation.



Graph the data and the regression equation.

Section 6−6

1. On this interval, the function is not one-to-one since, for example $\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$ but $\frac{5\pi}{6} \ne \frac{\pi}{6}$.

3. Yes because the range of $\tan^{-1}x$ is all real numbers.

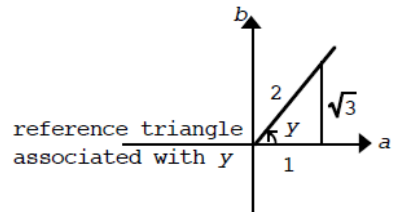
5. The graphs of f and f^{-1} are reflections of one another in the line $y = x$.

7. $y = \cos^{-1} 0$ is equivalent to $\cos y = 0 \quad 0 \le y \le \pi$

$y = \frac{\pi}{2}$

9. $y = \arcsin \frac{\sqrt{3}}{2}$ is equivalent to $\sin y = \frac{\sqrt{3}}{2} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$$y = \frac{\pi}{3}$$



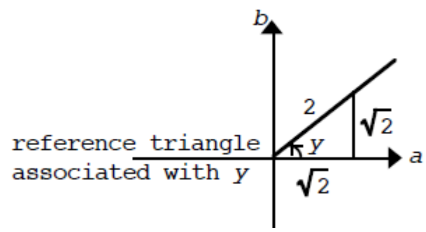
11. $y = \arctan \sqrt{3}$ is equivalent to $\tan y = \sqrt{3} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

$$y = \frac{\pi}{3}$$

(see sketch for problem 9)

13. $y = \sin^{-1} \frac{\sqrt{2}}{2}$ is equivalent to $\sin y = \frac{\sqrt{2}}{2} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$$y = \frac{\pi}{4}$$



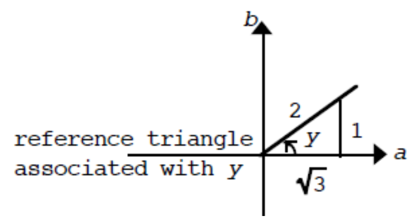
15. $y = \arccos 1$ is equivalent to $\cos y = 1 \quad 0 \le y \le \pi$

$$y = 0$$

17. $y = \sin^{-1} \frac{1}{2}$ is equivalent to

$\sin y = \frac{1}{2} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

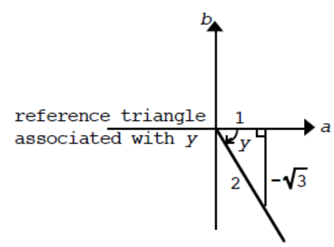
$y = \frac{\pi}{6}$



19. 1.144 **21.** 1.561 **23.** arccos x is not defined if $x > 1$.

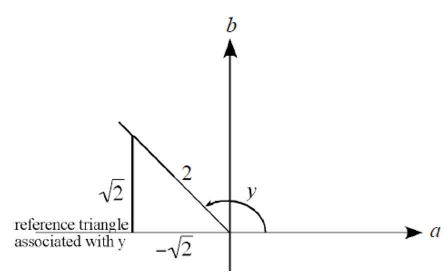
25. $y = \arctan(-\sqrt{3})$ is equivalent to $\tan y = -\sqrt{3} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

$$y = -\frac{\pi}{3}$$



27. $y = \cos^{-1}(-\frac{\sqrt{2}}{2})$ is equivalent to $\cos y = -\frac{\sqrt{2}}{2} \quad 0 \le y \le \pi$

$$y = \frac{3\pi}{4}$$



29. arcsin(−2) is not defined. −2 is not in the restricted domain of the sine function.

31. $\cot [\cos^{-1}(-0.7003)] = \frac{1}{\tan[\cos^{-1}(-0.7003)]} = -0.9810$ **33.** 2.645

35. $\tan(\tan^{-1} \sqrt{5}) = \sqrt{5}$ by the tangent-inverse tangent identity.

37. $\sin\left(\sin^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right) = -\frac{1}{\sqrt{3}}$ by the sine-inverse sine identity.

39. $\cos\left(\cos^{-1}\left(-\sqrt{2}\right)\right)$ is not defined. $-\sqrt{2}$ is not in the restricted domain of the cosine function.

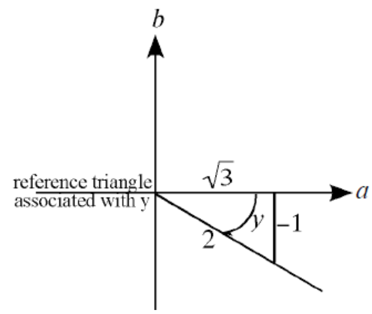
41. $\sin^{-1}(\sin 1.5) = 1.5$ by the sine-inverse sine identity.

43. $\cos^{-1}[\cos(-\pi)] = \cos^{-1}(-1) = \pi$

45. $\tan^{-1}\left[\tan\left(\frac{\pi}{2}\right)\right]$ is not defined. $\frac{\pi}{2}$ is not in the domain of the tangent function.

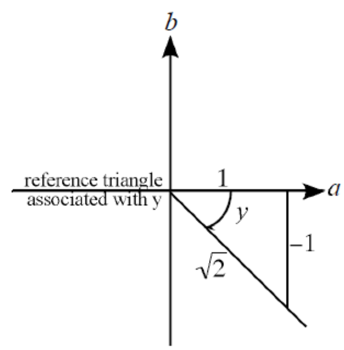
47. $y = \sin^{-1}(-\frac{1}{2})$ is equivalent to $\sin y = -\frac{1}{2}$ $\quad -90° \le y \le 90°$

$$y = -30°$$



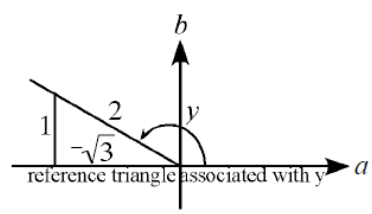
49. $y = \tan^{-1}(-1)$ is equivalent to $\tan y = -1$ $\quad -90° < y < 90°$

$$y = -45°$$



51. $y = \arccos(-\frac{\sqrt{3}}{2})$ is equivalent to $\cos y = -\frac{\sqrt{3}}{2}$ $\quad 0° \le y \le 180°$

$$y = 150°$$



Calculator in degree mode for problems 53 - 56.

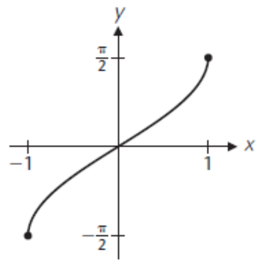
53. 43.51° **55.** −21.48°

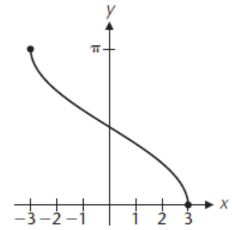
57. $\sin^{-1}(\sin 2) = 1.1416 \neq 2$. For the identity $\sin^{-1}(\sin x) = x$ to hold, x must be in the restricted domain of the sine function; that is, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. The number 2 is not in the restricted domain.

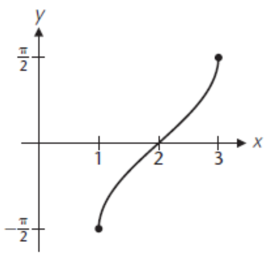
59. True. A periodic function cannot be one-to-one, since $f(x+p) = f(x)$ but $x + p \neq x$.

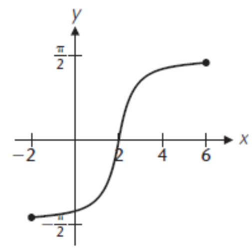
61. False. None of them are periodic.

63. True. $\sin^{-1}(-x) = -\sin^{-1}(x)$ for all x, $-1 \le x \le 1$

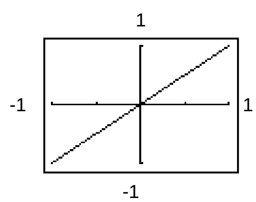
65. 

67. 

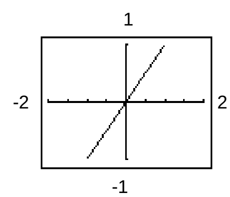
69. 

71. 

73. (A)



(B) The domain of $\cos^{-1}$ is restricted to $-1 \le x \le 1$; hence no graph will appear for other x.



75. Let $y = \sin^{-1} x$. Then $x = \sin y$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. If $\sin y = \frac{b}{r} = x = \frac{x}{1}$, then let $b = x$, $r = 1$.

$$a^2 + b^2 = r^2$$
$$a^2 + x^2 = 1^2$$
$$a^2 = 1 - x^2$$
$$a = \sqrt{1-x^2} \quad (a \text{ must be positive since } y \text{ is a I or IV quadrant angle})$$
$$\cos y = \cos(\sin^{-1} x) = \frac{a}{r} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

77. Let $y = \arctan x$. Then $x = \tan y$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$. If $\tan y = \frac{b}{a} = x = \frac{x}{1}$, then let $b = x$, $a = 1$.

$$a^2 + b^2 = r^2$$
$$1^2 + x^2 = r^2$$
$$r = \sqrt{1+x^2} \quad (r \text{ is always taken positive})$$
$$\cos y = \cos(\tan^{-1} x) = \frac{a}{r} = \frac{1}{\sqrt{1+x^2}}$$

79. $f(x) = 4 + 2\cos(x - 3)$. $3 \le x \le 3 + \pi$. With this restriction, f is one-to-one (proof omitted).

Solve $y = f(x)$ for x:

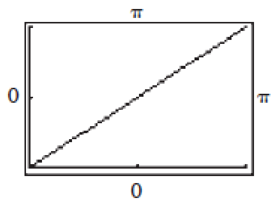
$$
\begin{aligned}
y &= 4 + 2\cos(x-3) & \qquad 3 \le x \le 3 + \pi \\
y - 4 &= 2\cos(x-3) & \qquad 0 \le x - 3 \le \pi \\
\frac{y-4}{2} &= \cos(x-3) & \qquad -1 \le \cos(x-3) \le 1 \\
\cos^{-1}\frac{y-4}{2} &= x - 3 & \qquad -2 \le 2\cos(x-3) \le 2 \\
x = 3 + \cos^{-1}\frac{y-4}{2} &= f^{-1}(y) & \qquad 2 \le 4 + 2\cos(x-3) \le 6
\end{aligned}
$$

range of f: $2 \le y \le 6$

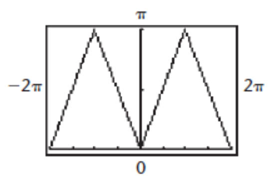
Interchange x and y

$f^{-1}(x) = 3 + \cos^{-1}\frac{x-4}{2}$ $\quad 2 \le 4 + 2\cos(y-3) \le 6$. Thus range of f = domain of f^{-1}: $2 \le x \le 6$.

81. (A)



(B) The domain for $\cos x$ is $(-\infty, \infty)$ and the range is $[-1, 1]$, which is the domain for $\cos^{-1} x$. Thus, $y = \cos^{-1}(\cos x)$ has a graph over the interval $(-\infty, \infty)$, but $\cos^{-1}(\cos x) = x$ only on the restricted domain of $\cos x$, $[0, \pi]$.



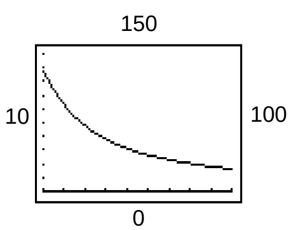
83. For a 28mm lens, $x = 28$, thus $\theta = 2\tan^{-1}\frac{21.634}{28} = 1.31567$ radians.

In decimal degrees, $\theta_D = \frac{180°}{\pi}(1.31567 \text{ rad}) = 75.38°$.

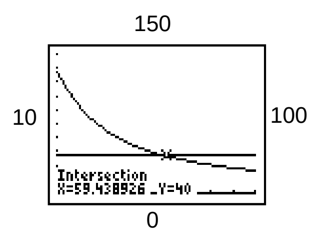
For a 100 mm lens, $x = 100$, thus $\theta = 2\tan^{-1}\frac{21.634}{100} = 0.42611$ radians.

In decimal degrees, $\theta_D = \frac{180°}{\pi}(0.42611 \text{ rad}) = 24.41°$.

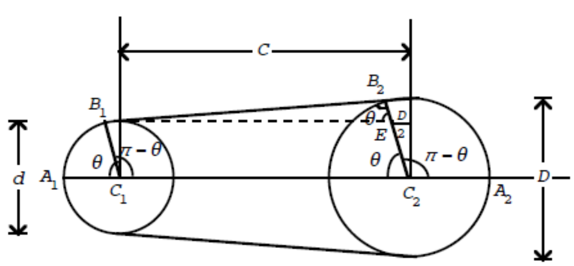
85. (A)



(B) 59.44 mm



87.



From the above figure, the following should be clear: Finally, Length of belt

Length of belt = $2[arc\ \widehat{A_1B_1} + B_1B_2 + arc\ B_2A_2]$

$arc\ A_1B_1 = \frac{d}{2}(\theta)$

$arc\ B_2A_2 = \frac{D}{2}(\pi - \theta)$

To find B_1B_2 we note: C_1C_2 has length C
B_1E is constructed parallel to C_1C_2. EC_2 is parallel to B_1C_1. Hence $EB_1C_1C_2$ is a parallelogram. EB_1 has length C. EB_1B_2 is a right triangle. Thus

(1) $\cos\theta = \frac{B_2E}{EB_1} = \frac{\frac{D}{2} - \frac{d}{2}}{C} = \frac{D-d}{2C}$

(2) $\sin\theta = \frac{B_1B_2}{EB_1}$, so $B_1B_2 = EB_1 \sin\theta = C\sin\theta$

$L = 2[arc\ A_1B_1 + B_1B_2 + arc\ B_2A_2]$

$= 2\left[\frac{d}{2}\theta + C\sin\theta + \frac{D}{2}(\pi - \theta)\right]$

$= d\theta + 2C\sin\theta + D(\pi - \theta)$

$L = \pi D + (d - D)\,\theta + 2C\sin\theta$ and (from (1) above)

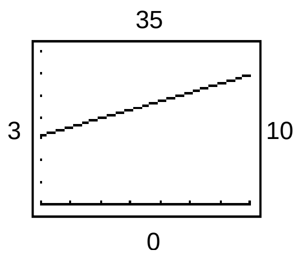
$\theta = \cos^{-1}\frac{D-d}{2C}$

Substituting the given values, we have $D = 4$, $d = 2$, $C = 6$ (calculator in radian mode)

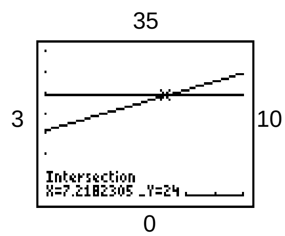
$\theta = \cos^{-1}\frac{4-2}{2\cdot 6} = \cos^{-1}\frac{1}{6}$

$L = 4\pi + (2 - 4)\cos^{-1}\frac{1}{6} + 2\cdot 6\sin\left(\cos^{-1}\frac{1}{6}\right) \approx 21.59$ inches

89. (A)



(B) 7.22 inches



91. (A) Following the hint, we draw AC. Then, since the central angle in a circle subtended by an arc is twice any inscribed angle subtended by the same arc, angle ACB has measure 2θ. Thus, $d = r\cdot 2\,\theta = 2r\,\theta$

In triangle ECP, $\tan\theta = \frac{x}{r}$, hence $\theta = \tan^{-1}\frac{x}{r}$

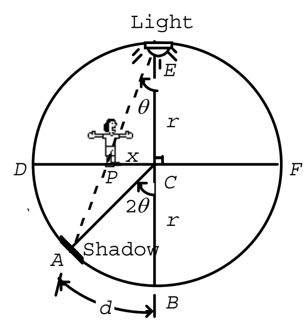
$d = 2r\tan^{-1}\frac{x}{r}$

(B) Substituting the given values, we have

$r = 100 \quad x = 40$

$d = 2\cdot 100\tan^{-1}\frac{40}{100} = 200\tan^{-1}\frac{40}{100}$

$= 200\tan^{-1}(0.4)$ (calculator in radian mode) = 76.10 feet



CHAPTER 6 REVIEW

1. $\theta = \frac{s}{r} = \frac{15 \text{ centimeters}}{6 \text{ centimeters}} = \frac{15}{6} = 2.5$ radians *(6−3)*

2. $s = r\,\theta$ = (3 centimeters)(2.5 radians) = 7.5 centimeters *(6−3)*

3. Solve for α:
$\alpha = 90° - 35.2° = 54.8°$

Solve for a:

$\cos\beta = \frac{a}{c}$

$\cos 35.2° = \frac{a}{20.2}$

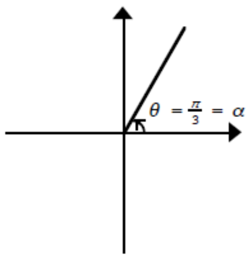
Solve for b:

$\sin\beta = \frac{b}{c}$

$\sin 35.2° = \frac{b}{20.2}$

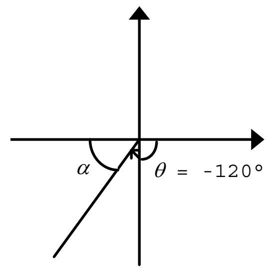
$a = 20.2 \cos 35.2° = 16.5$ ft $\qquad b = 20.2 \sin 35.2° = 11.6$ ft $\qquad$ *(6−3)*

4. (A)



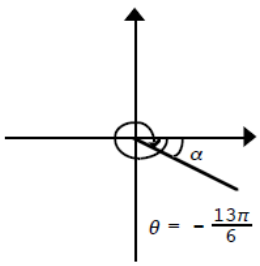
$\alpha = \dfrac{\pi}{3}$

(B)



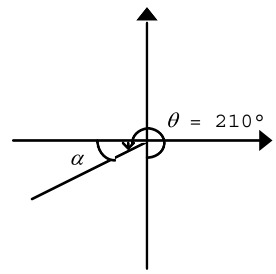
$\alpha = 180° + (-120°) = 60°$

(C)



$\alpha = \left|2\pi - \dfrac{13\pi}{6}\right| = \dfrac{\pi}{6}$

(D)



$a = 210° - 180° = 30°$ $\qquad$ *(6−4)*

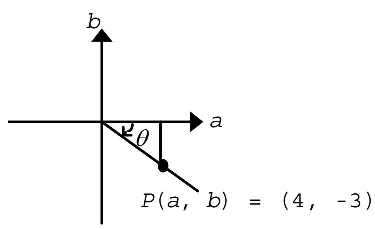
5. (A) $\dfrac{b}{r} = \sin\theta < 0$ if $b < 0$. This occurs in quadrants III, IV.

(B) $\dfrac{a}{r} = \cos\theta < 0$ if $a < 0$. This occurs in quadrants II, III.

(C) $\dfrac{b}{a} = \tan\theta < 0$ if a and b have opposite signs. This occurs in quadrants II, IV. $\qquad$ *(6−2)*

6.

$$a^2 + b^2 = r^2$$
$$4^2 + (-3)^2 = r^2$$
$$25 = r^2$$
$$r = 5$$
$$\sin\theta = \frac{b}{r} = \frac{-3}{5} = -\frac{3}{5}$$
$$\sec\theta = \frac{r}{a} = \frac{5}{4}$$
$$\cot\theta = \frac{a}{b} = \frac{4}{-3} = -\frac{4}{3}$$



(6−4)

7.

θ°	θ rad	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°	0	0	1	0	ND*	1	ND
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	2	$2/\sqrt{3}$	$\sqrt{3}$
45°	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2/\sqrt{3}$	2	$1/\sqrt{3}$
90°	$\pi/2$	1	0	ND	1	ND	0
180°	π	0	−1	0	ND	−1	ND
270°	$3\pi/2$	−1	0	ND	−1	ND	0
360°	2π	0	1	0	ND	1	ND

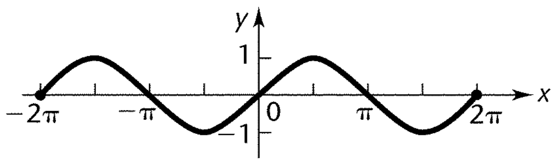
*ND = not defined $\qquad$ *(6−1, 6−2)*

8. (A) 2π (B) 2π (C) π *(6–4)*

9. (A) Domain = all real numbers, Range = $[-1, 1]$

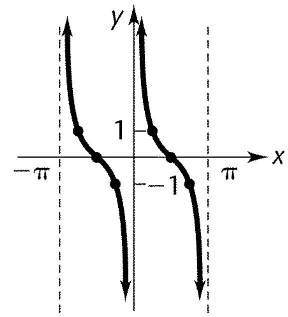
(B) Domain is set of all real numbers except $x = \frac{2k+1}{2}\pi$, k an integer, Range = all real numbers *(6–4)*

10.



(6–4)

11.



(6–4)

12. The central angle in a circle subtended by an arc of half the length of the radius. *(6–1)*

13. If the graph of $y = \sin x$ is shifted $\frac{\pi}{2}$ units to the left, the result will be the graph of $y = \cos x$. *(6–4)*

14. $\theta_D = \frac{180°}{\pi}\ \theta_R = \frac{180°}{\pi}\ (1.37) = 78.50°$ *(6–1)*

15. Solve for β:

$$\tan\beta = \frac{b}{a}$$

$$\tan\beta = \frac{13.3}{15.7}$$

$$\beta = \tan^{-1}\frac{13.3}{15.7} = 40.3°$$

Solve for α: $\alpha = 90° - 40.3° = 49.7°$

Solve for c:

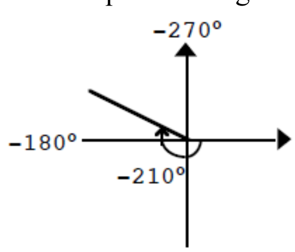
$$\sec\beta = \frac{c}{a}$$

$$\sec 40.3° = \frac{c}{15.7}$$

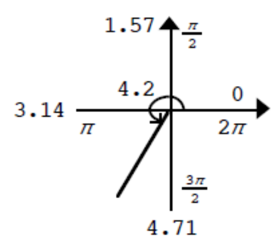
$$c = 15.7 \sec 40.3° = 20.6 \text{ cm}$$

(6–3)

16. (A) Since $-270° < -210° < -180°$, this is a II quadrant angle.



(C) Since $3.14 < 4.2 < 4.71$, this is a III quadrant angle.



(B) Since $\frac{5\pi}{2}$ is coterminal with $\frac{\pi}{2}$, this is a quadrantal angle. *(6–1)*

17. (A) Since $-240° + 360° = 120°$, this is coterminal with 120°.

(B) Since $-\frac{7\pi}{6} + 2\pi = \frac{5\pi}{6}$, which is equivalent to 150°, this is not coterminal with 120°.

(C) Since $840° - 2(360°) = 120°$, this is coterminal with 120°. *(6–1)*

18. (B) and (C), since 3 radians is equivalent to the real number 3, and cosine is periodic with period 2π. (A) is not the same as cos 3, since 3° is equivalent to $\frac{\pi}{180°}3°$, not 3. *(6−2)*

19. (A) $\frac{b}{a} = \tan x$ is not defined if $a = 0$. This occurs if $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$.

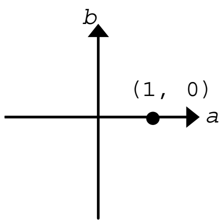
(B) $\frac{a}{b} = \cot x$ is not defined if $b = 0$. This occurs if $\theta = 0, \pi$.

(C) $\frac{r}{b} = \csc x$ is not defined if $b = 0$. This occurs if $\theta = 0, \pi$. *(6−4)*

20. Since the coordinates of a point on a unit circle are given by $P(a, b) = P(\cos x, \sin x)$, we evaluate $P(\cos(-8.305), \sin(-8.305))$--using a calculator set in radian mode--to obtain $P(-0.436, -0.900)$. Note that $x = -8.305$, since P is moving clockwise. The quadrant in which $P(a, b)$ lies can be determined by the signs of a and b. In this case P is in the third quadrant, since a is negative and b is negative. *(6−1, 6−2)*

21. $(a, b) = (1, 0)$ $r = 1$

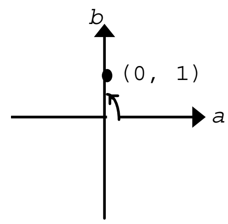
$\tan 0 = \frac{b}{a} = \frac{0}{1} = 0$



(6−2)

22. $(a, b) = (0, 1)$ $r = 1$

$\sec 90° = \frac{r}{a} = \frac{1}{0}$ Not defined

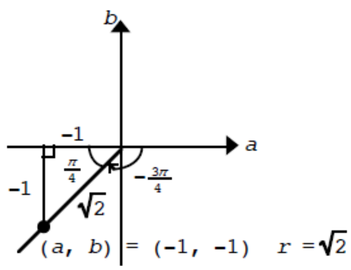


(6−2)

23. $y = \cos^{-1} 1$ is equivalent to $\cos y = 1$ $0 \le y \le \pi$

$y = 0$ *(6−6)*

24.



$$\cos\left(-\frac{3\pi}{4}\right) = \frac{a}{r} = \frac{-1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

(6−2)

25. $y = \sin^{-1}\frac{\sqrt{2}}{2}$ is equivalent to

$\sin y = \frac{\sqrt{2}}{2}$ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\sin y = \frac{b}{r} = \frac{\sqrt{2}}{2}$ $b = \sqrt{2}$ $r = 2$

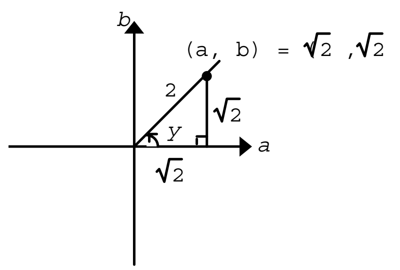
$a^2 + (\sqrt{2})^2 = 2^2$

$a^2 = 2$

$a = \sqrt{2}$

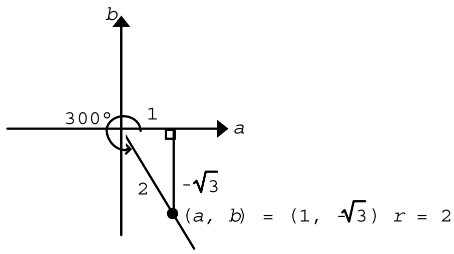
(positive since y is a quadrant I or IV angle)

$y = \frac{\pi}{4}$



(6−6)

26.



$$\csc 300° = \frac{r}{b} = \frac{2}{-\sqrt{3}} = -\frac{2}{\sqrt{3}} \text{ or } \frac{-2\sqrt{3}}{3}$$

(6−2)

27. $y = \arctan\sqrt{3}$ is equivalent to

$\tan y = \sqrt{3}$ $\qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$

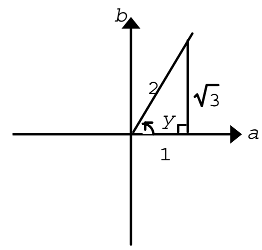
$\tan y = \frac{b}{a} = \frac{\sqrt{3}}{1}$ $\qquad b = \sqrt{3} \quad a = 1$

$r^2 = 1^2 + (\sqrt{3})^2$

$r^2 = 4$

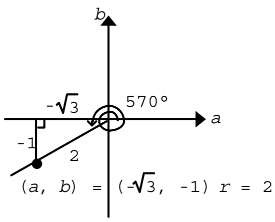
$r = 2$

$y = \frac{\pi}{3}$



(6−6)

28.



$$\sin 570° = \frac{b}{r} = -\frac{1}{2}$$

(6−2)

29. $y = \tan^{-1}(-1)$ is equivalent to

$\tan y = -1$ $\qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$

$\tan y = \frac{b}{a} = \frac{-1}{1}$ $\qquad b = -1 \quad a = 1$

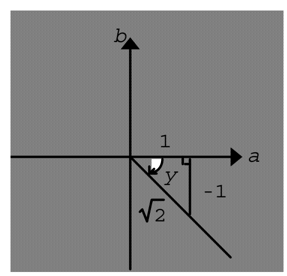
$r^2 = (-1)^2 + 1^2$

$r^2 = 2$

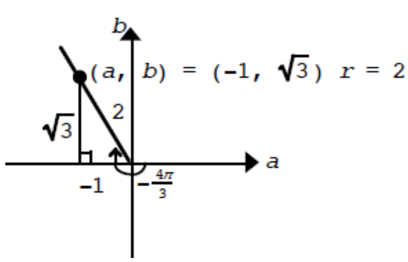
$r = \sqrt{2}$

$y = -\frac{\pi}{4}$

(6−6)



30.



$$\cot\left(-\frac{4\pi}{3}\right) = \frac{a}{b} = \frac{-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

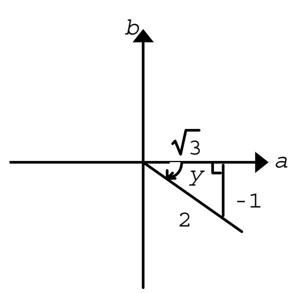
(6−2)

31. $y = \arcsin\left(-\frac{1}{2}\right)$ is equivalent to

$\sin y = -\frac{1}{2}$ $\qquad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

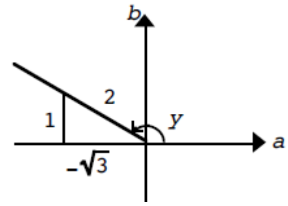
$\sin y = \frac{b}{r} = \frac{-1}{2}$ $\qquad b = -1 \quad r = 2 \quad a = \sqrt{3}$

$y = -\frac{\pi}{6}$



(6−6)

32. $y = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is equivalent to



$$\cos y = -\frac{\sqrt{3}}{2} \qquad 0 \le y \le \pi$$

$$\cos y = \frac{a}{r} = \frac{-\sqrt{3}}{2} \qquad a = -\sqrt{3} \quad r = 2 \quad b = 1$$

$$y = \frac{5\pi}{6}$$

(6−6)

33. $\cos(\cos^{-1} 0.33) = 0.33$ by the cosine-inverse cosine identity. (6−6)

34. Let $y = \tan^{-1}(-1)$, then $\tan y = -1$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Using the drawing in problem 29, we have $(a, b) = (1, -1)$, $r = \sqrt{2}$.

$\csc y = \csc[\tan^{-1}(-1)] = \frac{r}{b} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$ (6−6)

35. Let $y = \arccos\left(-\frac{1}{2}\right)$, then $\cos y = -\frac{1}{2}$, $0 \le y \le \pi$.

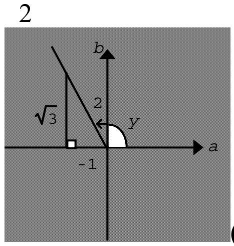
Draw the reference triangle associated with y,

then $\sin y = \sin\left[\arccos\left(-\frac{1}{2}\right)\right]$ can be determined

directly from the triangle.

$$\cos y = \frac{a}{r} = -\frac{1}{2} = \frac{-1}{2} \qquad a = -1 \quad r = 2 \quad b = \sqrt{3}$$

$$\sin y = \frac{b}{r} = \frac{\sqrt{3}}{2}$$



(6−6)

36. Let $y = \sin^{-1} \frac{-4}{5}$, then $\sin y = \frac{-4}{5}$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

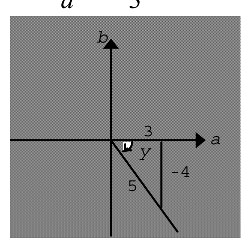
Draw the reference triangle associated with y,

then $\tan y = \tan\left[\sin^{-1}\left(-\frac{4}{5}\right)\right]$ can be determined

directly from the triangle.

$$\sin y = \frac{b}{r} = \frac{-4}{5} \qquad b = -4 \quad r = 5 \quad a = 3$$

$$\tan y = \frac{b}{a} = \frac{-4}{3}$$



(6−6)

37. 0.4431 (6−2)

38. $\tan\left(93 + \frac{46}{60} + \frac{17}{3600}\right)^\circ = -15.17$ (6−2)

39. $\sec(-2.073) = \frac{1}{\cos(-2.073)} = -2.077$ (6−2)

40. −0.9750 (6−6)

41. $\arccos x$ is not defined if $x < -1$ (6−6)

42. 1.557 (6−6)

43. 1.095 (6−6)

44. Since $\tan 1.345 = 4.353 > 1$, $\sin^{-1}(\tan 1.345)$ is not defined. (6−6)

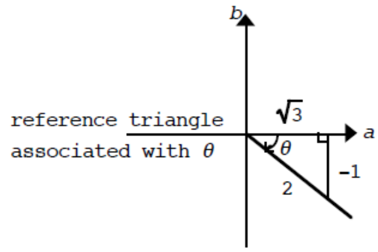
45. (A) $\theta = \arcsin^{-1}\left(-\frac{1}{2}\right)$ is equivalent to

$$\sin\theta = -\frac{1}{2} \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\theta = -\frac{\pi}{6}$$

Using the relation $\theta_D = \dfrac{180°}{\pi \text{ rad}}\ \theta_R$, we have

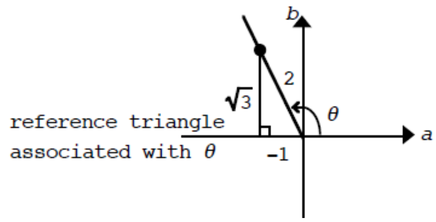
$$\sin^{-1}\left(-\frac{1}{2}\right) = \frac{180°}{\pi} \cdot \left(-\frac{\pi}{6}\right) = -30°$$



(B) $\theta = \arccos\left(-\dfrac{1}{2}\right)$ is equivalent to

$$\cos\theta = -\frac{1}{2} \qquad 0 \le \theta \le \pi$$

$$\theta = \frac{2\pi}{3}$$



(6−6)

Using the relation $\theta_D = \dfrac{180°}{\pi \text{ rad}}\ \theta_R$, we have

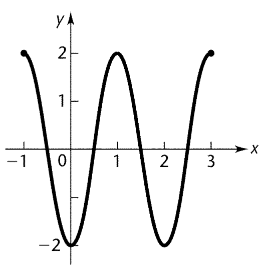
$$\arccos\left(-\frac{1}{2}\right) = \frac{180°}{\pi} \cdot \frac{2\pi}{3} = 120°.$$

46. Calculator in degree mode: (A) $\Theta = 151.20°$ (B) $\Theta = 82.28°$ (6−6)

47. $\cos^{-1}[\cos(-2)] = 2$ For the identity $\cos^{-1}(\cos x) = x$ to hold, x must be in the restricted domain of the cosine function; that is, $0 \le x \le \pi$. The number -2 is not in the restricted domain. (6−6)

48. Amplitude $= |-2| = 2$

Period $= \dfrac{2\pi}{\pi} = 2$

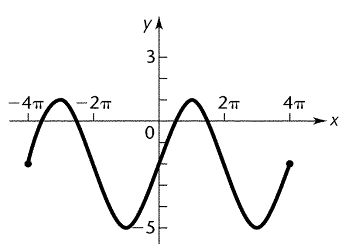


(6−5)

49. $y = -2 + 3\sin\left(\dfrac{x}{2}\right)$

For the graph of $y = 3\sin\dfrac{x}{2}$, we note: $A = 3$, $P = 2\pi \div \dfrac{1}{2} = 4\pi$, phase shift = 0.

We graph $y = 3\sin\dfrac{x}{2}$, then vertically translate the graph down 2 units.



(6−5)

50. $A = 6$ $P = \pi = 2\pi \div 2$.

Hence $B = 2$ $y = 6\cos 2x$; $-\dfrac{\pi}{2} \le x \le \pi$ (6−5)

51. $|A| = 0.5$ $P = 2 = 2\pi \div \pi$. Hence $B = \pi$ $A = -0.5$, since the graph has the form of the standard sine curve turned upside down. $y = -0.5\sin \pi x$; $-1 \le x \le 2$ (6−5)

52. If the graph of $y = \tan x$ is shifted $\dfrac{\pi}{2}$ units to the right and reflected in the x axis, the result will be the graph of $y = \cot x$. (6−4)

53. (A)

$\sin(-x)\cot(-x) = \sin(-x)\dfrac{\cos(-x)}{\sin(-x)}$	Quotient Identity
$= \cos(-x)$	Algebra
$= \cos x$	Identities for negatives

(B) $\dfrac{\sin^2 x}{1-\sin^2 x} = \dfrac{\sin^2 x}{\cos^2 x}$ Pythagorean Identity

$= \left(\dfrac{\sin x}{\cos x}\right)^2$ Algebra

$= \tan^2 x$ Quotient Identity *(6−4)*

54. $y = 3\sin\left(\dfrac{x}{2} + \dfrac{\pi}{2}\right)$

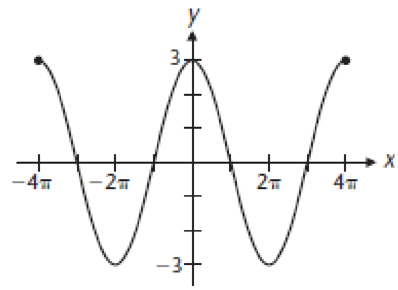
$A = 3$

Solve $\dfrac{x}{2} + \dfrac{\pi}{2} = 0$ $\qquad \dfrac{x}{2} + \dfrac{\pi}{2} = 2\pi$

$\dfrac{x}{2} = -\dfrac{\pi}{2}$ $\qquad \dfrac{x}{2} = -\dfrac{\pi}{2} + 2\pi$

$x = -\pi$ $\qquad x = -\pi + 4\pi$

Phase shift Period



The graph completes one full cycle as x varies over the interval $[-\pi, 3\pi]$.

(6−5)

55. $y = -2\cos\left(\dfrac{\pi}{2}x - \dfrac{\pi}{4}\right)$ amplitude $= |A| = |-2| = 2$

Solve $\dfrac{\pi}{2}x - \dfrac{\pi}{4} = 0$ $\qquad \dfrac{\pi}{2}x - \dfrac{\pi}{4} = 2\pi$

$\dfrac{\pi}{2}x = \dfrac{\pi}{4}$ $\qquad \dfrac{\pi}{2}x = \dfrac{\pi}{4} + 2\pi$

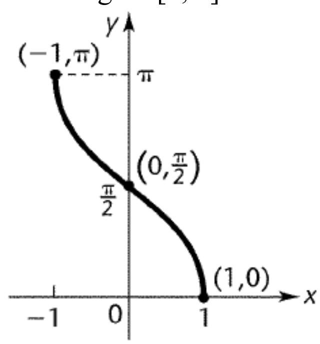
$x = \dfrac{\pi}{4} \div \dfrac{\pi}{2}$ $\qquad x = \left(\dfrac{\pi}{4} + 2\pi\right) \div \dfrac{\pi}{2}$

$x = \dfrac{1}{2}$ $\qquad x = \dfrac{1}{2} + 4$

Phase shift Period

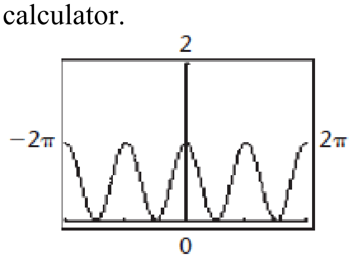
(6−5)

56. Domain $= [-1, 1]$
Range $= [0, \pi]$



(6−6)

57. Here is the graph of $y = \dfrac{1}{1+\tan^2 x}$ in a graphing calculator.



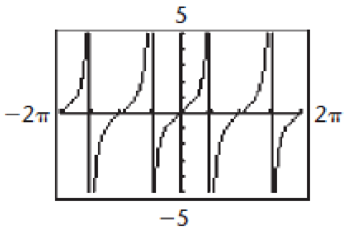
The graph has amplitude $\dfrac{1-0}{2} = \dfrac{1}{2}$ and period π.

It appears to be the graph of $y = \dfrac{1}{2}\cos 2x$ shifted up $\dfrac{1}{2}$ unit, that is, $y = \dfrac{1}{2}\cos 2x + \dfrac{1}{2}$.

(6−5)

58. (A) Here is the graph of $y = \dfrac{2\sin^2 x}{\sin 2x}$ in a graphing calculator.

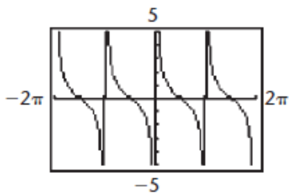
The graph appears to have the form $y = A\tan Bx$.
Since the period is π, $B = 1$.
The graph of $y = A\tan x$ shown appears to pass through $\left(\dfrac{\pi}{4}, 1\right)$, thus $1 = A\tan\dfrac{\pi}{4} = A$



The equation of the graph can be written $y = \tan x$.

59.

(B) Here is the graph of $y = \dfrac{2\cos^2 x}{\sin 2x}$ in a graphing calculator.



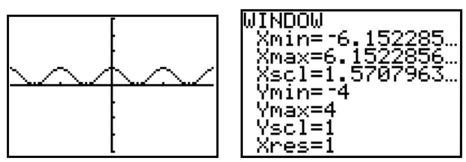
The graph appears to have the form $y = A \cot Bx$.
Since the period is π, $B = 1$.
The graph of $y = A \cot x$ shown appears to pass through $\left(\frac{\pi}{4}, 1\right)$, thus $1 = A \cot \frac{\pi}{4} = A$
The equation of the graph can be written $y = \cot x$.
(6−5)

59. (A) $f(-x) = \dfrac{1}{1+\tan^2(-x)}$

$= \dfrac{1}{1+(-\tan x)^2}$

$= \dfrac{1}{1+\tan^2 x} = f(x)$

$f(x)$ is even.

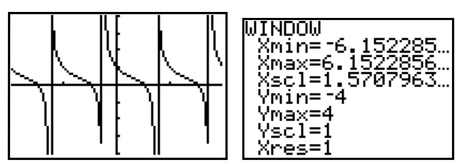


The graph is symmetric about the y-axis so $f(x)$ is even.

(B) $g(-x) = \dfrac{1}{1+\tan(-x)} = \dfrac{1}{1-\tan x}$

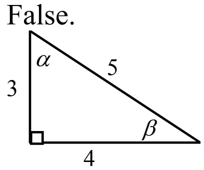
This is not equal to $g(x)$ nor is it equal to $-g(x)$ which would be $\dfrac{1}{-1-\tan x}$.

So $g(x)$ is neither even nor odd.



The graph is not symmetric about the y-axis nor is it symmetric about the origin. (6−4)

60. False.



According to the diagram,

$\sin \alpha = \dfrac{\text{Opp}}{\text{Hyp}} = \dfrac{4}{5}$ while $\csc \beta = \dfrac{\text{Hyp}}{\text{Opp}} = \dfrac{5}{4}$.

(6−3)

61. True. The two acute angles in a right triangle add up to 90° so if they're equal, both are 45°.

$\sin 45° = \dfrac{1}{\sqrt{2}} \approx 0.71$ $\quad$ $\cos 45° = \dfrac{1}{\sqrt{2}} \approx 0.71$

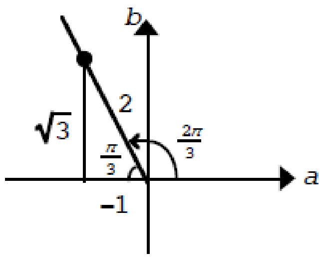
$\tan 45° = 1$ $\quad$ $\cot 45° = 1$

$\sec 45° = \sqrt{2} \approx 1.4$ $\quad$ $\csc 45° = \sqrt{2} \approx 1.4$ (6−3)

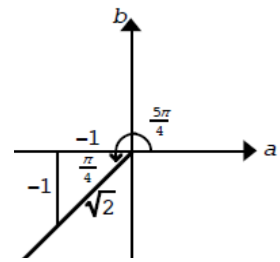
62. (A) Since $\theta_R = \dfrac{s}{r}$ and r = radius of circle = distance of A from center = 8, we have $\theta_R = \dfrac{20}{8} = 2.5$ radians.

(B) Since $\sin\theta = \dfrac{b}{r}$ and $\cos\theta = \dfrac{a}{r}$, we can write $a = r\cos\theta = 8\cos 2.5 = -6.41$ (calculator in radian mode)
$b = r\sin\theta = 8\sin 2.5 = 4.79$ $\quad (a, b) = (-6.41, 4.79)$ (6−1, 6−2)

63. (A) $\cos x = \frac{a}{r} = -\frac{1}{2} = \frac{-1}{2}$. a is negative in the II and III quadrants. The least positive x is associated with a $\frac{\pi}{3}$ reference triangle in the II quadrant as drawn: $x = \frac{2\pi}{3}$

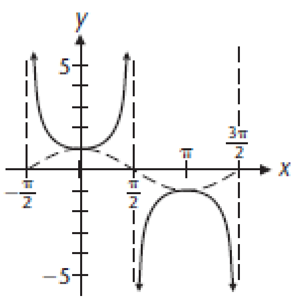


(B) $\csc x = -\sqrt{2} = \frac{\sqrt{2}}{-1} = \frac{r}{b}$, b is negative in the III and IV quadrants. The least positive x is associated with a $\frac{\pi}{4}$ reference triangle in the III quadrant as drawn: $x = \frac{5\pi}{4}$



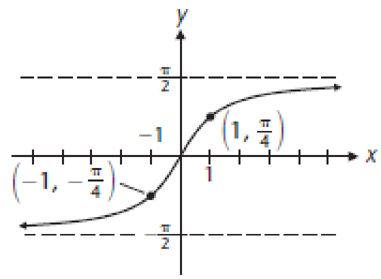
(6−2)

64. The dashed curve is the graph of $y = \cos x$. The solid curve is the required graph of $y = \sec x$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$



(6−4)

65.



Domain = all real numbers

Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *(6−6)*

66. $y = -5\tan\left(\pi x + \frac{\pi}{2}\right)$

Solve $\pi x + \frac{\pi}{2} = 0$ $\qquad$ $\pi x + \frac{\pi}{2} = \pi$

$$\pi x = -\frac{\pi}{2} \qquad \pi x = -\frac{\pi}{2} + \pi$$

$$x = -\frac{1}{2} \qquad x = -\frac{1}{2} + 1$$

Phase Shift = $-\frac{1}{2}$ $\quad$ Period $P = 1$ *(6−5)*

67. One cycle of $y = 3\csc\left(\frac{x}{2} - \frac{\pi}{4}\right)$ is completed as $\frac{x}{2} - \frac{\pi}{4}$ varies from 0 to 2π. Solve each equation for x:

$$\frac{x}{2} - \frac{\pi}{4} = 0 \qquad \frac{x}{2} - \frac{\pi}{4} = 2\pi$$

$$\frac{x}{2} = \frac{\pi}{4} \qquad \frac{x}{2} = \frac{\pi}{4} + 2\pi$$

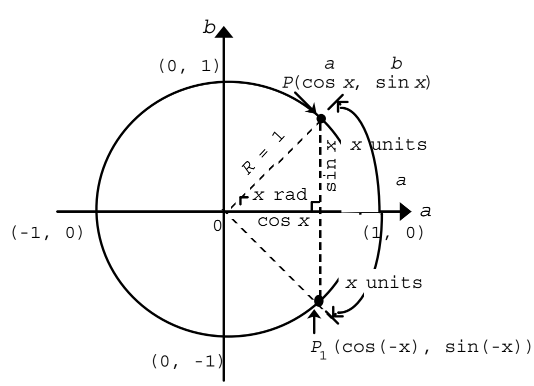
$$x = \frac{\pi}{2} \qquad x = \frac{\pi}{2} + 4\pi$$

Phase shift = $\frac{\pi}{2}$ $\quad$ Period = 4π *(6−5)*

68. From the figure, it should be clear that $\cos(-x) = \cos x$ (P and P_1 have the same x-coordinate) $\sin(-x) = -\sin x$ (P and P_1 have opposite y-coordinates)

Therefore $\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$

It follows that the graph of
(A) sine has origin symmetry
(B) cosine has y axis symmetry
(C) tangent has origin symmetry



(6−4)

69. Let $y = \sin^{-1} x$. Then $x = \sin y$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. If $\sin y = \frac{b}{r} = \frac{x}{1}$, then let $b = x$, $r = 1$

$$a^2 + x^2 = 1^2$$
$$a^2 = 1 - x^2$$
$$a = \sqrt{1-x^2}$$

We choose the positive sign because y is a I or IV quadrant angle.

$\sec y = \sec(\sin^{-1} x) = \frac{r}{a} = \frac{1}{\sqrt{1-x^2}}$ *(6−6)*

70. For each case, the number is not in the domain of the function and an error message of some type will appear. *(6−2, 6−6)*

71. $|A| = 2$. Hence $A = 2$ or -2. The graph completes one full cycle as x varies over the interval $\left[-\frac{5}{4}, \frac{3}{4}\right]$.

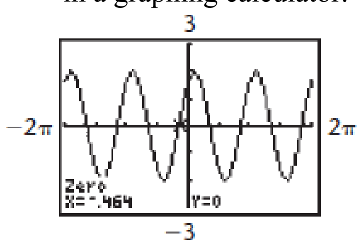
Since $-\frac{C}{B}$ is required between -1 and 0, we cannot simply set $-\frac{C}{B} = -\frac{5}{4}$. We must (mentally) extend the curve so that the (extended) graph is that of a standard sine curve that completes one full cycle as x varies over the interval $\left[-\frac{1}{4}, \frac{7}{4}\right]$. Hence $A = 2$

$$-\frac{C}{B} = -\frac{1}{4} \qquad -\frac{C}{B} + \frac{2\pi}{B} = \frac{7}{4}$$
$$C = \frac{B}{4} \qquad \frac{2\pi}{B} = 2 \quad B = \frac{2\pi}{2} = \pi \quad C = \frac{\pi}{4}$$

The equation is then $y = A\sin(Bx + C)$

$y = 2\sin\left(\pi x + \frac{\pi}{4}\right)$ *(6−5)*

72. Here is the graph of $y = 1.2\sin 2x + 1.6\cos 2x$ in a graphing calculator.



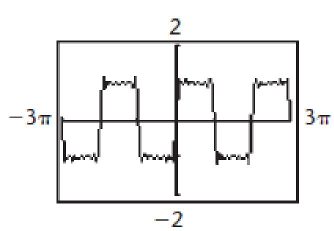
It appears that this is a sine curve shifted to the left, with $A = 2$ and, since $P = \frac{2\pi}{B}$ and P appears to be π, $B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$.

From the graphing calculator, we find that the x intercept closest to the origin, to three decimal places, is -0.464. To find C, substitute $B = 2$ and $x = -0.464$ into the phase-shift formula $x = -\frac{C}{B}$ and solve for C.

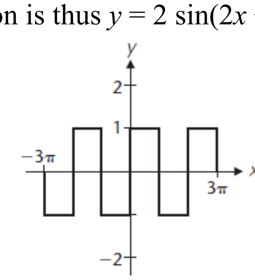
$$x = -\frac{C}{B}$$
$$-0.464 = -\frac{C}{2}$$
$$C = 0.928$$

The equation is thus $y = 2\sin(2x + 0.928)$.

73. (A)



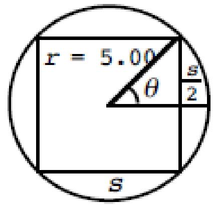
(B)



(6−5)

74. In one year the line sweeps out one full revolution, or 2π radians in 365 days. In 73 days the line sweeps out $\frac{73}{365}$ of one full revolution, or $\frac{73}{365} \cdot 2\pi = \frac{2\pi}{5}$ radians. *(6−1)*

75. Sketch a figure.



From geometry we know that

$\theta = \frac{1}{8}(360°) = 45°$

$\frac{s}{2} = r \sin 45° = r\frac{1}{\sqrt{2}}$

Hence $P = 4s = 8\left(\frac{s}{2}\right) = 8r\left(\frac{1}{\sqrt{2}}\right) = 8(5.00)\left(\frac{1}{\sqrt{2}}\right) = 28.3$ cm *(6−2)*

76. $d = 40$ feet, so $c = \pi d = 40\pi$ feet

In 80 revolutions, the wheel turns a total of $80 \times 2\pi$ radians $= 160\pi$ radians.

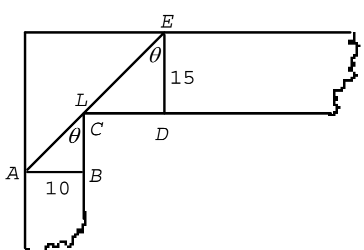
$$\frac{160\pi \text{ rad}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 8.38 \frac{\text{rad}}{\text{sec}}$$

In 80 revolutions, the linear distance traveled is $80 \times 40\pi$ feet $= 3200\pi$ feet.

$$\frac{3200\pi \text{ feet}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 167.55 \frac{\text{ft}}{\text{sec}}$$ *(6−1)*

77. When $t = 0$, $I = 30$. Hence $30 = A \cos B(0)$, that is, $A = 30$. Since the period is $\frac{1}{60}$ second, $\frac{2\pi}{B} = \frac{1}{60}$, $B = 2\pi(60) = 120\pi$. Hence the equation is $I = 30 \cos 120\pi t$. *(6−5)*

78.



(A) In the figure as labeled, $L = AC + CE$.

In triangle ABC, since $\sin\theta = \frac{10}{AC}$, $AC \sin\theta = 10$, $AC = \frac{10}{\sin\theta} = 10 \csc\theta$

In triangle CDE, since $\cos\theta = \frac{15}{CE}$, $CE \cos\theta = 15$, $CE = \frac{15}{\cos\theta} = 15 \sec\theta$

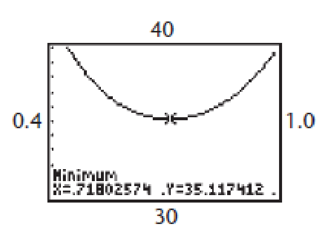
Then $L = AC + CE = 10 \csc\theta + 15 \sec\theta \quad 0 < \theta < \frac{\pi}{2}$

(B)

θ radians	0.4	0.5	0.6	0.7	0.8	0.9	1.0
L feet	42.0	38.0	35.9	35.1	35.5	36.9	39.6

From the table, the shortest distance L to the nearest foot, is 35 feet. This is the length of the longest log that can make the corner.

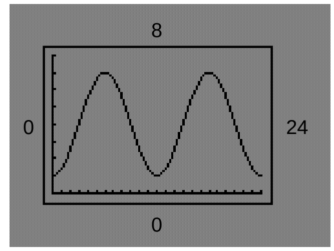
(C) Here is a graph of $L = 10 \csc\theta + 15 \sec\theta$ from a graphing calculator. The minimum value of L is shown as 35.1 feet, to one decimal place.



(D) As $\theta \to 0$, $L = 10 \csc\theta + 15 \sec\theta$ approaches an asymptote of $\csc\theta$; L increases without bound.

As $\theta \to \frac{\pi}{2}$, $L = 10 \csc\theta + 15 \sec\theta$ approaches an asymptote of $\sec\theta$; L increases without bound. *(6−2, 6−3)*

79. (A) $|A| = \dfrac{R_{\max} - R_{\min}}{2} = \dfrac{7-1}{2} = 3$. Hence $A = 3$ or -3. The graph appears to be shifted up from a graph of an upside down cosine curve that completes one full cycle as t varies over the interval [0, 12]. Thus $A = -3$.

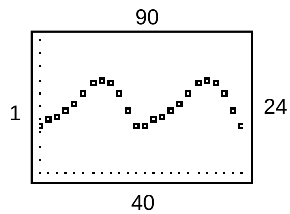


Then $P = 12 = 2\pi \cdot \dfrac{6}{\pi} = 2\pi \div \dfrac{\pi}{6}$. Hence $B = \dfrac{\pi}{6}$.

$k = \dfrac{R_{\max} + R_{\min}}{2} = \dfrac{7+1}{2} = 4$. Thus $R(t) = 4 - 3\cos\dfrac{\pi}{6}t$.

(B) The graph shows the seasonal changes in soft drink consumption. Most is consumed in August and the least in February. *(6−5)*

80. (A)



(B) $|A| = \dfrac{y_{\max} - y_{\min}}{2} = \dfrac{75-58}{2} = 8.5$. Hence $A = 8.5$ or -8.5.

$k = \dfrac{y_{\max} + y_{\min}}{2} = \dfrac{75+58}{2} = 66.5$. $P = 12 = 2\pi \cdot \dfrac{6}{\pi} = 2\pi \div \dfrac{\pi}{6}$. Hence $B = \dfrac{\pi}{6}$.

The x intercept closest to the origin is estimated from the graph as 4.5.

To find C, substitute $B = \dfrac{\pi}{6}$ and $x = 4.5$ into the phase shift formula $x = -\dfrac{C}{B}$ and solve for C.

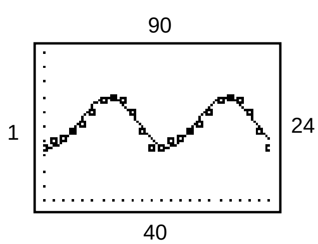
$$x = -\frac{C}{B}$$

$$4.5 = -C \div \frac{\pi}{6}$$

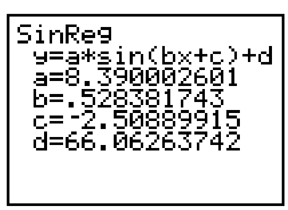
$$C = -4.5\left(\frac{\pi}{6}\right) = -2.4$$

With this value of C, the graph is seen to be shifted up from the graph of a standard sine curve, thus $A = 8.5$. The equation is thus $y = 66.5 + 8.5\sin\left(\dfrac{\pi}{6}x - 2.4\right)$.

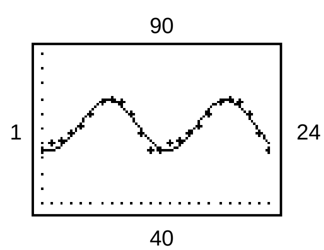
(C)



(D)



Compute the regression equation.



Graph the data and the regression equation.

(6−5)