CHAPTER 3

Section 3-1

1. No. A correspondence between two sets is a function only if exactly one element of the second set corresponds to each element of the first set.

3. The domain of a function is the set of all first components in the ordered pairs defining the function; the range is the set of all second components.

5. The symbol $f(x)$ does not denote multiplication of x by f.

7. A function

9. Not a function (two range values correspond to domain values 3 and 5)

11. A function

13. A function; domain = {2, 3, 4, 5}; range = {4, 6, 8, 10}

15. Not a function (two range values correspond to domain values 5 and 10)

17. A function; domain = {Ohio, Alabama, West Virginia, California}; range = {Obama, McCain}

19. A function

21. Not a function (fails vertical line test since the y axis crosses the graph three times.)

23. Not a function (fails vertical line test since the graph itself is vertical)

25. (A) A function. (B) Not a function, as long as there is more than one student in any Math 125 class.

27. $f(x) = 3x - 5$
(A) $f(3) = 3(3) - 5 = 4$
(B) $f(h) = 3h - 5$
(C) $f(3) + f(h) = 4 + (3h - 5) = 3h - 1$
(D) $f(3 + h) = 3(3 + h) - 5 = 9 + 3h - 5 = 3h + 4$

Common Error: $f(3 + h)$ is not the same as $f(3) + h$ or $f(3) + f(h)$.

29. $F(w) = -w^2 + 2w$
(A) $F(4) = -(4)^2 + 2(4) = -16 + 8 = -8$
(B) $F(-4) = -(-4)^2 + 2(-4) = -16 - 8 = -24$
(C) $F(4 + a) = -(4 + a)^2 + 2(4 + a)$
$= -(16 + 8a + a^2) + 8 + 2a$
$= -16 - 8a - a^2 + 8 + 2a$
$= -a^2 - 6a - 8$
(D) $F(2 - a) = -(2 - a)^2 + 2(2 - a)$
$= -(4 - 4a + a^2) + 4 - 2a$
$= -4 + 4a - a^2 + 4 - 2a$
$= -a^2 + 2a$

31. $f(t) = 2 - 3t^2$
(A) $f(-2) = 2 - 3(-2)^2 = 2 - 12 = -10$
(B) $f(-t) = 2 - 3(-t)^2 = 2 - 3t^2$
(C) $-f(t) = -(2 - 3t^2) = -2 + 3t^2$
(D) $-f(-t) = -(2 - 3t^2) = -2 + 3t^2$

33. $F(u) = u^2 - u - 1$
(A) $F(10) = (10)^2 - (10) - 1 = 89$
(B) $F(u^2) = (u^2)^2 - (u^2) - 1 = u^4 - u^2 - 1$
(C) $F(5u) = (5u)^2 - (5u) - 1 = 25u^2 - 5u - 1$
(D) $5F(u) = 5(u^2 - u - 1) = 5u^2 - 5u - 5$

35. (A) When $x = -2$, $y = 6$, so $f(-2) = 6$.
(B) When $y = -4$, x can equal –6, 2, or 4.

37. Solving for the dependent variable y, we have
$y - x^2 = 1$
$y = x^2 + 1$
Since $x^2 + 1$ is a real number for each real number x, the equation defines a function with domain all real numbers.

39. Solving for the dependent variable y, we have

$$2x^3 + y^2 = 4$$
$$y^2 = 4 - 2x^3$$
$$y = \pm\sqrt{4-2x^3}$$

Since each positive number has two real square roots, the equation does not define a function. For example, when $x = 0$, $y = \pm\sqrt{4-2(0)^3} = \pm\sqrt{4} = \pm 2$.

41. Solving for the dependent variable y, we have

$$x^3 - y = 2$$
$$-y = 2 - x^3$$
$$y = x^3 - 2$$

Since $x^3 - 2$ is a real number for each real number x, the equation defines a function with domain all real numbers.

43. If $2x + |y| = 7$, then $|y| = -2x + 7$. This equation does not define a function. For example, if $x = 0$, then $|y| = 7$, so $y = 7$ or $y = -7$.

45. Solving for the dependent variable y, we have

$$3y + 2|x| = 12$$
$$3y = 12 - 2|x|$$
$$y = \frac{12-2|x|}{3}$$

Since $\frac{12-2|x|}{3}$ is a real number for each real number x, the equation defines a function with domain all real numbers.

47. Since $f(x)$ is a polynomial, the domain is the set of all real numbers R, $-\infty < x < \infty$, $(-\infty, \infty)$.

49. Since $3u^2 + 4$ is never negative, $\sqrt{3u^2+4}$ represents a real number for all replacements of u by real numbers.
The domain is R, $-\infty < u < \infty$, $(-\infty, \infty)$.

51. If the denominator of the fraction is zero, the function will be undefined, since division by zero is undefined.
For any other values of z, $h(z)$ represents a real number. Solve the equation $4 - z = 0$; $z = 4$. Thus, the domain is all real numbers except 4 or $(-\infty, 4) \cup (4, \infty)$; $z < 4$ or $z > 4$.

53. The square root of a number is defined only if the number is nonnegative.
Thus, $\sqrt{t-4}$ is defined if $t - 4 \geq 0$ and undefined if $t - 4 < 0$.
Note that $t - 4 \geq 0$ when $t \geq 4$, so the domain is all real numbers greater than or equal to 4 or $[4, \infty)$.

55. The formula $\sqrt{7+3w}$ is defined only if $7 + 3w \geq 0$, since the square root is only defined if the number inside is nonnegative.
We solve the inequality:

$$7 + 3w \geq 0$$
$$3w \geq -7$$
$$w \geq -\frac{7}{3}$$

Thus, the domain is all real numbers greater than or equal to $-\frac{7}{3}$ or $\left[-\frac{7}{3}, \infty\right)$.

57. The fraction $\frac{u}{u^2+4}$ is defined for any value of u that does not make the denominator zero, so we solve the equation $u^2 + 4 = 0$ to find values that make the function undefined.

$$u^2 + 4 = 0$$
$$u^2 = -4$$

This equation has no solution since the square of a number can't be negative. So there are no values of u that make the fraction undefined and the domain is all real numbers or $(-\infty, \infty)$.

59. There are two issues to consider: we need to make certain that $x + 4 \geq 0$ so that the number inside the square root is nonnegative, and we need to avoid x-values that make the denominator zero.
First, $x + 4 \geq 0$ whenever $x \geq -4$. So x must be greater than or equal to -4 to avoid a negative under the root. Also, $x - 1 = 0$ when $x = 1$, so x cannot be 1. The domain is all real numbers greater than or equal to -4 except 1, or $[-4, 1) \cup (1, \infty)$; $-4 \leq x < 1$ or $x > 1$.

61. There are two issues to consider: the number inside the square root has to be nonnegative, and the denominator of the fraction has to be nonzero. Since we just have t inside the square root, t has to be greater than or equal to zero. Next, we solve $3 - \sqrt{t} = 0$ to find any t-values that make the denominator zero.

$$3 - \sqrt{t} = 0$$
$$3 = \sqrt{t}$$
$$9 = t$$

Thus, t cannot be 9. The domain is all real numbers greater than or equal to zero except 9, or $[0, 9) \cup (9, \infty)$; $0 \le t < 9$ or $t > 9$.

63. $g(x) = 2x^3 - 5$

65. $G(x) = 8\sqrt{x} - 4(x + 2)$

67. Function f multiplies the square of the domain element by 2, then adds 5 to the result.

69. Function Z multiplies the domain element by 4, adds 5 to the result, then divides <u>this</u> result by the square root of the domain element.

71.
$$F(s) = 3s + 15$$
$$F(2 + h) = 3(2 + h) + 15$$
$$F(2) = 3(2) + 15$$
$$\frac{F(2+h)-F(2)}{h} = \frac{[3(2+h)+15]-[3(2)+15]}{h} = \frac{[6+3h+15]-[21]}{h} = \frac{3h+21-21}{h} = \frac{3h}{h} = 3$$

73.
$$g(x) = 2 - x^2$$
$$g(3 + h) = 2 - (3 + h)^2$$
$$g(3) = 2 - (3)^2$$
$$\frac{g(3+h)-g(3)}{h} = \frac{[2-(3+h)]^2-[2-(3)^2]}{h} = \frac{[2-9-6h-h^2]-[-7]}{h}$$
$$= \frac{-6h-h^2}{h} = \frac{h(-6-h)}{h} = -6 - h$$

75. (A)
$$f(x) = 4x - 7$$
$$f(x + h) = 4(x + h) - 7$$
$$\frac{f(x+h)-f(x)}{h} = \frac{[4(x+h)-7]-[4x-7]}{h}$$
$$= \frac{4x+4h-7-4x+7}{h}$$
$$= \frac{4h}{h} = 4$$

Common Errors: $f(x + h) \neq f(x) + f(h)$ or $4x - 7 + 4h - 7$ $f(x + h) \neq f(x) + h$ or $4x - 7 + h$ Also note: $-f(x) \neq -4x - 7$ Parentheses must be supplied.

(B)
$$f(x) = 4x - 7$$
$$f(a) = 4a - 7$$
$$\frac{f(x)-f(a)}{x-a} = \frac{(4x-7)-(4a-7)}{x-a}$$
$$= \frac{4x-7-4a+7}{x-a}$$
$$= \frac{4x-4a}{x-a}$$
$$= \frac{4(x-a)}{x-a} = 4$$

77. (A)

$$f(x) = 2x^2 - 4$$
$$f(x+h) = 2(x+h)^2 - 4$$

$$\frac{f(x+h)-f(x)}{h} = \frac{[2(x+h)^2-4]-[2x^2-4]}{h}$$
$$= \frac{2(x^2+2xh+h^2)-4-2x^2+4}{h}$$
$$= \frac{2x^2+4xh+2h^2-4-2x^2+4}{h}$$
$$= \frac{4xh+2h^2}{h}$$
$$= \frac{h(4x+2h)}{h}$$
$$= 4x + 2h$$

(B)

$$f(x) = 2x^2 - 4$$
$$f(a) = 2a^2 - 4$$

$$\frac{f(x)-f(a)}{x-a} = \frac{(2x^2-4)-(2a^2-4)}{x-a}$$
$$= \frac{2x^2-4-2a^2+4}{x-a}$$
$$= \frac{2x^2-2a^2}{x-a}$$
$$= \frac{2(x-a)(x+a)}{x-a}$$
$$= 2(x+a)$$
$$= 2x + 2a$$

79. (A)

$$f(x) = -4x^2 + 3x - 2$$
$$f(x+h) = -4(x+h)^2 + 3(x+h) - 2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{[-4(x+h)^2+3(x+h)-2]-[-4x^2+3x-2]}{h}$$
$$= \frac{-4(x^2+2xh+h^2)+3(x+h)-2+4x^2-3x+2}{h}$$
$$= \frac{-4x^2-8xh-4h^2+3x+3h-2+4x^2-3x+2}{h}$$
$$= \frac{-8xh-4h^2+3h}{h}$$
$$= \frac{h(-8x-4h+3)}{h}$$
$$= -8x - 4h + 3$$

(B)

$$f(x) = -4x^2 + 3x - 2$$
$$f(a) = -4a^2 + 3a - 2$$

$$\frac{f(x)-f(a)}{x-a} = \frac{(-4x^2+3x-2)-(-4a^2+3a-2)}{x-a}$$
$$= \frac{-4x^2+3x-2+4a^2-3a+2}{x-a}$$
$$= \frac{-4x^2+4a^2+3x-3a}{x-a}$$
$$= \frac{-4(x-a)(x+a)+3(x-a)}{x-a}$$
$$= \frac{(x-a)[-4(x+a)+3]}{x-a}$$
$$= -4(x+a) + 3$$
$$= -4x - 4a + 3$$

81. (A)

$$f(x) = \sqrt{x+2}$$

$$f(x+h) = \sqrt{x+h+2}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}$$

$$= \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$= \frac{(\sqrt{x+h+2})^2-(\sqrt{x+2})^2}{h(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$= \frac{x+h+2-(x+2)}{h(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$= \frac{h}{h(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$= \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$$

(B)

$$f(x) = \sqrt{x+2}$$

$$f(a) = \sqrt{a+2}$$

$$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a}$$

$$= \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a} \cdot \frac{\sqrt{x+2}+\sqrt{a+2}}{\sqrt{x+2}+\sqrt{a+2}}$$

$$= \frac{(\sqrt{x+2})^2-(\sqrt{a+2})^2}{(x-a)(\sqrt{x+2}+\sqrt{a+2})}$$

$$= \frac{x+2-(a+2)}{(x-a)(\sqrt{x+2}+\sqrt{a+2})} = \frac{x-a}{(x-a)(\sqrt{x+2}+\sqrt{a+2})}$$

$$= \frac{1}{\sqrt{x+2}+\sqrt{a+2}}$$

83. (A)

$$f(x) = \frac{4}{x}$$

$$f(x+h) = \frac{4}{x+h}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{4}{x+h}-\frac{4}{x}}{h}$$

$$= \frac{4x-4(x+h)}{hx(x+h)}$$

$$= \frac{-4h}{hx(x+h)}$$

$$= \frac{-4}{x(x+h)}$$

(B)

$$f(x) = \frac{4}{x}$$

$$f(a) = \frac{4}{a}$$

$$\frac{f(x)-f(a)}{x-a} = \frac{\frac{4}{x}-\frac{4}{a}}{x-a}$$

$$= \frac{4a-4x}{ax(x-a)}$$

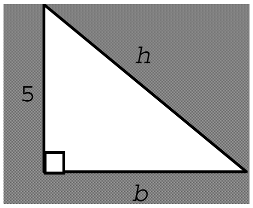
$$= \frac{-4(x-a)}{ax(x-a)}$$

$$= \frac{-4}{ax}$$

85. Given w = width and Area = 64, we use $A = \ell w$ to write $\ell = \frac{A}{w} = \frac{64}{w}$.

Then $P = 2w + 2\ell = 2w + 2\left(\frac{64}{w}\right) = 2w + \frac{128}{w}$. Since w must be positive, the domain of $P(w)$ is $w > 0$.

87.



Using the given letters, the Pythagorean theorem gives

$h^2 = b^2 + 5^2$

$h^2 = b^2 + 25$

$h = \sqrt{b^2 + 25}$ (since h is positive)

Since b must be positive, the domain of $h(b)$ is $b > 0$.

89. Daily cost = fixed cost + variable cost

$C(x)$ = \$300 + (\$1.75 per dozen doughnuts) × (number of dozen doughnuts)

$C(x) = 300 + 1.75x$

91. The cost is a flat \$17 per month, plus \$2.40 for each hour of airtime.

93. (A) $S(0) = 16(0)^2 = 0$; $S(1) = 16(1)^2 = 16$; $S(2) = 16(2)^2 = 16(4) = 64$;
$S(3) = 16(3)^2 = 16(9) = 144$ (Note: Remember that the order of operations requires that we apply the exponent first then multiply by 16.)

(B)
$$\frac{S(2+h)-S(2)}{h} = \frac{16(2+h)^2-16(2)^2}{h} = \frac{16(4+4h+h^2)-16(4)}{h}$$
$$= \frac{64+64h+16h^2-64}{h} = \frac{64h+16h^2}{h} = \frac{\cancel{h}(64+16h)}{\cancel{h}} = 64 + 16h$$

(Note: Be careful when evaluating $S(2 + h)$! You need to replace the variable t in the function with $(2 + h)$. $S(2 + h)$ is <u>not</u> the same thing as $S(2) + h$!)

(C) $h = 1$: $64 + 16(1) = 80$

$h = -1$: $64 + 16(-1) = 64 - 16 = 48$

$h = 0.1$: $64 + 16(0.1) = 64 + 1.6 = 65.6$;

$h = -0.1$: $64 + 16(-0.1) = 64 - 1.6 = 62.4$

$h = 0.01$: $64 + 16(0.01) = 64 + 0.16 = 64.16$:

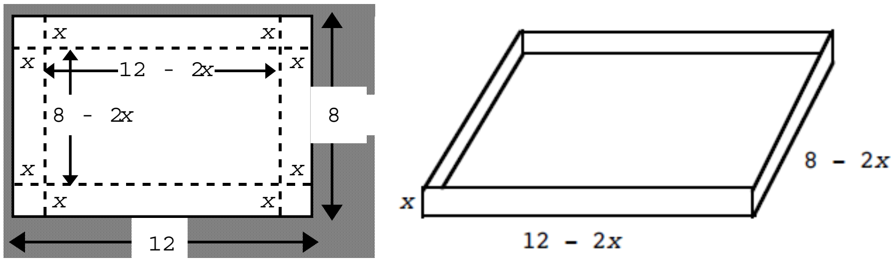
$h = -0.01$: $64 + 16(-0.01) = 64 - 0.16 = 63.84$

$h = 0.001$: $64 + 16(0.001) = 64 + 0.016 = 64.016$

$h = -0.001$: $64 + 16(-0.001) = 64 - 0.016 = 63.984$

(D) The smaller h gets the closer the result is to 64. The numerator of the fraction, $S(2 + h) - S(2)$, is the difference between how far an object has fallen after $2 + h$ seconds and how far it's fallen after 2 seconds. This difference is how far the object falls in the small period of time from 2 to $2 + h$ seconds. When you divide that distance by the time (h), you get the average velocity of the object between 2 and $2 + h$ seconds. Part (C) shows that this average velocity approaches 64 feet per second as h gets smaller.

95.



From the above figures it should be clear that
V = length × width × height = $(12 - 2x)(8 - 2x)x$.
Since all distances must be positive, $x > 0$, $8 - 2x > 0$, $12 - 2x > 0$.
Thus, $0 < x$, $4 > x$, $6 > x$, or $0 < x < 4$ (the last condition, $6 > x$, will be automatically satisfied if $x < 4$.)
Domain: $0 < x < 4$.

97. From the text diagram, since each pen must have area 50 square feet, we see

Area = (length)(width) or 50 = (length)x. Thus, the length of each pen is $\frac{50}{x}$ ft.

The total amount of fencing = 4(width) + 5(length) + 4(width – gate width)

$$F(x) = 4x + 5\left(\frac{50}{x}\right) + 4(x - 3)$$

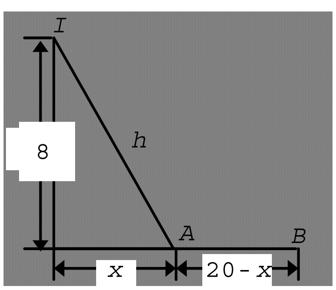
$$F(x) = 4x + \frac{250}{x} + 4x - 12$$

$$F(x) = 8x + \frac{250}{x} - 12$$

Then $F(4) = 8(4) + \frac{250}{4} - 12 = 82.5$ $\quad F(5) = 8(5) + \frac{250}{5} - 12 = 78$

$F(6) = 8(6) + \frac{250}{6} - 12 = 77.7$ $\quad F(7) = 8(7) + \frac{250}{7} - 12 = 79.7$

99.



We note that the pipeline consists of the lake section IA, and shore section AB.
The shore section has length $20 - x$.
The lake section has length h, where $h^2 = 8^2 + x^2$, thus $h = \sqrt{64 + x^2}$.

$$\text{The cost of all the pipeline} = \left(\begin{matrix} \textit{cost of shore} \\ \textit{section per mile} \end{matrix}\right)\left(\begin{matrix} \textit{number of} \\ \textit{shore miles} \end{matrix}\right) + \left(\begin{matrix} \textit{cost of lake} \\ \textit{section per mile} \end{matrix}\right)\left(\begin{matrix} \textit{number of} \\ \textit{lake miles} \end{matrix}\right)$$

$C(x) = 10{,}000(20 - x) + 15{,}000\sqrt{64 + x^2}$

From the diagram we see that x must be non-negative, but no more than 20.

Domain: $0 \le x \le 20$

Section 3-2

1. The graph of a function $f(x)$ is the set of all points whose first coordinate is an element of the domain of f and whose second coordinate is the associated element of the range.

3. The graph of a function can have one y intercept (when $y = f(0)$) or none (if 0 is not in the domain). The graph can have any number of x intercepts.

5. A function is increasing on an interval if for any choice x_1, x_2 of x values on that interval, if $x_2 > x_1$, then $f(x_2) > f(x_1)$.

7. A function is defined piecewise if it is defined by different expressions for different parts of its domain.

9. (A) $[-4, 4)$ (B) $[-3, 3)$ (C) 0 (D) 0 (E) $[-4, 4)$ (F) None (G) None (H) None

11. (A) $(-\infty, \infty)$ (B) $[-4, \infty)$ (C) -3, 1 (D) -3 (E) $[-1, \infty)$ (F) $(-\infty, -1]$ (G) None (H) None

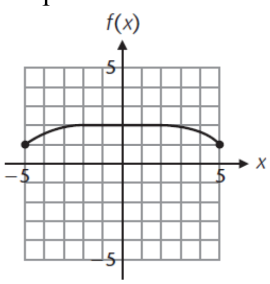
13. (A) $(-\infty, 2) \cup (2, \infty)$ (The function is not defined at $x = 2$.) (B) $(-\infty, -1) \cup [1, \infty)$ (C) None (D) 1 (E) None (F) $(-\infty, -2] \cup (2, \infty)$ (G) $[-2, 2)$ (H) $x = 2$

15. $f(-4) = -3$ since the point $(-4, -3)$ is on the graph; $f(0) = 0$ since the point (0,0) is on the graph; $f(4)$ is undefined since there is no point on the graph at $x = 4$.

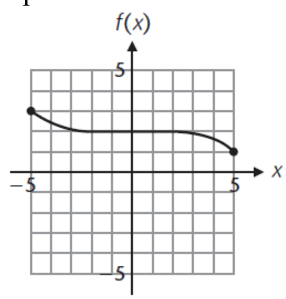
17. $h(-3) = 0$ since the point $(-3, 0)$ is on the graph; $h(0) = -3$ since the point $(0, -3)$ is on the graph; $h(2) = 5$ since the point (2, 5) is on the graph.

19. $p(-2) = 1$ since the point $(-2, 1)$ is on the graph; $p(2)$ is undefined since there is no point on the graph at $x = 2$; $p(5) = -4$ since the point $(5, -4)$ is on the graph.

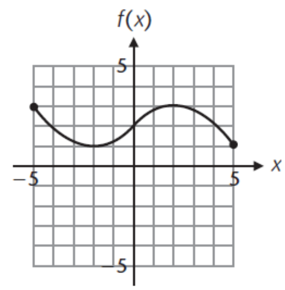
21. One possible answer:



23. One possible answer:



25. One possible answer:



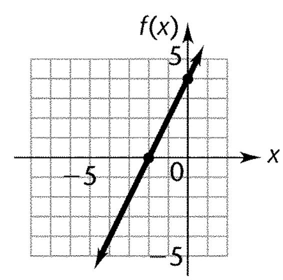
27.

$$f(x) = \underbrace{2}_{\text{slope}}x + 4$$

The y intercept is $f(0) = 4$, and the slope is 2.
To find the x intercept, we solve the equation $f(x) = 0$ for x.

$$\begin{aligned} f(x) &= 0 \\ 2x + 4 &= 0 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

The x intercept is -2.

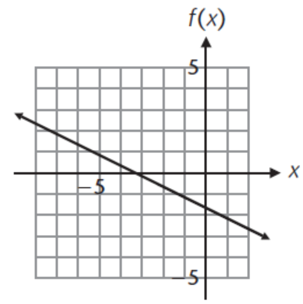


29. $f(x) = -\frac{1}{2}x - \frac{5}{3}$

The y intercept is $f(0) = -\frac{5}{3}$, and the slope is $-\frac{1}{2}$. To find the x intercept, we solve the equation $f(x) = 0$ for x.

$$f(x) = 0$$
$$-\frac{1}{2}x - \frac{5}{3} = 0$$
$$-\frac{1}{2}x = \frac{5}{3}$$
$$x = (-2)\frac{5}{3} = \frac{-10}{3}$$

The x intercept is $\frac{-10}{3}$.

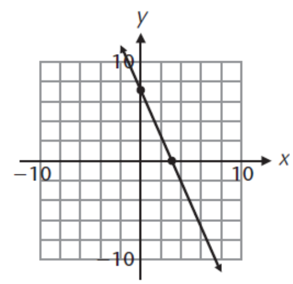


31. $f(x) = -2.3x + 7.1$

The y intercept is $f(0) = 7.1$, and the slope is -2.3. To find the x intercept, we solve the equation $f(x) = 0$ for x.

$$f(x) = 0$$
$$-2.3x + 7.1 = 0$$
$$-2.3x = -7.1$$
$$x = \frac{-7.1}{-2.3}$$
$$x = 3.1$$

The x intercept is 3.1.



33. A linear function must have the form $f(x) = mx + b$. We are given $f(0) = 10$, hence $10 = f(0) = m(0) + b$. Thus $b = 10$. Since $f(-2) = 2$, we also have $2 = f(-2) = m(-2) + b = -2m + 10$. Solving for m, we have

$$2 = -2m + 10$$
$$-8 = -2m$$
$$m = 4$$

Hence $f(x) = mx + b$ becomes $f(x) = 4x + 10$.

35. A linear function must have the form $f(x) = mx + b$. We are given $f(-2) = 7$, hence $7 = f(-2) = m(-2) + b$. Thus $b = 7 + 2m$. Since $f(4) = -2$, we also have $-2 = f(4) = 4m + b$. Substituting, we have

$$b = 7 + 2m$$
$$-2 = 4m + b$$
$$-2 = 4m + 7 + 2m$$
$$-2 = 6m + 7$$
$$-9 = 6m$$
$$m = -\frac{3}{2}$$
$$b = 7 + 2m = 7 + 2\left(-\frac{3}{2}\right) = 4$$

Hence $f(x) = mx + b$ becomes $f(x) = -\frac{3}{2}x + 4$.

37. The rational expression $\frac{3x-12}{2x+4}$ is defined everywhere except at the zero of the denominator:

$$2x + 4 = 0$$
$$2x = -4$$
$$x = -2$$

The domain of f is $\{x \mid x \neq -2\}$.

A rational expression is 0 if and only if the numerator is zero:

$$3x - 12 = 0$$
$$3x = 12$$
$$x = 4$$

The x intercept of f is 4.

The y intercept of f is $f(0) = \frac{3(0)-12}{2(0)+4} = -3$.

39. The rational expression $\frac{3x-2}{4x-5}$ is defined everywhere except at the zero of the denominator:

$$4x - 5 = 0$$
$$4x = 5$$
$$x = \frac{5}{4}$$ The domain of f is $\left\{x \mid x \neq \frac{5}{4}\right\}$.

A rational expression is 0 if and only if the numerator is zero:

$$3x - 2 = 0$$
$$3x = 2$$
$$x = \frac{2}{3}$$

The x intercept of f is $\frac{2}{3}$.

The y intercept of f is $f(0) = \frac{3(0)-2}{4(0)-5} = \frac{2}{5}$.

41. The rational expression $\frac{4x}{(x-2)^2}$ is defined everywhere except at the zero of the denominator:

$$(x-2)^2 = 0$$
$$x - 2 = 0$$
$$x = 2$$

The domain of f is $\{x \mid x \neq 2\}$.
A rational expression is 0 if and only if the numerator is zero:

$$4x = 0$$
$$x = 0$$

The x intercept of f is 0.

The y intercept of f is $f(0) = \frac{4(0)}{(0-2)^2} = 0$.

43. The rational expression $\frac{x^2-16}{x^2-9}$ is defined everywhere except at the zeros of the denominator:

$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$
$$x = 3 \text{ or } -3$$

The domain of f is $\{x \mid x \neq -3, 3\}$.
A rational expression is 0 if and only if the numerator is zero:

$$x^2 - 16 = 0$$
$$(x-4)(x+4) = 0$$
$$x = 4 \text{ or } -4$$

The x intercepts of f are ± 4.

The y intercept of f is $f(0) = \frac{0^2-16}{0^2-9} = \frac{16}{9}$.

45. The rational expression $\frac{x^2+7}{x^2-25}$ is defined everywhere except at the zeros of the denominator:

$$x^2 - 25 = 0$$
$$(x-5)(x+5) = 0$$
$$x = 5 \text{ or } -5$$

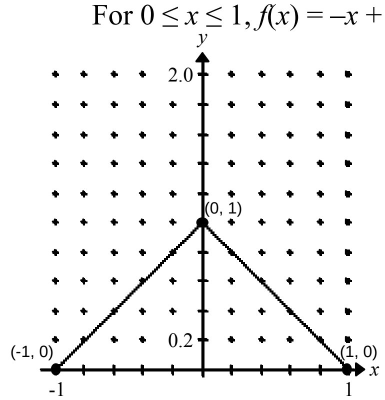
The domain of f is $\{x \mid x \neq -5, 5\}$.
A rational expression is 0 if and only if the numerator is zero.

$x^2 + 7 = 0$ has no real solutions, hence f has no x intercept.

The y intercept of f is $f(0) = \frac{0^2+7}{0^2-25} = -\frac{7}{25}$.

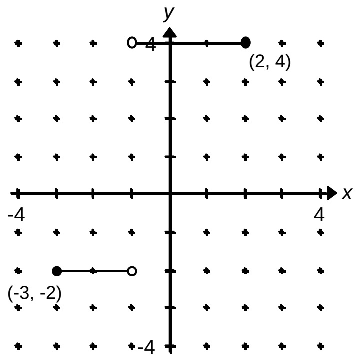
47. (A) For $-1 \le x < 0$, $f(x) = x + 1$, so $f(-1) = -1 + 1 = 0$.
For $0 \le x \le 1$, $f(x) = -x + 1$, so $f(0) = -0 + 1 = 1$ and $f(1) = -1 + 1 = 0$.

(B)



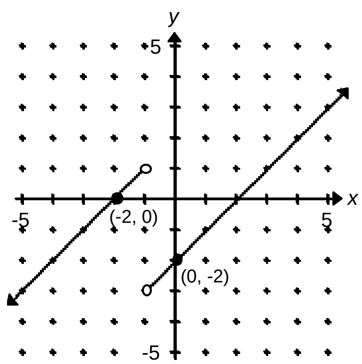
(C) The domain is the union of the intervals used in the definition of f: $[-1, 1]$.
From the graph, the range is $[0, 1]$.
The function is continuous on its domain.

49. (A) For $-3 \le x < -1$, $f(x) = -2$, so $f(-3) = -2$.
$f(-1)$ is not defined.
For $-1 < x \le 2$, $f(x) = 4$, so $f(2) = 4$.

(B)


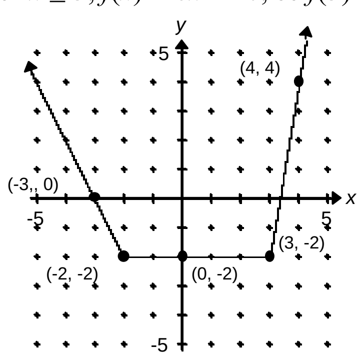
(C) The domain is the union of the intervals used in the definition of f:
$[-3, -1) \cup (-1, 2]$.
From the graph, the range is the set of numbers $\{-2, 4\}$.
The function is discontinuous at $x = -1$.

51. (A) For $x < -1$, $f(x) = x + 2$, so $f(-2) = -2 + 2 = 0$.
$f(-1)$ is not defined.
For $x > -1$, $f(x) = x - 2$, so $f(0) = 0 - 2 = -2$.

(B)


(C) The domain is the union of the intervals used in the definition of f:
$(-\infty, -1) \cup (-1, \infty)$.
From the graph, the range is R.
The function is discontinuous at $x = -1$.

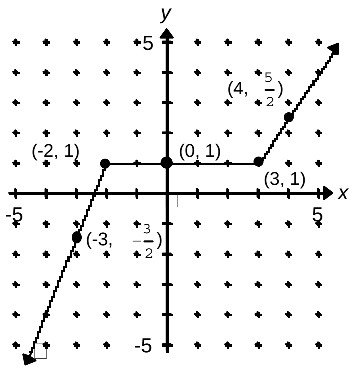
53. (A) For $x < -2$, $f(x) = -2x - 6$, so $f(-3) = -2(-3) - 6 = 0$.
For $-2 \le x < 3$, $f(x) = -2$, so $f(-2) = f(0) = -2$.
For $x \ge 3$, $f(x) = 6x - 20$, so $f(3) = 6 \cdot 3 - 20 = -2$ and $f(4) = 6 \cdot 4 - 20 = 4$.

(B)


(C) The domain is the union of the intervals used in the definition of f.
The domain is therefore R.
From the graph, the range is $[-2, \infty)$.
The function is continuous on its domain.

55. (A) For $x < -2$, $f(x) = \frac{5}{2}x + 6$, so $f(-3) = \frac{5}{2}(-3) + 6 = -\frac{3}{2}$.
For $-2 \le x \le 3$, $f(x) = 1$, so $f(-2) = f(0) = f(3) = 1$.
For $x > 3$, $f(x) = \frac{3}{2}x - \frac{7}{2}$, so $f(4) = \frac{3}{2}(4) - \frac{7}{2} = \frac{5}{2}$.

(B)



(C) The domain is the union of the intervals used in the definition of *f*.
The domain is therefore *R*.
From the graph, the range is *R*.
The function is continuous on its domain.

57. (A) For $x < 0$, $f(x) = \frac{2}{3}x + 4$, so $f(-1) = \frac{2}{3}(-1) + 4 = \frac{10}{3}$.

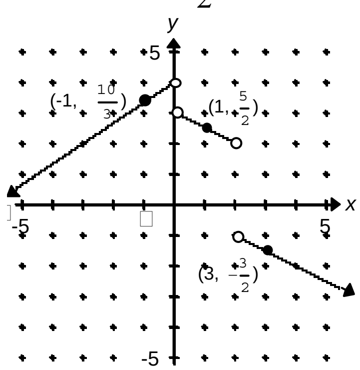
$f(0)$ is not defined.

For $0 < x < 2$, $f(x) = -\frac{1}{2}x + 3$, so $f(1) = -\frac{1}{2}(1) + 3 = \frac{5}{2}$.

$f(2)$ is not defined.

For $x > 2$, $f(x) = -\frac{1}{2}x$, so $f(3) = -\frac{1}{2}(3) = -\frac{3}{2}$.

(B)



(C) The domain is the union of the intervals used in the definition of *f*.
$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$.
From the graph, the range is $(-\infty, 4)$.
The function is discontinuous at $x = 0$ and $x = 2$.

59. For $x \le 0$, the graph is a line with slope $\frac{1-3}{0-(-2)} = -1$ and *y* intercept 1, that is, $y = -x + 1$.

For $x > 0$, the graph is a line with slope $\frac{(-3)-(-1)}{2-0} = -1$ and *y* intercept -1, that is, $y = -x - 1$.

Therefore,

$$f(x) = \begin{cases} -x+1 & \text{if} \quad x \le 0 \\ -x-1 & \text{if} \quad x > 0 \end{cases}$$

61. For $x \le -1$, the graph is the horizontal line $y = 3$. For $x > -1$, the graph is a line with slope $\frac{-3-3}{1-(-1)} = -3$ passing through $(1, -3)$. The point-slope form yields $y - (-3) = -3(x - 1)$ or $y = -3x$.
Therefore,

$$f(x) = \begin{cases} 3 & \text{if} \quad x \le -1 \\ -3x & \text{if} \quad x > -1 \end{cases}$$

An equally valid solution would be

$$f(x) = \begin{cases} 3 & \text{if} \quad x < -1 \\ -3x & \text{if} \quad x \ge -1 \end{cases}$$

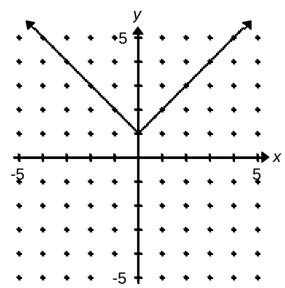
63. For $x < -2$, the graph is the horizontal line $y = 3$. The function is not defined at $x = -2$ or at $x = 1$.
For $-2 < x < 1$, the graph is a line with slope $\frac{(-4)-2}{1-(-2)} = -2$ which would, if extended, pass through (–2, 2).
The point-slope form yields $y - 2 = -2[x - (-2)]$, that is, $y = -2x - 2$. For $x > 1$, the graph is the horizontal line $y = -1$.
Therefore,

$$f(x) = \begin{cases} 3 & \text{if} \quad x < -2 \\ -2x-2 & \text{if} \quad -2 < x < 1 \\ -1 & \text{if} \quad x > 1 \end{cases}$$

65. If $x < 0$, then $|x| = -x$ and $f(x) = 1 + |x| = 1 + (-x) = 1 - x$.
If $x \geq 0$, then $|x| = x$ and $f(x) = 1 + x$.
Therefore,

$$f(x) = \begin{cases} 1-x & \text{if} \quad x < 0 \\ 1+x & \text{if} \quad x \geq 0 \end{cases}$$

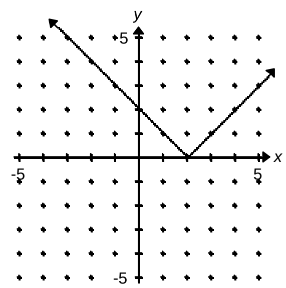
The function is defined for all real numbers; the domain is R. From the graph, the range is $[1, \infty)$. The function is continuous on its domain.



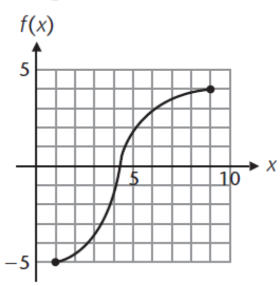
67. If $x - 2 < 0$, that is, $x < 2$, $f(x) = |x - 2| = -(-x - 2) = -x + 2$.
If $x - 2 \geq 0$, that is, $x \geq 2$, $f(x) = |x - 2| = x - 2$.
Therefore,

$$f(x) = \begin{cases} -x+2 & \text{if} \quad x < 2 \\ x-2 & \text{if} \quad x \geq 2 \end{cases}$$

The function is defined for all real numbers; the domain is R. From the graph, the range is $[0, \infty)$. The function is continuous on its domain.

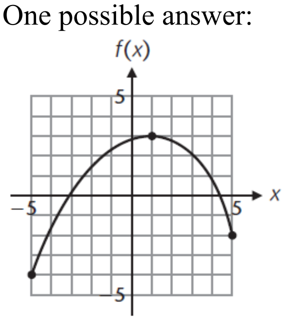


69. (A) One possible answer:



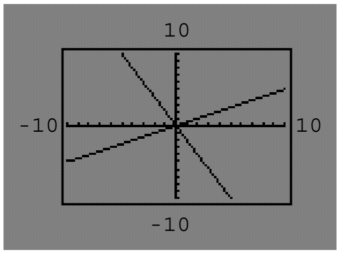
(B) This graph crosses the x axis once. To meet the conditions specified a graph must cross the x axis exactly once. If it crossed more times the function would have to be decreasing somewhere; if it did not cross at all the function would have to be discontinuous somewhere.

71. (A) One possible answer:

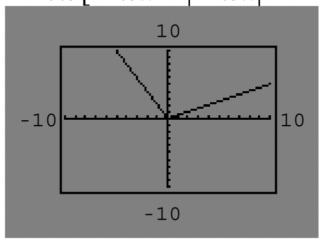


(B) This graph crosses the x axis twice. To meet the conditions specified a graph must cross the x axis at least twice. If it crossed fewer times the function would have to be discontinuous somewhere. However, the graph could cross more times; in fact there is no upper limit on the number of times it can cross the x axis.

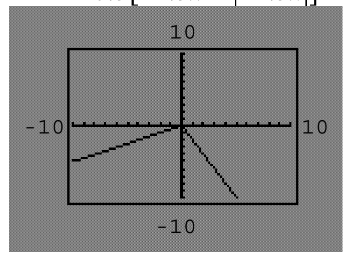
73. Graphs of f and g



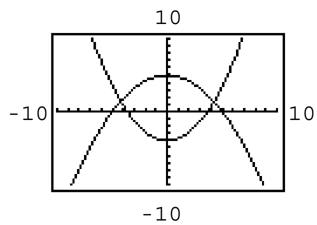
Graph of
$m(x) = 0.5[-2x + 0.5x + |-2x - 0.5x|]$
$= 0.5[-1.5x + |-2.5x|$



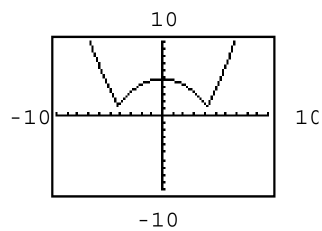
Graph of
$n(x) = 0.5[-2x + 0.5x - |-2x - 0.5x|]$
$= 0.5[-1.5x - |-2.5x|]$



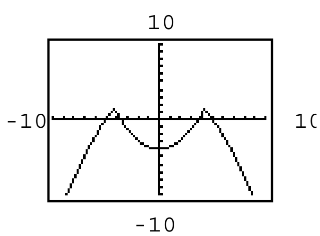
75. Graphs of f and g



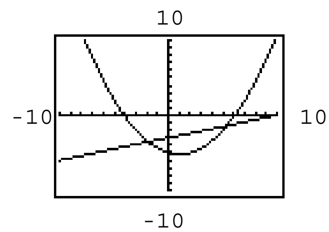
Graph of
$m(x) = 0.5[5 - 0.2x^2 + 0.3x^2 - 4 + |5 - 0.2x^2 - (0.3x^2 - 4)|]$
$= 0.5[1 + 0.1x^2 + |9 - 0.5x^2|]$



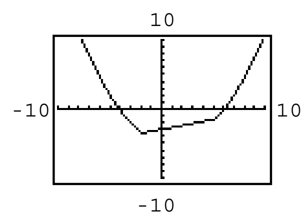
Graph of
$n(x) = 0.5[5 - 0.2x^2 + 0.3x^2 - 4 - |5 - 0.2x^2 - (0.3x^2 - 4)|]$
$= 0.5[1 + 0.1x^2 - |9 - 0.5x^2|]$



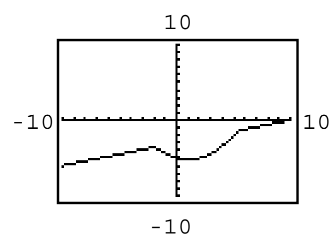
77. Graphs of f and g



Graph of
$m(x) = 0.5[0.2x^2 - 0.4x - 5 + 0.3x - 3$
$+ |0.2x^2 - 0.4x - 5 - (0.3x - 3)|]$
$= 0.5[0.2x^2 - 0.1x - 8 + |0.2x^2 - 0.7x - 2|]$



Graph of
$n(x) = 0.5[0.2x^2 - 0.4x - 5 + 0.3x - 3$
$- |0.2x^2 - 0.4x - 5 - (0.3x - 3)|]$
$= 0.5[0.2x^2 - 0.1x - 8 - |0.2x^2 - 0.7x - 2|]$



79. The graphs of $m(x)$ show that the value of $m(x)$ is always the larger of the two values for $f(x)$ and $g(x)$. In other words, $m(x) = \max[f(x), g(x)]$.

81. Since 100 miles are included, only the daily charge of \$32 applies for mileage between 0 and 100. So if $R(x)$ is the daily cost of rental where x is miles driven, $R(x) = 32$ if $0 \le x \le 100$. After 100 miles, the charge is an extra \$0.16 for each mile: the mileage charge will be 0.16 times the number of miles over 100 which is $x - 100$. So the mileage charge is $0.16(x - 100)$ or $0.16x - 16$. The \$32 charge still applies so when $x \ge 100$ the rental charge is $32 + 0.16x - 16$, or $16 + 0.16x$.

$$R(x) = \begin{cases} 32 & 0 \le x \le 100 \\ 16 + 0.16x & x > 100 \end{cases}$$

83. If $0 \le x \le 3{,}000$, $E(x) = 200$

		Base Salary	+	Commission on Sales Over \$3,000
If \$3,000 < x < 8,000, $E(x)$		= 200	+	$0.04(x - 3000)$
		= 200	+	$0.04x - 120$
		= 80	+	$0.04x$

Common Error: Commission is not $0.04x$ (4% of sales) nor is it $0.04x + 200$ (base salary plus 4% of sales).

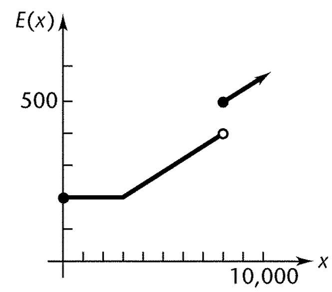
There is a point of discontinuity at $x = 8{,}000$.

	Salary	+	Bonus
If $x \ge 8{,}000$. $E(x)$	$= 80 + 0.04x$	+	100
	= 180		$+ 0.04x$

Summarizing, $E(x) = \begin{cases} 200 & \text{if} \quad 0 \le x \le 3{,}000 \\ 80 + 0.04x & \text{if} \quad 3{,}000 < x < 8{,}000 \\ 180 + 0.04x & \text{if} \quad 8{,}000 \le x \end{cases}$

$E(5{,}750) = 80 + 0.04(5{,}750) = \$310$
$E(9{,}200) = 180 + 0.04(9{,}200) = \$548$

x	$y = 200$	x	$y = 80 + 0.04x$	x	$y = 180 + 0.04x$
0	200	3,000	200	8,000	500
2,000	200	7,000	360	10,000	580



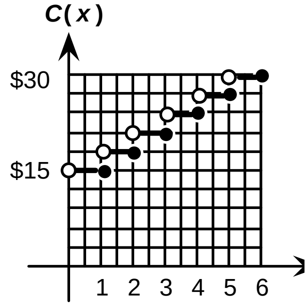
85.

$$\left.\begin{array}{rcrcrcr} f(4) &=& 10[\![0.5+0.4]\!] &=& 10(0) &=& 0 \\ f(-4) &=& 10[\![0.5-0.4]\!] &=& 10(0) &=& 0 \\ f(6) &=& 10[\![0.5+0.6]\!] &=& 10(1) &=& 10 \\ f(-6) &=& 10[\![0.5-0.6]\!] &=& 10(-1) &=& -10 \\ f(24) &=& 10[\![0.5+2.4]\!] &=& 10(2) &=& 20 \\ f(25) &=& 10[\![0.5+2.5]\!] &=& 10(3) &=& 30 \\ f(247) &=& 10[\![0.5+24.7]\!] &=& 10(25) &=& 250 \\ f(-243) &=& 10[\![0.5-24.3]\!] &=& 10(-24) &=& -240 \\ f(-245) &=& 10[\![0.5-24.5]\!] &=& 10(-24) &=& -240 \\ f(-246) &=& 10[\![0.5-24.6]\!] &=& 10(-25) &=& -250 \end{array}\right\} f \text{ rounds numbers to the tens place}$$

87. Since $f(x) = [\![10x + 0.5]\!]/10$ rounds numbers to the nearest tenth, (see text example 6) we try $[\![100x + 0.5]\!]/100 = f(x)$ to round to the nearest hundredth.
$f(3.274) = [\![327.9]\!]/100 = 3.27$
$f(7.846) = [\![785.1]\!]/100 = 7.85$
$f(-2.8783) = [\![-287.33]\!]/100 = -2.88$
A few examples suffice to convince us that this is probably correct.
(A proof would be out of place in this book.)
$f(x) = [\![100x + 0.5]\!]/100$.

89. (A)

$$C(x) = \begin{cases} 15 & 0 < x \le 1 \\ 18 & 1 < x \le 2 \\ 21 & 2 < x \le 3 \\ 24 & 3 < x \le 4 \\ 27 & 4 < x \le 5 \\ 30 & 5 < x \le 6 \end{cases}$$



(B) The two functions appear to coincide, for example

$C(3.5) = 24$ $f(3.5) = 15 + 3[\![3.5]\!] = 15 + 3 \cdot 3 = 24$

However,

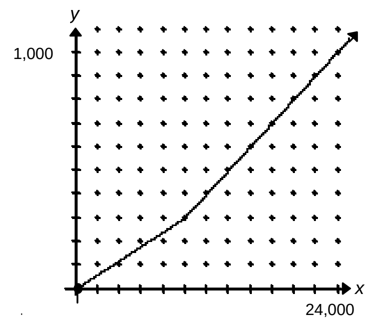
$C(1) = 15$ $f(1) = 15 + 3[\![1]\!] = 15 + 3 \cdot 1 = 18$

The functions are not the same, therefore. In fact, $f(x) \neq C(x)$ at $x = 1, 2, 3, 4, 5, 6$.

91. On the interval [0, 10,000], the tax is $0.03x$. On the interval $(10{,}000, \infty)$, the tax is $0.03(10{,}000) + 0.05(x - 10{,}000)$ or $300 + 0.05x - 500$, that is, $0.05x - 200$.

Combining the intervals with the above linear expressions, we have

$$T(x) = \begin{cases} 0.03x & \text{if} \quad 0 \le x \le 10{,}000 \\ 0.05x - 200 & \text{if} \quad x > 10{,}000 \end{cases}$$



93. A tax of 5.35% on any amount up to \$19,890 is calculated by multiplying 0.0535 by the income, which is represented by x. So the tax due, $t(x)$, is $0.0535x$ if x is between 0 and 19,890. For x-values between 19,890 and 65,330, the percentage is computed only on the portion over 19,890. To find that portion we subtract 19,890 from the income $(x - 19{,}890)$; then multiply by 0.0705 to get the percentage portion of the tax. The total tax is \$1,064 plus the percentage portion, so $t(x) = 1{,}064 + 0.0705(x - 19{,}890)$ if x is between 19,890 and 65,330. The tax for incomes over \$65,340 is computed in a similar manner: \$4,268 plus 7.85% of the portion over 65,330, which is $x - 65{,}330$. We get $\$4{,}268 + 0.0785(x - 65{,}330)$ if x is over 65,330.

Combined we get

$$t(x) = \begin{cases} 0.0535x & \text{if} \quad 0 \le x \le 19{,}890 \\ 1{,}064 + 0.0705(x - 19{,}890) & \text{if} \quad 19{,}890 < x \le 65{,}330 \\ 4{,}268 + 0.0785(x - 65{,}330) & \text{if} \quad x > 65{,}330 \end{cases}$$

or, after simplifying,

$$t(x) = \begin{cases} 0.0535x & \text{if} \quad 0 \le x \le 19{,}890 \\ 0.0705x - 338.25 & \text{if} \quad 19{,}890 < x \le 65{,}330 \\ 0.0785x - 860.41 & \text{if} \quad x > 65{,}330 \end{cases}$$

$t(10{,}000) = 0.0535(10{,}000) = 535$; the tax is \$535
$t(30{,}000) = 0.0705(30{,}000) - 338.25 = \$1{,}776.75$
$t(100{,}000) = 0.0785(100{,}000) - 860.41 = \$6{,}989.59$

Section 3-3

1. For each point with coordinates $(x, f(x))$ on the graph of $y = f(x)$ there is a corresponding point with coordinates $(x, f(x) + k)$ on the graph of $y = f(x) + k$. Since this point is k units above the first point, each point, and thus the entire graph, has been moved upward k units.

3. For each point with coordinates $(x, f(x))$ on the graph of $y = f(x)$ there is a corresponding point with coordinates $(x, -f(x))$ on the graph of $y = -f(x)$. Since this point is the reflection of the first point with respect to the x axis, the entire graph is a reflection of the graph of $y = f(x)$. Similarly for $y = f(-x)$.

5. Domain: Since $x \geq 0$, the domain is $[0, \infty)$

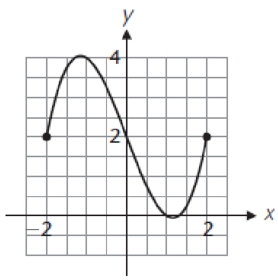
Range: Since the range of $f(x) = \sqrt{x}$ is $y \geq 0$, for $h(x) = -\sqrt{x}$, $y \leq 0$.
Thus, the range of h is $(-\infty, 0]$.

7. Domain: R

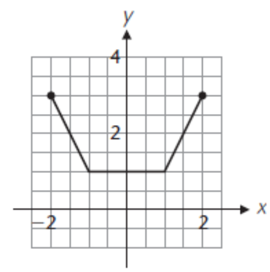
Range: Since the range of $f(x) = x^2$ is $y \geq 0$, for $g(x) = -2x^2$, $y \leq 0$.
Thus, the range of g is $(-\infty, 0]$.

9. Domain: R; Range: R

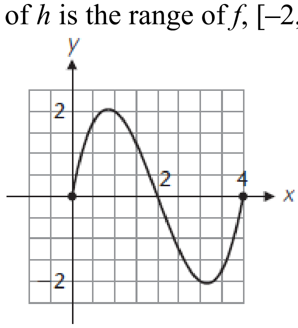
11. The graph of $y = h(x)$ is the graph of $y = f(x)$ shifted up 2 units.
The domain of h is the domain of f, $[-2, 2]$.
The range of h is the range of f shifted up 2 units, $[0, 4]$.



13. The graph of $y = h(x)$ is the graph of $y = g(x)$ shifted up 2 units.
The domain of h is the domain of g, $[-2, 2]$.
The range of h is the range of g shifted up 2 units, $[1, 3]$.

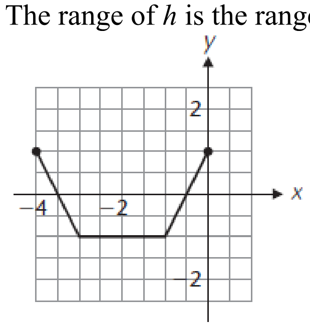


15. The graph of h is the graph of f shifted right 2 units.
The domain of h is the domain of f shifted right 2 units, $[0, 4]$.
The range of h is the range of f, $[-2, 2]$.

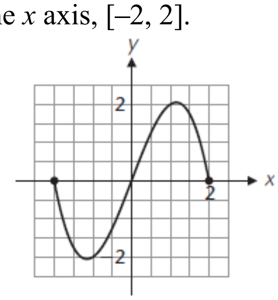


Common Error:
$x - 2$ does not indicate shifting left.

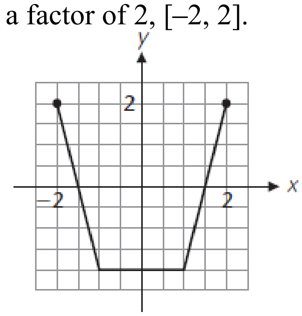
17. The graph of h is the graph of g shifted left 2 units.
The domain of h is the domain of g shifted left 2 units, $[-4, 0]$.
The range of h is the range of g, $[-1, 1]$.



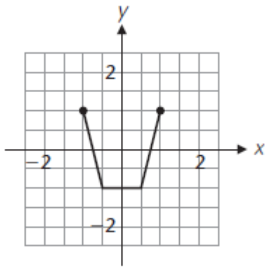
19. The graph of h is the graph of f reflected through the x axis.
The domain of h is the domain of f, $[-2, 2]$.
The range of h is the range of f reflected through the x axis, $[-2, 2]$.



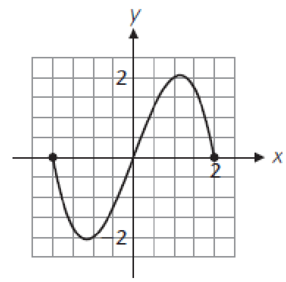
21. The graph of h is the graph of g stretched vertically by a factor of 2.
The domain of h is the domain of g, $[-2, 2]$.
The range of f is the range of g stretched vertically by a factor of 2, $[-2, 2]$.



23. The graph of h is the graph of g shrunk horizontally by a factor of $\frac{1}{2}$.
The domain of h is the domain of g shrunk horizontally by a factor of $\frac{1}{2}$, $[-1, 1]$.
The range of h is the range of g, $[-1, 1]$.



25. The graph of h is the graph of f reflected through the y axis.
The domain of h is the domain of f reflected through the y axis, $[-2, 2]$.
The range of h is the range of f, $[-2, 2]$.



27. $g(-x) = (-x)^3 + (-x) = -x^3 - x = -(x^3 + x) = -g(x)$. Odd

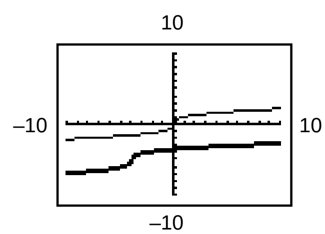
29. $m(-x) = (-x)^4 + 3(-x)^2 = x^4 + 3x^2 = m(x)$. Even

31. $F(-x) = (-x)^5 + 1 = -x^5 + 1$
$-F(x) = -(x^5 + 1) = -x^5 - 1$
Therefore $F(-x) \neq F(x)$. $F(-x) \neq -F(x)$.
$F(x)$ is neither even nor odd.

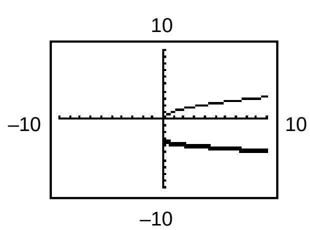
33. $G(-x) = (-x)^4 + 2 = x^4 + 2 = G(x)$. Even

35. $q(-x) = (-x)^2 + (-x) - 3 = x^2 - x - 3$.
$-q(x) = -(x^2 + x - 3) = -x^2 - x + 3$
Therefore $q(-x) \neq q(x)$. $q(-x) \neq -q(x)$.
$q(x)$ is neither even nor odd.

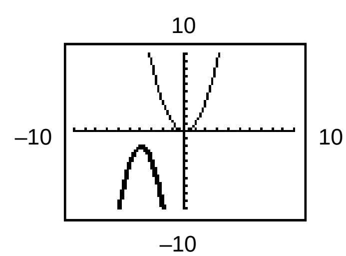
37. $g(x) = \sqrt[3]{x+4} - 5$.
The graphs of $f(x) = \sqrt[3]{x}$ (thin curve) and $g(x)$ (thick curve) are shown.



39. $g(x) = -0.5(6 + \sqrt{x})$
The graphs of $f(x) = \sqrt{x}$ (thin curve) and $g(x)$ (thick curve) are shown.

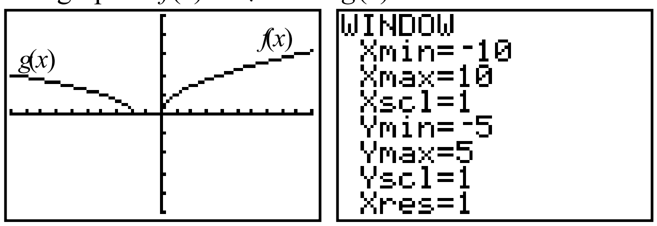


41. $g(x) = -2(x + 4)^2 - 2$
The graphs of $f(x) = x^2$ (thin curve) and $g(x)$ (thick curve) are shown.

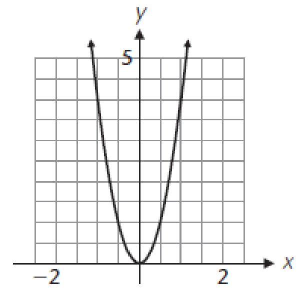


43. $g(x) = \sqrt{-\frac{1}{2}(x+2)}$.

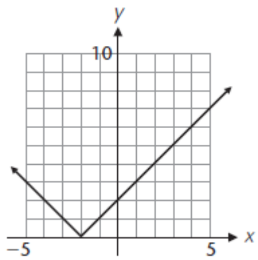
The graph of $f(x) = \sqrt{x}$ and $g(x)$ are shown.



45. The graph of $y = x^2$ is stretched vertically by a factor of 4 (**or** shrunk horizontally by a factor of 2.)

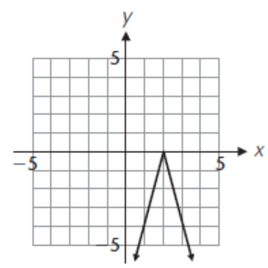


47. The graph of $y = |x|$ is shifted left 2 units.

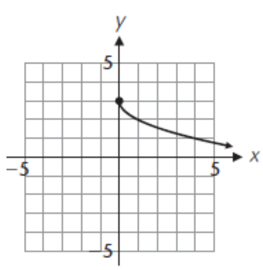


49. $m(x) = -|4x - 8| = -|4(x - 2)|$

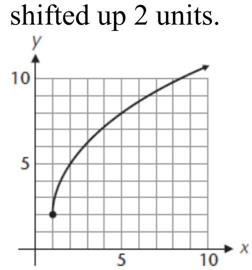
The graph of $y = |x|$ is shifted right 2 units, stretched vertically by a factor of 4, and reflected through the x axis.



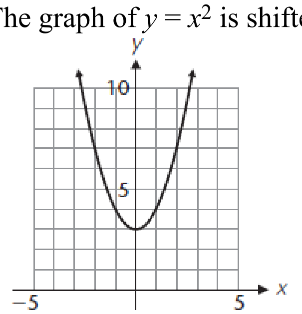
51. The graph of $y = \sqrt{x}$ is reflected through the x axis and shifted up 3 units.



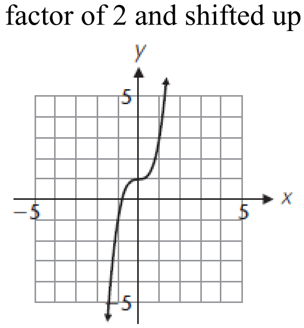
53. The graph of $y = \sqrt{x}$ is shifted right 1 unit, stretched vertically by a factor of 3, and shifted up 2 units.



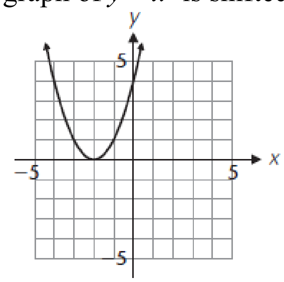
55. The graph of $y = x^2$ is shifted up 3 units.



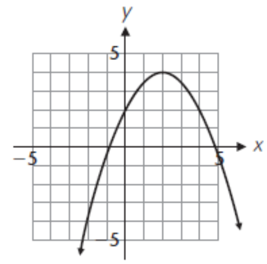
57. The graph of $y = x^3$ is stretched vertically by a factor of 2 and shifted up 1 unit.



59. The graph of $y = x^2$ is shifted left 2 units.



61. The graph of $y = x^2$ is shifted right 2 units, shrunk vertically by a factor of $\frac{1}{2}$, reflected through the x axis, and shifted up 4 units.



63. The graph of $y = x^2$ has been shifted 2 units right. $y = (x - 2)^2$

65. The graph of $y = x^3$ has been shifted 2 units down. $y = x^3 - 2$

67. The graph of $y = |x|$ has been shrunk vertically by a factor of $\frac{1}{4}$. $y = \frac{1}{4}|x|$ or $y = 0.25|x|$

69. The graph of $y = x^3$ has been reflected through the y axis, (or the x axis). $y = -x^3$

71. The graph of $y = |x|$ has been shifted left 2 units and up 2 units. $y = |x + 2| + 2$

73. The graph of $y = \sqrt{x}$ has been reflected through the x axis and shifted up 4 units. $y = 4 - \sqrt{x}$

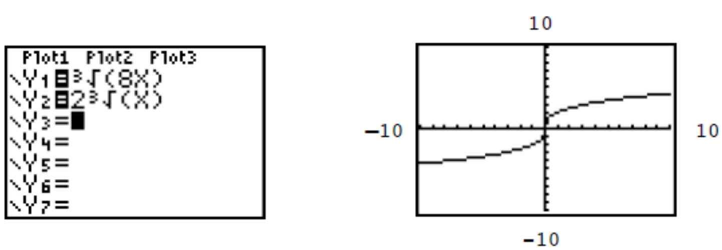
75. The graph of $y = x^2$ has been reflected through the x axis and shifted right 1 unit and up 4 units. $y = 4 - (x - 1)^2$

77. The graph of $y = x^3$ has been shrunk vertically by a factor of $\frac{1}{2}$ and shifted right 3 units and up 1 unit.

$y = \frac{1}{2}(x - 3)^3 + 1$ or $y = 0.5(x - 3)^3 + 1$

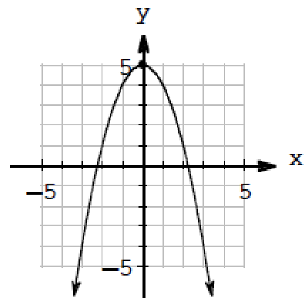
79. (A) The function f is a horizontal shrink of $y = \sqrt[3]{x}$ by a factor of 1/8, while g is a vertical stretch of $y = \sqrt[3]{x}$ by a factor of 2.

(B) The graphs are shown below in a standard window: they are identical.

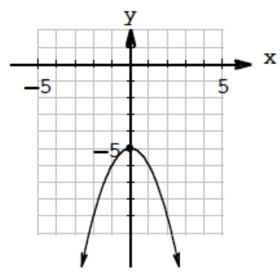


(C) $\sqrt[3]{8x} = \sqrt[3]{8} \cdot \sqrt[3]{x} = 2\sqrt[3]{x}$

81. (A) The graphs are shown below.



(i)



(ii)

The graphs are different, so order is significant when performing multiple transformations.

(B) (i): $y = -\left(x^2 - 5\right)$; (ii): $y = -x^2 - 5$

These functions are different. In the second one, order of operations tells us to first multiply by –1, then subtract 5. In the first, the parentheses indicate that this order should be reversed.

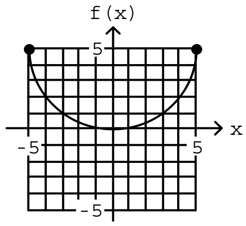
83. $f(x) = x$ is an odd function, since $f(-x) = -x = -f(x)$.
$g(x) = |x|$ is an even function, since $g(-x) = |-x| = |x| = g(x)$.
$h(x) = x^2$ is an even function, since $h(-x) = (-x)^2 = x^2 = h(x)$.
$m(x) = x^3$ is an odd function, since $m(-x) = (-x)^3 = -x^3 = -m(x)$.
$n(x) = \sqrt{x}$ is neither even nor odd. $n(-x) = \sqrt{-x} \neq \sqrt{x}$ and $n(-x) = \sqrt{-x} \neq -\sqrt{x}$.
$p(x) = \sqrt[3]{x}$ is an odd function, since $p(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -p(x)$

85. The graph of $y = f(x - h)$ represents a horizontal shift from the graph of $y = f(x)$. The graph of $y = f(x) + k$ represents a vertical shift from the graph of $y = f(x)$. The graph of $y = f(x - h) + k$ represents both a horizontal and a vertical shift but the order does not matter:
Vertical first then horizontal: $y = f(x) \to y = f(x) + k \to y = f(x - h) + k$
Horizontal first then vertical: $y = f(x) \to y = f(x - h) \to y = f(x - h) + k$
The same result is achieved; reversing the order does not change the result.

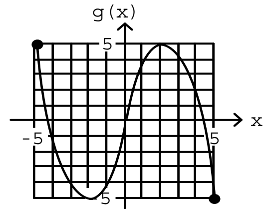
87. Consider the graph of $y = x^2$.
If a vertical shift is performed the equation becomes $y = x^2 + k$.
If a reflection is now performed the equation becomes $y = -(x^2 + k)$ or $y = -x^2 - k$.
If the reflection is performed first the equation becomes $y = -x^2$.
If the vertical shift is now performed the equation becomes $y = -x^2 + k$.
Since $y = -x^2 - k$ and $y = -x^2 + k$ differ (unless $k = 0$), reversing the order changes the result.

89. The graph of $y = f(x - h)$ represents a horizontal shift from the graph of $y = f(x)$. The graph of $y = -f(x)$ represents a reflection of the graph of $y = f(x)$ in the x axis. The graph of $y = -f(x - h)$ represents both a horizontal shift and a reflection but the order does not matter:
Shift first then reflection: $y = f(x) \to y = f(x - h) \to y = -f(x - h)$
Reflection first then shift: $y = f(x) \to y = -f(x) \to y = -f(x - h)$
The same result is achieved; reversing the order does not change the result.

91.



93.



95. (A) $E(x) = \frac{1}{2}[f(x) + f(-x)]$

$E(-x) = \frac{1}{2}[f(-x) + f\{-(-x)\}]$

$= \frac{1}{2}[f(-x) + f(x)]$

$= \frac{1}{2}[f(x) + f(-x)] = E(x).$

Thus, $E(x)$ is even.

(B) $O(x) = \frac{1}{2}[f(x) - f(-x)]$

$O(-x) = \frac{1}{2}[f(-x) - f\{-(-x)\}]$

$= \frac{1}{2}[f(-x) - f(x)]$

$= -\frac{1}{2}[f(x) - f(-x)] = -O(x).$

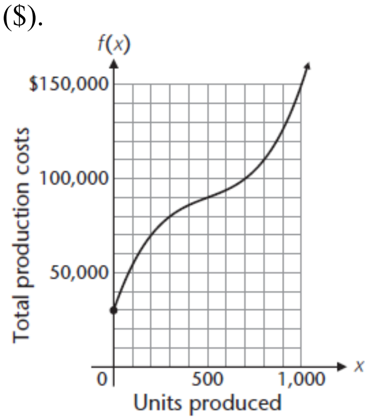
Thus, $O(x)$ is odd.

(C) $E(x) + O(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$

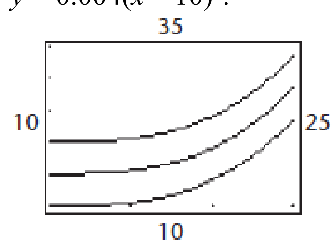
$= \frac{1}{2}f(x) + \frac{1}{2}f(-x) + \frac{1}{2}f(x) - \frac{1}{2}f(-x) = f(x)$

Conclusion: Any function can be written as the sum of two other functions, one even and the other odd.

97. The graph of the function $C(x) = 30{,}000 + f(x)$ is the same as the given graph of the function $f(x)$ shifted up 30,000 units ($).



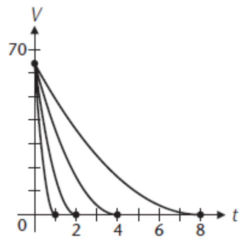
99. $y = 10 + 0.004(x - 10)^3$,
$y = 15 + 0.004(x - 10)^3$,
$y = 20 + 0.004(x - 10)^3$.
Each graph is a vertical shift of the graph of $y = 0.004(x - 10)^3$.



101. $V(t) = \frac{64}{C^2}(C - t)^2 \quad 0 \le t \le C$

t	$C = 1$ $V = 64(1 - t)^2$	$C = 2$ $V = 16(2 - t)^2$	$C = 4$ $V = 4(4 - t)^2$	$C = 8$ $V = (8 - t)^2$
0	64	64	64	64
1	0	16	36	49
2	not defined for $t>1$	0	16	36
4		not defined for $t>2$	0	16
8			not defined for $t > 4$	0

Each graph is a portion of the graph of a horizontal translation followed by a vertical stretch (except for $C = 8$) of the graph of $y = t^2$.



The height of the graph represents the volume of water left in the tank, so we see that for larger values of C, the water stays in the tank longer. We can conclude that larger values of C correspond to a smaller opening.

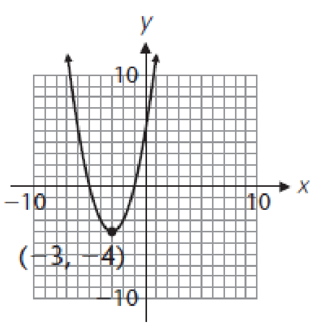
Section 3-4

1. The graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola with vertex at $x = -b/2a$, opening upward if a is positive, and opening downward if a is negative.

3. False. A quadratic function $f(x) = ax^2 + bx + c$ has a maximum if and only if a is negative; if a is positive it has no maximum.

5. If a is positive the graph opens upward and has a minimum at the vertex; if a is negative the graph opens downward and has a maximum at the vertex.

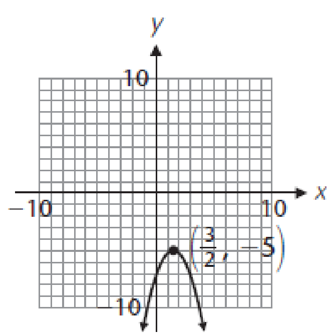
7. $f(x) = (x + 3)^2 - 4$
$= [x - (-3)]^2 - 4$
Vertex: $(-3, -4)$
axis: $x = -3$



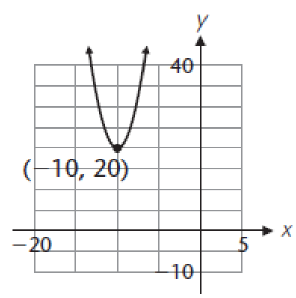
9. $f(x) = -\left(x - \frac{3}{2}\right)^2 - 5$

Vertex: $\left(\frac{3}{2}, -5\right)$

axis: $x = \frac{3}{2}$



11. $f(x) = 2(x + 10)^2 + 20$
$= 2[x - (-10)]^2 + 20$
Vertex: $(-10, 20)$
axis: $x = -10$



13. The graph of $y = x^2$ is shifted right 2 units and up 1 unit.

15. The graph of $y = x^2$ is shifted left 1 unit and reflected in the x axis.

17. The graph of $y = x^2$ is shifted right 2 units and down 3 units.

19. The graph of $y = x^2$ has been shifted to the right 2 units. This is the graph of $y = (x - 2)^2$, corresponding to the function k.

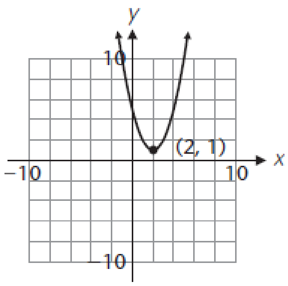
21. The graph of $y = x^2$ has been shifted to the right 2 units and down 3 units. This is the graph of $y = (x - 2)^2 - 3$, corresponding to the function m.

23. The graph of $y = x^2$ has been reflected in the x axis and shifted to the left 1 unit, corresponding to the function h.

25. Begin by grouping the first two terms with parentheses:

$f(x)=\left(x^2-4x\right)+5$	Find the number needed to complete the square
$=\left(x^2-4x+?\right)+5$	$(-4/2)^2=4$; add and subtract 4
$=\left(x^2-4x+4\right)+5-4$	Factor parentheses, combine like terms
$=(x-2)^2+1$	

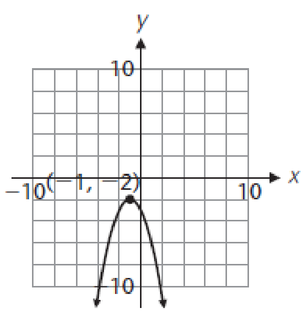
The vertex form is $f(x)=(x-2)^2+1$. The vertex is (2, 1) and the axis is $x = 2$.



27. Begin by grouping the first two terms with parentheses, then factoring –1 out of those two terms so that the coefficient of x^2 is 1:

$h(x)=-1\left(x^2+2x\right)-3$	Find the number needed to complete the square
$=-1\left(x^2+2x+?\right)-3$	$(2/2)^2=1$; add 1 inside the parentheses
$=-1\left(x^2+2x+1\right)-3+?$	We actually added –1(1), so add 1 as well
$=-1\left(x^2+2x+1\right)-3+1$	Factor parentheses, combine like terms
$=-1(x+1)^2-2$	

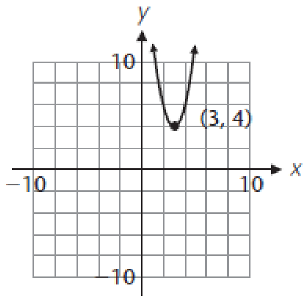
The vertex form is $h(x)=-1(x+1)^2-2$. The vertex is (–1, –2) and the axis is $x = –1$.



29. Begin by grouping the first two terms with parentheses, then factoring 2 out of those two terms so that the coefficient of x^2 is 1:

$m(x)=2\left(x^2-6x\right)+22$	Find the number needed to complete the square
$=2\left(x^2-6x+?\right)+22$	$(-6/2)^2=9$; add 9 inside the parentheses
$=2\left(x^2-6x+9\right)+22+?$	We actually added 2(9), so subtract 18 as well
$=2\left(x^2-6x+9\right)+22-18$	Factor parentheses, combine like terms
$=2(x-3)^2+4$	

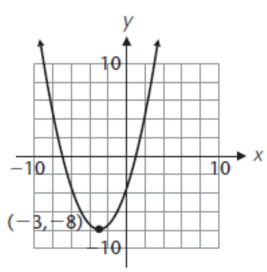
The vertex form is $m(x) = 2(x-3)^2 + 4$. The vertex is (3, 4) and the axis is $x = 3$.



31. Begin by grouping the first two terms with parentheses, then factoring 1/2 out of those two terms so that the coefficient of x^2 is 1:

$f(x) = \frac{1}{2}(x^2 + 6x) - \frac{7}{2}$	Find the number needed to complete the square
$= \frac{1}{2}(x^2 + 6x + ?) - \frac{7}{2}$	$(6/2)^2 = 9$; add 9 inside the parentheses
$= \frac{1}{2}(x^2 + 6x + 9) - \frac{7}{2} + ?$	We actually added $\frac{1}{2}(9)$; subtract $\frac{9}{2}$ as well
$= \frac{1}{2}(x^2 + 6x + 9) - \frac{7}{2} - \frac{9}{2}$	Factor the parentheses, combine like terms
$= \frac{1}{2}(x+3)^2 - 8$	

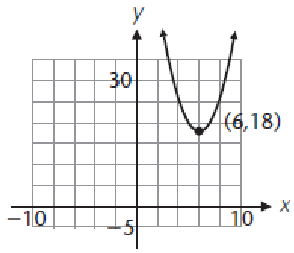
The vertex form is $f(x) = \frac{1}{2}(x+3)^2 - 8$. The vertex is (–3, –8) and the axis is $x = –3$.



33. Begin by grouping the first two terms with parentheses, then factoring 2 out of those two terms so that the coefficient of x^2 is 1:

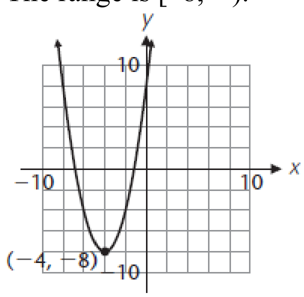
$f(x) = 2(x^2 - 12x) + 90$	Find the number needed to complete the square
$= 2(x^2 - 12x + ?) + 90$	$(-12/2)^2 = 36$; add 36 inside the parentheses
$= 2(x^2 - 12x + 36) + 90 + ?$	We actually added 2(36); subtract 72 as well
$= 2(x^2 - 12x + 36) + 90 - 72$	Factor the parentheses, combine like terms
$= 2(x-6)^2 + 18$	

The vertex form is $f(x)=2(x-6)^2+18$. The vertex is (6, 18) and the axis is $x = 6$.



35. $x=-\dfrac{b}{2a}=-\dfrac{8}{2(1)}=-4;\ f(-4)=(-4)^2+8(-4)+8=16-32+8=-8$

The vertex is (–4,–8). The coefficient of x^2 is positive, so the parabola opens up. The graph is symmetric about its axis, x = –4. It decreases until reaching a minimum at (–4, –8), then increases.
The range is [–8, ∞).

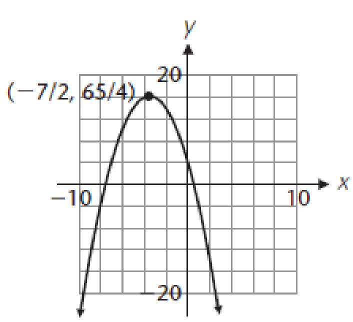


37. $x=-\dfrac{b}{2a}=-\dfrac{-7}{2(-1)}=-\dfrac{7}{2}$

$$f\left(-\frac{7}{2}\right)=-\left(-\frac{7}{2}\right)^2-7\left(-\frac{7}{2}\right)+4=-\frac{49}{4}+\frac{49}{2}+4=-\frac{49}{4}+\frac{98}{4}+\frac{16}{4}=\frac{65}{4}$$

The vertex is $\left(-\dfrac{7}{2},\dfrac{65}{4}\right)$. The coefficient of x^2 is negative, so the parabola opens down. The graph is symmetric about its axis, $x=-\dfrac{7}{2}$. It increases until reaching a maximum at $\left(-\dfrac{7}{2},\dfrac{65}{4}\right)$, then decreases.

The range is $\left(-\infty,\dfrac{65}{4}\right]$.

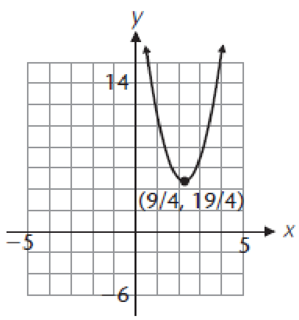


39. $x = -\frac{b}{2a} = -\frac{-18}{2(4)} = \frac{18}{8} = \frac{9}{4}$

$$f\left(\frac{9}{4}\right) = 4\left(\frac{9}{4}\right)^2 - 18\left(\frac{9}{4}\right) + 25 = 4\left(\frac{81}{16}\right) - \frac{162}{4} + 25 = \frac{81}{4} - \frac{162}{4} + \frac{100}{4} = \frac{19}{4}$$

The vertex is $\left(\frac{9}{4}, \frac{19}{4}\right)$. The coefficient of x^2 is positive, so the parabola opens up. The graph is symmetric about its axis, $x = \frac{9}{4}$. It decreases until reaching a minimum at $\left(\frac{9}{4}, \frac{19}{4}\right)$, then increases.

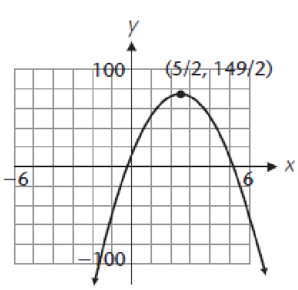
The range is $\left[\frac{19}{4}, \infty\right)$.



41. $x = -\frac{b}{2a} = -\frac{50}{2(-10)} = \frac{50}{20} = \frac{5}{2}$

$$f\left(\frac{5}{2}\right) = -10\left(\frac{5}{2}\right)^2 + 50\left(\frac{5}{2}\right) + 12 = -10\left(\frac{25}{4}\right) + 125 + 12 = -\frac{125}{2} + \frac{250}{2} + \frac{24}{2} = \frac{149}{2}$$

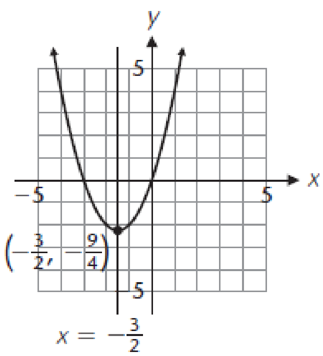
The vertex is $\left(\frac{5}{2}, \frac{149}{2}\right)$. The coefficient of x^2 is negative, so the parabola opens down. The graph is symmetric about its axis, $x = \frac{5}{2}$. It increases until reaching a maximum at $\left(\frac{5}{2}, \frac{149}{2}\right)$, then decreases. The range is $\left(-\infty, \frac{149}{2}\right]$.



43. $x = -\frac{b}{2a} = -\frac{3}{2(1)} = \frac{-3}{2}$

$$f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} = \frac{9}{4} - \frac{18}{4} = -\frac{9}{4}$$

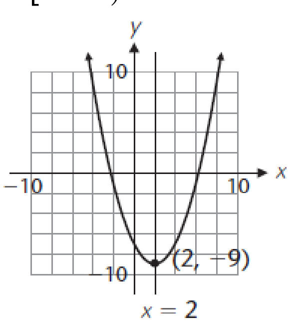
The vertex is $\left(-\frac{3}{2}, -\frac{9}{4}\right)$. The coefficient of x^2 is positive, so the parabola opens up. The graph is symmetric about its axis, $x = -\frac{3}{2}$. It decreases until reaching a minimum at $\left(-\frac{3}{2}, -\frac{9}{4}\right)$, then increases. The range is $\left[-\frac{9}{4}, \infty\right)$.



45. $x = -\frac{b}{2a} = -\frac{-2}{2(0.5)} = 2$

$$f(2) = 0.5(2)^2 - 2(2) - 7 = 0.5(4) - 4 - 7 = 2 - 4 - 7 = -9$$

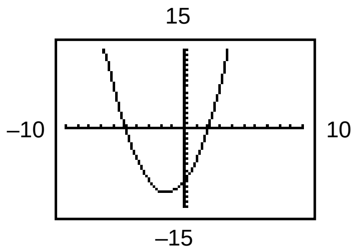
The vertex is $(2, -9)$. The coefficient of x^2 is positive, so the parabola opens up. The graph is symmetric about its axis, $x = 2$. It decreases until reaching a minimum at $(2, -9)$, then increases. The range is $[-9, \infty)$.



47. $x^2 < 10 - 3x$

$$f(x) = x^2 + 3x - 10 < 0$$
$$f(x) = (x + 5)(x - 2) < 0$$

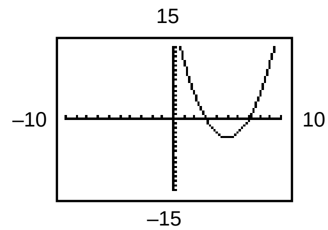
The zeros of f are –5 and 2. Plotting the graph of $f(x)$, we see that $f(x) < 0$ for $-5 < x < 2$, or $(-5, 2)$.



49. $x^2 + 21 > 10x$

$$f(x) = x^2 - 10x + 21 > 0$$
$$f(x) = (x - 3)(x - 7) > 0$$

The zeros of f are 3 and 7. Plotting the graph of $f(x)$, we see that $f(x) > 0$ for $x < 3$ and $x > 7$, or $(-\infty, 3) \cup (7, \infty)$.

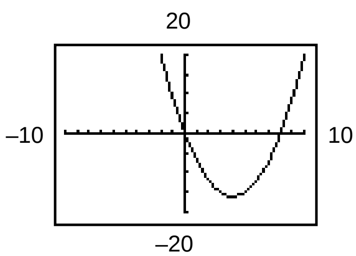


51. $x^2 \le 8x$

$f(x) = x^2 - 8x \le 0$

$f(x) = x(x - 8) \le 0$

The zeros of f are 0 and 8. Plotting the graph of $f(x)$, we see that $f(x) \le 0$ for $0 \le x \le 8$ or $[0, 8]$.

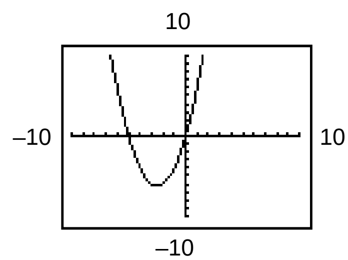


53. $x^2 + 5x \le 0$

$f(x) = x^2 + 5x \le 0$

$f(x) = x(x + 5) \le 0$

The zeros of f are –5 and 0. Plotting the graph of $f(x)$, we see that $f(x) \le 0$ for $-5 \le x \le 0$ or $[-5, 0]$.



55.

$$x^2 + 1 < 2x$$
$$x^2 - 2x + 1 < 0$$
$$(x - 1)^2 < 0$$

Since the square of no real number is negative, these statements are never true for any real number x. No solution; $\varnothing$ is the solution set.

57. $x^2 < 3x - 3$

$x^2 - 3x + 3 < 0$

We attempt to find all real zeros of the polynomial.

$x^2 - 3x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 1, b = -3, c = 3$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{-3}}{2}$$

The polynomial has no real zeros. Hence the statement is either true for all real x or for no real x. To determine which, we choose a test number, say 0.

59. $x^2 - 1 \ge 4x$

$f(x) = x^2 - 4x - 1 \ge 0$

Find all real zeros of $f(x)$.

$x^2 - 4x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 1, b = -4, c = -1$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2}$$

$$x = 2 \pm \sqrt{5} \approx -0.236, 4.236$$

Common Error:
$x \ne 2 \pm \sqrt{20}$

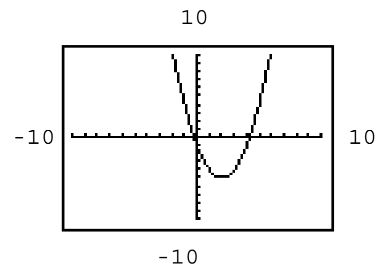
$x^2 < 3x - 3$

$0^2 \overset{?}{<} 3(0) - 3$

$0 \overset{?}{<} -3$ False.

The statement is never true for any real number x. No solution. ∅ is the solution set.

Plotting the graph of $f(x)$ we see that $f(x) \geq 0$ for $x \leq 2 - \sqrt{5}$ and $x \geq 2 + \sqrt{5}$,
or $(-\infty, 2 - \sqrt{5}\,] \cup [2 + \sqrt{5}, \infty)$.



61. The vertex of the parabola is at (1, –4).
Therefore the equation must have form
$y = a(x - 1)^2 - 4$
Since the parabola passes through (3, 4), these coordinates must satisfy the equation
$4 = a(3 - 1)^2 - 4$
$8 = 4a$
$a = 2.$
The equation is
$y = 2(x - 1)^2 - 4$
$y = 2(x^2 - 2x + 1) - 4$
$y = 2x^2 - 4x + 2 - 4$
$y = 2x^2 - 4x - 2$

63. The vertex of the parabola is at (–1, 4).
Therefore the equation must have form
$y = a(x + 1)^2 + 4$
Since the parabola passes through (1, 2), these coordinates must satisfy the equation.
$2 = a(1 + 1)^2 + 4$
$2 = 4a + 4$
$-2 = 4a$
$a = -0.5$
The equation is
$y = -0.5(x + 1)^2 + 4$
$y = -0.5(x^2 + 2x + 1) + 4$
$y = -0.5x^2 - x - 0.5 + 4$
$y = -0.5x^2 - x + 3.5$

65. Notice that the graph does not provide the exact coordinates of the vertex, so we can't tell for certain what they are. We know that $f(-1)$ and $f(3)$ are both zero, so the axis of symmetry is halfway between $x = -1$ and $x = 3$. In other words, the x-coordinate of the vertex is 1; the equation looks like
$f(x) = a(x - 1)^2 + k.$
Plug in $x = -1$:
$f(-1) = a(-1 - 1)^2 + k = a(-2)^2 + k = 4a + k = 0$ (since (–1, 0) is on the graph)
$4a + k = 0$
$k = -4a$
Substitute $-4a$ in for k: $f(x) = a(x - 1)^2 - 4a$
Plug in $x = 0$:
$f(0) = a(0 - 1)^2 - 4a = a - 4a = -3a = -3$ (since (0, –3) is on the graph)
$-3a = -3$
$a = 1$
$f(x) = (x - 1)^2 - 4$ or $f(x) = x^2 - 2x - 3$

67. Notice that the graph does not provide the exact coordinates of the vertex, so we can't tell for certain what they are. We know that $f(-1)$ and $f(5)$ are equal, so the axis of symmetry is halfway between $x = -1$ and $x = 5$. In other words, the x-coordinate of the vertex is 2; the equation looks like
$f(x) = a(x - 2)^2 + k$
Plug in $x = -1$: $f(-1) = a(-1 - 2)^2 + k = a(-3)^2 + k = 9a + k = 0$ (since (–1, 0) is on the graph)
$9a + k = 0$
$k = -9a$

Substitute $-9a$ in for k: $f(x) = a(x-2)^2 - 9a$

Plug in $x = 0$: $f(0) = a(0-2)^2 - 9a = a(-2)^2 - 9a = 4a - 9a = -5a = 2.5$ (since (0, 2.5) is on the graph)

$-5a = 2.5$

$a = -0.5$

$f(x) = -0.5(x-2)^2 + 4.5$ or $f(x) = -0.5x^2 + 2x + 2.5$

69. The vertex of the parabola is at (4, 8).
Therefore the equation must have form
$y = a(x-4)^2 + 8$
Since the x intercept is 6, (6, 0) must satisfy the equation
$0 = a(6-4)^2 + 8$
$0 = 4a + 8$
$a = -2$
The equation is
$y = -2(x-4)^2 + 8$
$y = -2(x^2 - 8x + 16) + 8$
$y = -2x^2 + 16x - 32 + 8$
$y = -2x^2 + 16x - 24$

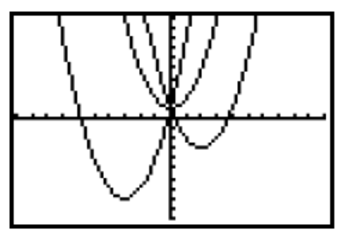
71. The vertex of the parabola is at (–4, 12).
Therefore the equation must have form
$y = a(x+4)^2 + 12$
Since the y intercept is 4, (0, 4) must satisfy the equation.
$4 = a(0+4)^2 + 12$
$4 = 16a + 12$
$-8 = 16a$
$a = -0.5$
The equation is
$y = -0.5(x+4)^2 + 12$
$y = -0.5(x^2 + 8x + 16) + 12$
$y = -0.5x^2 - 4x - 8 + 12 = -0.5x^2 - 4x + 4$

73. The vertex of the parabola is at (–5, –25).
Therefore the equation must have form
$y = a(x+5)^2 - 25$
Since the parabola passes through (–2, 20), these coordinates must satisfy the equation.
$20 = a(-2+5)^2 - 25$
$20 = 9a - 25$
$45 = 9a$
$a = 5$

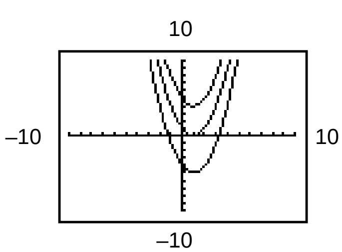
The equation is $y = 5(x+5)^2 - 25$
$y = 5(x^2 + 10x + 25) - 25$
$y = 5x^2 + 50x + 125 - 25$
$y = 5x^2 + 50x + 100$

75. $a(x-h)^2 + k = a(x^2 - 2xh + h^2) + k$
$= ax^2 - 2axh + ah^2 + k$
$= ax^2 - (2ah)x + (ah^2 + k)$

77. The graphs shown are $f(x) = x^2 + 6x + 1$,
$f(x) = x^2 + 1$, and $f(x) = x^2 - 4x + 1$.
These correspond to $f(x) = x^2 + kx + 1$ with $k = 6$, 0, and –4 respectively.
Note that all have the same shape but a different vertex. In fact, all three are translations of the graph $y = x^2$.



79. The graphs of $f(x) = (x-1)^2$,
$g(x) = (x-1)^2 + 4$, and
$h(x) = (x-1)^2 - 5$ are shown.



It is clear that $f(x)$ has one x intercept (at $x = 1$), $g(x)$ has no x intercepts and $h(x)$ has two x intercepts.
In general, for $a > 0$, the graph of $f(x) = a(x-h)^2 + k$ can be expected to have no intercepts for $k > 0$, one intercept at $x = h$ for $k = 0$, and two intercepts for $k < 0$.

81. Let one number = x. Then the other number is $x - 30$. The product is a function of x given by $f(x) = x(x - 30) = x^2 - 30x$. This is a quadratic function with $a > 0$, therefore it has a minimum value at the vertex of its graph (a parabola). Completing the square yields

$f(x) = x^2 - 30x$ $\qquad \frac{1}{2}(-30) = -15; \quad (-15)^2 = 225$

$= x^2 - 30x + 225 - 225$
$= (x - 15)^2 - 225$

Thus the minimum product is –225, when $x = 15$ and $x - 30 = -15$. There is no "highest point" on this parabola and no maximum product.

83. Find the vertex, using $a = -1.2$ and $b = 62.5$:

$$x = -\frac{b}{2a} = -\frac{62.5}{2(-1.2)} \approx 26; \quad P(26) = -1.2(26)^2 + 62.5(26) - 491 = 322.8$$

The company should hire 26 employees to make a maximum profit of $322,800.

85. (A) Find the first coordinate of the vertex, using $a = -0.19$ and $b = 1.2$:

$$x = -\frac{b}{2a} = -\frac{1.2}{2(-0.19)} = 3.2$$

The maximum box office revenue was three years after 2002, which is 2005.

(B) The function specifies yearly totals for revenue, so the domain should be restricted to whole numbers. The exact vertex occurs at $x = 3.2$, so we needed to round down to 3.

87. (A) Since the four sides needing fencing are x, y, $x + 50$, and y, we have $x + y + (x + 50) + y = 250$.
Solving for y, we get
$y = 100 - x$.
Therefore the area
$A(x) = (x + 50)y = (x + 50)(100 - x) = -x^2 + 50x + 5000$.

(Since both x and $100 - x$ must be nonnegative, the domain of $A(x)$ is $0 \le x \le 100$.)

(B) This is a quadratic function with $a < 0$, so it has a maximum value at the vertex:

$$x = -\frac{b}{2a} = -\frac{50}{2(-1)} = 25; \quad A(25) = -(25)^2 + 50(25) + 5,000 = -625 + 1,250 + 5,000 = 5,625$$

The maximum area is 5,625 square feet when $x = 25$.
(C) When $x = 25$, $y = 100 - 25 = 75$. The dimensions of the corral are then $x + 50$ by y, or 75 ft by 75 ft.

89. According to Example 7, the function describing the height of the sandbag is $h(t) = 10{,}000 - 16t^2$ (since the initial height is 10,000 feet). We want to know when it reaches ground level, so plug in zero for $h(t)$, then solve for t.

$0 = 10{,}000 - 16t^2$
$16t^2 = 10{,}000$
$t^2 = 625$
$t = 25$

It hits the ground 25 seconds after it's dropped.

91. According to Example 7, the function describing the height of the diver is $h(t) = h_0 - 16t^2$, where h_0 is the initial height in feet. (This initial height is the height of the cliff.) We know that $h(2.5) = 0$ since it takes 2.5 seconds to reach the water; we plug in 2.5 for t and 0 for $h(t)$, which allows us to solve for h_0.

$0 = h_0 - 16(2.5)^2$
$0 = h_0 - 100$
$100 = h_0$

The cliff is 100 feet high.

93. (A) Since $d(t)$ is a quadratic function with maximum value 484 when $t = 5.5$, an equation for $d(t)$ must be of the form

$$d(t) = a(t - 5.5)^2 + 484$$

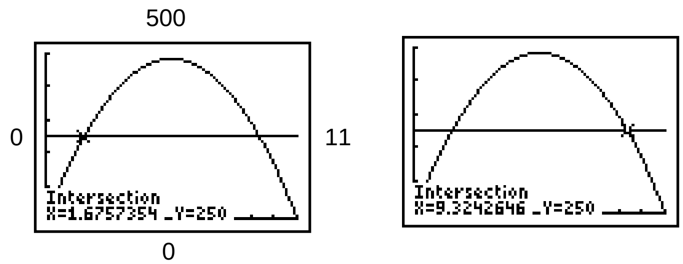
Since $d(0) = 0$,

$$0 = d(0) = a(0 - 5.5)^2 + 484$$
$$0 = 30.25a + 484$$
$$a = -16$$

Hence
$$d(t) = -16(t - 5.5)^2 + 484$$
$$= -16(t^2 - 11t + 30.25) + 484$$
$$= -16t^2 + 176t - 484 + 484$$
$$= -16t^2 + 176t$$

Since the graph of d must be symmetric with respect to $t = 5.5$, and $d(0) = 0$, $d(11)$ must also equal 0. The distance above the ground will be nonnegative only for values of t between 0 and 11, hence the domain of the function is $0 \le t \le 11$.

(B) Solve $250 = -16t^2 + 176t$ by graphing Y1 = 250 and Y2 = $-16x^2 + 176x$ and applying a built-in routine.



From the graphs $t = 1.68$ sec and $t = 9.32$ sec, to two decimal places.

95. (A) If coordinates are chosen with origin at the center of the base, the parabola is the graph of a quadratic function $h(x)$ with maximum value 14 when $x = 0$. The equation must be of form $h(x) = ax^2 + 14$

Since $h(10) = 0$ (why?)

$$0 = h(10) = a(10)^2 + 14$$
$$0 = 100a + 14$$
$$a = -0.14$$

Hence $h(x) = -0.14x^2 + 14 \quad -10 \le x \le 10$

(B) Suppose the truck were to drive so as to maximize its clearance, that is, in the center of the roadway. Then half its width, or 4 ft, would extend to each side. But if $x = 4$, $h(x) = -0.14(4)^2 + 14 = 11.76$ ft. The arch is only 11.76 feet high, but the truck is 12 feet high. The truck cannot pass through the arch.

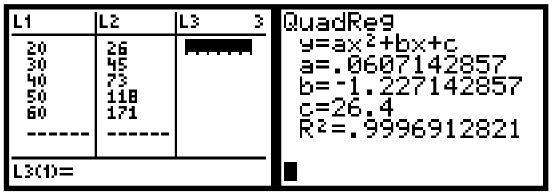
(C) From part (B), if $x = 4$, $h(x) = 11.76$ ft is the height of the tallest truck.

(D) Find x so that $h(x) = 12$. Solve the equation

$$12 = 0.14x^2 + 14$$
$$-2 = -0.14x^2 \qquad \text{(The negative solution doesn't make sense)}$$
$$x = \sqrt{\frac{2}{0.14}} = 3.78$$

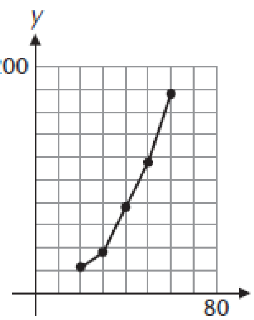
The width of the truck is at most $2x = 2(3.78) = 7.56$ feet

97. (A) The entered data is shown here along with the results of the quadratic regression calculation.



The quadratic model for the skid mark length is $L(x) = 0.061x^2 - 1.2x + 26$

(B)



(C) Solve $150 = 0.06x^2 - 1.2x + 26$

$$0 = 0.06x^2 - 1.2x - 124$$

$$x = \frac{-(-1.2) \pm \sqrt{(-1.2)^2 - 4(0.06)(-124)}}{2(0.06)}$$

$$x = \frac{1.2 \pm 5.59}{0.12}$$

$x = 57$ mph

(the negative answer doesn't make sense)

99. (A) Beer consumption in 1960 is given as 0.99.

Solve $0.99 = -0.0006x^2 + 0.03x + 1$

$$0 = -0.0006x^2 + 0.03x + 0.01$$

$0 = -6x^2 + 300x + 100$ (for convenience)

$$x = \frac{-300 \pm \sqrt{(300)^2 - 4(-6)(100)}}{2(-6)}$$

$x = 50$ (discarding the negative answer)

This represents the year 2010.

(B) Substitute $x = 45$ to obtain

$$B(45) = -0.0006(45)^2 + 0.03(45) + 1$$
$$= 1.14 \text{ gallons}$$

101. A profit will result if $C(x) < R(x)$.

Solve $245 + 1.6x < 10x - 0.04x^2$

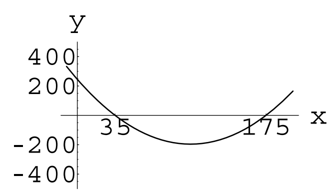
$$0.04x^2 - 8.4x + 245 < 0$$

$$f(x) = 0.04x^2 - 8.4x + 245 < 0$$

Find the zeros of $f(x)$.

$$x = \frac{-(-8.4) \pm \sqrt{(-8.4)^2 - 4(0.04)(245)}}{2(0.04)}$$

$x = 35$ or 175



Plotting the graph of $f(x)$ we see that $f(x) < 0$ for $35 < x < 175$. The break-even points are therefore $(35, R(35)) = (35, 301)$ and $(175, R(175)) = (175, 525)$.

103. The revenue function is $R(x) = xd(x) = x(9.3 - 0.15x)$ or $R(x) = -0.15x^2 + 9.3x$.

$R(x)$ has a maximum value at the vertex of this parabola, which is given by

$$x = -\frac{b}{2a} = -\frac{9.3}{2(-0.15)} = 31$$

Then $p = d(31) = 9.3 - 0.15(31) = \$4.65$ is the price which maximizes the revenue.

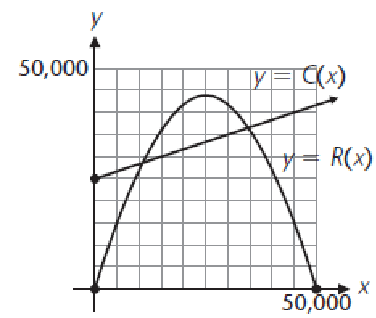
105. (A) The revenue function is $R(x) = xd(x) = x(3.5 - 0.00007x)$
$= 3.5x - 0.00007x^2$.

The domain is given by $x(3.5 - 0.00007x) \geq 0$ or $0 \leq x \leq 50{,}000$.
The cost function $C(x)$ is given by
$C(x)$ = Fixed Cost + Variable Cost = $24{,}500 + 0.35x$.
The domain is given by $x \geq 0$ or $[0, \infty)$.
The company will break even when $R(x) = C(x)$.
Solve

$$3.5x - 0.00007x^2 = 24{,}5000 + 0.35x$$
$$0 = 0.00007x^2 - 3.15x + 24{,}500$$
$$x = \frac{-(-3.15) \pm \sqrt{(-3.15)^2 - 4(0.00007)(24{,}500)}}{2(0.00007)}$$
$$x = 10{,}000 \text{ or } 35{,}000.$$

The company will break even for sales of 10,000 or 35,000 gallons.

(B)



(C) The company makes a profit for those sales levels for which the graph of the revenue function is above the graph of the cost function, that is, if the sales are between 10,000 and 35,000 gallons. The company suffers a loss for those sales levels for which the graph of the revenue function is below the graph of the cost function, that is, if the sales are between 0 and 10,000 gallons or between 35,000 and 50,000 gallons.

(D) The profit function is given by $P(x) = R(x) - C(x)$. Thus
$P(x) = (3.5x - 0.00007x^2) - (24{,}500 + 0.35x)$
$= -0.00007x^2 + 3.15x - 24{,}500$
The maximum value of this function occurs at the vertex of its parabola graph. This is given by the formulas (h, k) where

$$h = -\frac{b}{2a} = -\frac{3.15}{2(-0.00007)} = 22{,}500$$

and

$$k = C - \frac{b^2}{4a} = -24{,}500 - \frac{3.15^2}{4(-0.00007)} = 10{,}937.5$$

That is, the maximum profit is \$10,937.50 when 22,500 gallons are sold. Substitute $x = 22{,}500$ to find $p = d(22{,}500) = 3.5 - 0.00007(22{,}500) = \$1.92$ per gallon.

Section 3-5

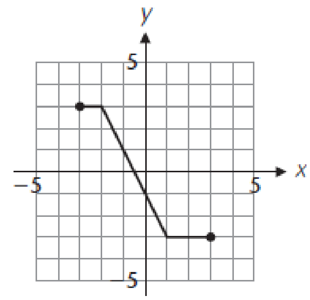
1. The sum of two functions is found by adding the expressions for the two functions and finding the intersection of their domains.

3. Answers will vary.

5. The simplification may obscure values that are not in the domain of one of the functions.

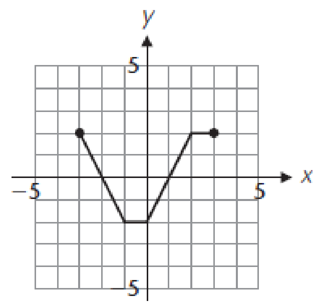
7. Construct a table of values of $f(x)$ and $g(x)$ from the graph, then add to obtain $(f+g)(x)$.

x	–3	–2	–1	0	1	2	3
$f(x)$	1	0	–1	–2	–3	–2	–1
$g(x)$	2	3	2	1	0	–1	–2
$(f+g)(x)$	3	3	1	–1	–3	–3	–3



9. Construct a table of values of $f(x)$ and $g(x)$ from the graph, then multiply to obtain $(fg)(x)$.

x	–3	–2	–1	0	1	2	3
$f(x)$	1	0	–1	–2	–3	–2	–1
$g(x)$	2	3	2	1	0	–1	–2
$(fg)(x)$	2	0	–2	–2	0	2	2



11. $(f \circ g)(-1) = f[g(-1)]$. From the graph of g, $g(-1) = 2$. From the graph of f, $f[g(-1)] = f(2) = -2$.

13. $(g \circ f)(-2) = g[f(-2)]$. From the graph of f, $f(-2) = 0$. From the graph of g, $g[f(-2)] = g(0) = 1$.

15. From the graph of g, $g(1) = 0$. From the graph of f, $f[g(1)] = f(0) = -2$.

17. From the graph of f, $f(2) = -2$. From the graph of g, $g[f(2)] = g(-2) = 3$.

19. $(f+g)(-3) = f(-3) + g(-3) = [2-(-3)] + \sqrt{3-(-3)} = 5 + \sqrt{6}$

21. $(fg)(-1) = f(-1)g(-1)$
$= [2-(-1)]\sqrt{3-(-1)}$
$= 3\sqrt{4} = 3 \cdot 2 = 6$

23. $f \circ g(-2) = f(g(-2))$
$= f(\sqrt{3-(-2)})$
$= f(\sqrt{5}) = 2 - \sqrt{5}$

25. $g \circ f(1) = g(f(1))$
$= g(2-1)$
$= g(1)$
$= \sqrt{3-1} = \sqrt{2}$

27. Using values from the table, $g(-7) = 4$, so $(f \circ g)(-7) = f(g(-7)) = f(4) = 3$. Similarly, $(f \circ g)(0) = f(g(0)) = f(-2) = 9$, and $(f \circ g)(4) = f(g(4)) = f(6) = -10$.

29.
$(f+g)(x) = f(x) + g(x) = 4x + x + 1 = 5x + 1$ Domain: $(-\infty, \infty)$
$(f-g)(x) = f(x) - g(x) = 4x - (x+1) = 3x - 1$ Domain: $(-\infty, \infty)$
$(fg)(x) = f(x)g(x) = 4x(x+1) = 4x^2 + 4x$ Domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{4x}{x+1}$ Domain: $\{x \mid x \neq -1\}$, or $(-\infty, -1) \cup (-1, \infty)$

Common Error:
$f(x) - g(x) \neq 4x - x + 1$. The parentheses are necessary.

31. $(f+g)(x) = f(x) + g(x) = 2x^2 + x^2 + 1 = 3x^2 + 1$ Domain: $(-\infty, \infty)$

$(f-g)(x) = f(x) - g(x) = 2x^2 - (x^2 + 1) = x^2 - 1$ Domain: $(-\infty, \infty)$

$(fg)(x) = f(x)g(x) = 2x^2(x^2 + 1) = 2x^4 + 2x^2$ Domain: $(-\infty, \infty)$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x^2}{x^2+1}$ Domain: $(-\infty, \infty)$ (since $g(x)$ is never 0.)

33. $(f+g)(x) = f(x) + g(x) = 3x + 5 + x^2 - 1$
$= x^2 + 3x + 4$ Domain: $(-\infty, \infty)$

$(f-g)(x) = f(x) - g(x) = 3x + 5 - (x^2 - 1)$
$= 3x + 5 - x^2 + 1 = -x^2 + 3x + 6$ Domain: $(-\infty, \infty)$

$(fg)(x) = f(x)g(x) = (3x + 5)(x^2 - 1)$
$= 3x^3 - 3x + 5x^2 - 5 = 3x^3 + 5x^2 - 3x - 5$ Domain: $(-\infty, \infty)$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x+5}{x^2-1}$ Domain: $\{x \mid x \neq \pm 1\}$, or $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

35. $(f+g)(x) = f(x) + g(x) = \sqrt{2-x} + \sqrt{x+3}$

$(f-g)(x) = f(x) - g(x) = \sqrt{2-x} - \sqrt{x+3}$

$(fg)(x) = f(x)g(x) = \sqrt{2-x}\,\sqrt{x+3} = \sqrt{(2-x)(3+x)} = \sqrt{6-x-x^2}$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{2-x}}{\sqrt{x+3}} = \sqrt{\dfrac{2-x}{x+3}}$

The domains of f and g are:
Domain of $f = \{x \mid 2 - x \geq 0\} = (-\infty, 2]$ Domain of $g = \{x \mid x + 3 \geq 0\} = [-3, \infty)$
The intersection of these domains is $[-3, 2]$. This is the domain of the functions of $f + g$, $f - g$, and fg.
Since $g(-3) = 0$, $x = -3$ must be excluded from the domain of $\dfrac{f}{g}$, so its domain is $(-3, 2]$.

37. $(f+g)(x) = f(x) + g(x) = \sqrt{x} + 2 + \sqrt{x} - 4 = 2\sqrt{x} - 2$

$(f-g)(x) = f(x) - g(x) = \sqrt{x} + 2 - (\sqrt{x} - 4) = \sqrt{x} + 2 - \sqrt{x} + 4 = 6$

$(fg)(x) = f(x)g(x) = (\sqrt{x} + 2)(\sqrt{x} - 4) = x - 2\sqrt{x} - 8$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}+2}{\sqrt{x}-4}$

The domains of f and g are both $\{x \mid x \geq 0\} = [0, \infty)$ This is the domain of $f + g$, $f - g$, and fg.
We note that in the domain of $\dfrac{f}{g}$, $g(x) \neq 0$. Thus $\sqrt{x} - 4 \neq 0$. To solve this, we solve

$$\sqrt{x} - 4 = 0$$
$$\sqrt{x} = 4$$
$$x = 16$$

Common Error: $x \neq \sqrt{4}$

Hence, 16 must be excluded from $\{x \mid x \geq 0\}$ to find the domain of $\dfrac{f}{g}$.

Domain of $\dfrac{f}{g} = \{x \mid x \geq 0, x \neq 16\} = [0, 16) \cup (16, \infty)$.

39. $(f+g)(x) = f(x) + g(x) = \sqrt{x^2+x-6} + \sqrt{7+6x-x^2}$

$(f-g)(x) = f(x) - g(x) = \sqrt{x^2+x-6} - \sqrt{7+6x-x^2}$

$(fg)(x) = f(x)g(x) = \sqrt{x^2+x-6}\,\sqrt{7+6x-x^2} = \sqrt{-x^4+5x^3+19x^2-29x-42}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x^2+x-6}}{\sqrt{7+6x-x^2}} = \sqrt{\frac{x^2+x-6}{7+6x-x^2}}$

The domains of f and g are:
Domain of $f = \{x \mid x^2 + x - 6 \ge 0\} = \{x \mid (x+3)(x-2) \ge 0\} = (-\infty, -3] \cup [2, \infty)$
Domain of $g = \{x \mid 7 + 6x - x^2 \ge 0\} = \{x \mid (7-x)(1+x) \ge 0\} = [-1, 7]$
The intersection of these domains is [2, 7]. This is the domain of the functions $f + g$, $f - g$, and fg.
Since $g(x) = 7 + 6x - x^2 = (7 - x)(1 + x)$, $g(7) = 0$ and $g(-1) = 0$, hence 7 must be excluded from the domain of $\frac{f}{g}$, so its domain is [2, 7).

41. $(f+g)(x) = f(x) + g(x) = x + \frac{1}{x} + x - \frac{1}{x} = 2x$

Common Error:
Domain is not $(-\infty, \infty)$. See below.

$(f-g)(x) = f(x) - g(x) = x + \frac{1}{x} - \left(x - \frac{1}{x}\right) = \frac{2}{x}$

$(fg)(x) = f(x)g(x) = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = x^2 - \frac{1}{x^2}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x + \frac{1}{x}}{x - \frac{1}{x}} = \frac{x^2+1}{x^2-1}$

The domains of f and g are both $\{x \mid x \ne 0\} = (-\infty, 0) \cup (0, \infty)$.

This is therefore the domain of $f + g$, $f - g$, and fg. To find the domain of $\frac{f}{g}$, we must exclude from this domain the set of values of x for which $g(x) = 0$.

$$x - \frac{1}{x} = 0$$
$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = -1, 1$$

Hence, the domain of $\frac{f}{g}$ is $\{x \mid x \ne 0, -1, \text{ or } 1\}$ or $(-\infty,-1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$.

43. $(f \circ g)(x) = f[g(x)] = f(x^2 - x + 1) = (x^2 - x + 1)^3$ Domain: $(-\infty, \infty)$
$(g \circ f)(x) = g[f(x)] = g(x^3) = (x^3)^2 - x^3 + 1 = x^6 - x^3 + 1$ Domain: $(-\infty, \infty)$

45. $(f \circ g)(x) = f[g(x)] = f(2x + 3) = |2x + 3 + 1| = |2x + 4|$ Domain: $(-\infty, \infty)$
$(g \circ f)(x) = g[f(x)] = g(|x + 1|) = 2|x + 1| + 3$ Domain: $(-\infty, \infty)$

47. $(f \circ g)(x) = f[g(x)] = f(2x^3 + 4) = (2x^3 + 4)^{1/3}$ Domain: $(-\infty, \infty)$
$(g \circ f)(x) = g[f(x)] = g(x^{1/3}) = 2(x^{1/3})^3 + 4 = 2x + 4$ Domain: $(-\infty, \infty)$

49. $(f \circ g)(x) = f[g(x)] = f(x - 4) = \sqrt{x-4}$ Domain: $\{x \mid x \ge 4\}$ or $[4, \infty)$
$(g \circ f)(x) = g[f(x)] = g(\sqrt{x}) = \sqrt{x} - 4$ Domain: $\{x \mid x \ge 0\}$ or $[0, \infty)$

51. $(f \circ g)(x) = f[g(x)] = f\left(\frac{1}{x}\right) = \frac{1}{x} + 2$ Domain: $\{x \mid x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$

$(g \circ f)(x) = g[f(x)] = g(x + 2) = \frac{1}{x+2}$ Domain: $\{x \mid x \neq -2\}$ or $(-\infty, -2) \cup (-2, \infty)$

53. $(f \circ g)(x) = f[g(x)] = f(x^2) = \sqrt{4-x^2}$

The domain of f is $(-\infty, 4]$. The domain of g is all real numbers. Hence the domain of $f \circ g$ is the set of those real numbers x for which $g(x)$ is in $(-\infty, 4]$, that is, for which $x^2 \le 4$, or $-2 \le x \le 2$.

Domain of $f \circ g = \{x \mid -2 \le x \le 2\} = [-2, 2]$

$(g \circ f)(x) = g[f(x)] = g(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4 - x$

The domain of $g \circ f$ is the set of those numbers x in $(-\infty, 4]$ for which $f(x)$ is in $(-\infty, \infty)$, that is, $(-\infty, 4]$.

55. $(f \circ g)(x) = f[g(x)] = f\left(\frac{x}{x-2}\right) = \frac{\frac{x}{x-2}+5}{\frac{x}{x-2}} = \frac{x+5(x-2)}{x} = \frac{x+5x-10}{x} = \frac{6x-10}{x}$

The domain of f is $\{x \mid x \neq 0\}$. The domain of g is $\{x \mid x \neq 2\}$. Hence the domain of $f \circ g$ is the set of those numbers in $\{x \mid x \neq 2\}$ for which $g(x)$ is in $\{x \mid x \neq 0\}$. Thus we must exclude from $\{x \mid x \neq 2\}$ those numbers x for which $\frac{x}{x-2} = 0$, or $x = 0$. Hence the domain of $f \circ g$ is $\{x \mid x \neq 0, x \neq 2\}$, or $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$.

$(g \circ f)(x) = g[f(x)] = g\left(\frac{x+5}{x}\right) = \frac{\frac{x+5}{x}}{\frac{x+5}{x}-2} = \frac{x+5}{x+5-2x} = \frac{x+5}{5-x}$

The domain of $g \circ f$ is the set of those numbers in $\{x \mid x \neq 0\}$ for which $f(x)$ is in $\{x \mid x \neq 2\}$. Thus we must exclude from $\{x \mid x \neq 0\}$ those numbers x for which $\frac{x+5}{x} = 2$, or $x + 5 = 2x$, or $x = 5$. Hence the domain of $g \circ f$ is $\{x \mid x \neq 0, x \neq 5\}$ or $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.

57. $(f \circ g)(x) = f[g(x)] = f\left(\frac{1}{x-2}\right) = \frac{2\left(\frac{1}{x-2}\right)+1}{\frac{1}{x-2}} = \frac{2+x-2}{1} = x$

The domain of f is $\{x \mid x \neq 0\}$. The domain of g is $\{x \mid x \neq 2\}$. Hence the domain of $f \circ g$ is the set of those numbers in $\{x \mid x \neq 2\}$ for which $g(x)$ is in $\{x \mid x \neq 0\}$. Thus we must exclude from $\{x \mid x \neq 2\}$ those numbers x for which $\frac{1}{x-2} = 0$; however, there are none. Hence the domain of $f \circ g$ is $\{x \mid x \neq 2\}$ or $(-\infty, 2) \cup (2, \infty)$.

$(g \circ f)(x) = g[f(x)] = g\left(\frac{2x+1}{x}\right) = \frac{1}{\frac{2x+1}{x}-2} = \frac{x}{2x+1-2x} = \frac{x}{1} = x$

The domain of $g \circ f$ is the set of those numbers in $\{x \mid x \neq 0\}$ for which $f(x)$ is in $\{x \mid x \neq 2\}$. Thus we must exclude from $\{x \mid x \neq 0\}$ those numbers x for which $\frac{2x+1}{x} = 2$; however, there are none. Hence the domain of $g \circ f$ is $\{x \mid x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$.

59. $(f \circ g)(x) = f[g(x)] = f(\sqrt{9+x^2}) = \sqrt{25-(\sqrt{9+x^2})^2} = \sqrt{25-(9+x^2)} = \sqrt{16-x^2}$

The domain of f is [–5, 5]. The domain of g is $(-\infty, \infty)$. Hence the domain of $f \circ g$ is the set of those real numbers x for which $g(x)$ is in [–5, 5], that is, $\sqrt{9+x^2} \le 5$, or $9 + x^2 \le 25$, or $x^2 \le 16$, or $-4 \le x \le 4$. Hence the domain of $f \circ g$ is $\{x \mid -4 \le x \le 4\}$ or [–4, 4].

$(g \circ f)(x) = g[f(x)] = g(\sqrt{25-x^2}) = \sqrt{9+(\sqrt{25-x^2})^2} = \sqrt{9+25-x^2} = \sqrt{34-x^2}$

The domain of $g \circ f$ is the set of those numbers x in [–5, 5] for which $g(x)$ is real. Since $g(x)$ is real for all x, the domain of $g \circ f$ is [–5, 5].

Common Error: The domain of $g \circ f$ is not evident from the final form $\sqrt{34-x^2}$. It is not $\left[-\sqrt{34}, \sqrt{34}\right]$

In Problems #61 through 63, f and g are linear functions. f has slope –2 and y intercept 2, so f(x) = –2x + 2. g has slope 1 and y intercept –2, so g(x) = x – 2.

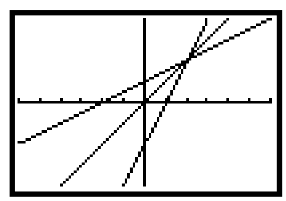
61. $(f + g)(x) = f(x) + g(x) = (-2x + 2) + (x - 2) = -x$.
The graph of $f + g$ is a straight line with slope –1 passing through the origin. This corresponds to graph (d).

63. $(g - f)(x) = g(x) - f(x) = (x - 2) - (-2x + 2) = x - 2 + 2x - 2 = 3x - 4$.
The graph of $g - f$ is a straight line with slope 3 and y intercept –4. This corresponds to graph (a).

65. $(f \circ g)(x) = f[g(x)] = f(2x - 2) = \frac{1}{2}(2x - 2) + 1 = x - 1 + 1 = x$

$(g \circ f)(x) = g[f(x)] = g\left(\frac{1}{2}x+1\right) = 2\left(\frac{1}{2}x+1\right) - 2 = x + 2 - 2 = x$

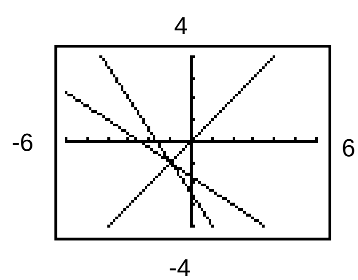
Graphing f, g, $f \circ g$, and $g \circ f$, we obtain the graph at the right.
The graphs of f and g are reflections of each other in the line $y = x$, which is the graph of $f \circ g$ and $g \circ f$.



67. $(f \circ g)(x) = f[g(x)] = f\left(-\frac{3}{2}x - \frac{5}{2}\right) = -\frac{2}{3}\left(-\frac{3}{2}x - \frac{5}{2}\right) - \frac{5}{3} = x + \frac{5}{3} - \frac{5}{3} = x$

$(g \circ f)(x) = g[f(x)] = g\left(-\frac{2}{3}x - \frac{5}{3}\right) = -\frac{3}{2}\left(-\frac{2}{3}x - \frac{5}{3}\right) - \frac{5}{2} = x + \frac{5}{2} - \frac{5}{2} = x$

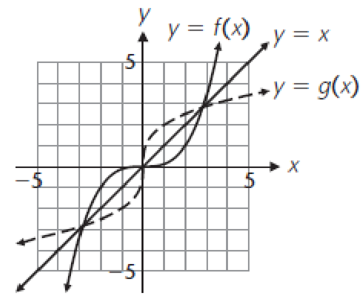
Graphing f, g, $f \circ g$, and $g \circ f$, we obtain the graph at the right.
The graphs of f and g are reflections of each other in the line $y = x$, which is the graph of $f \circ g$ and $g \circ f$.



69. $f \circ g(x) = f[g(x)] = f(2\sqrt[3]{x}\,) = \dfrac{\left(2\sqrt[3]{x}\right)^3}{8} = \dfrac{8x}{8} = x$

$g \circ f(x) = g[f(x)] = g\left(\dfrac{x^3}{8}\right) = 2\sqrt[3]{\dfrac{x^3}{8}} = 2\left(\dfrac{x}{2}\right) = x$

Graphing f, g, $f \circ g$, and $g \circ f$, we obtain the graph at the right.
The graphs of f and g are reflections of each other in the line $y = x$, which is the graph of $f \circ g$ and $g \circ f$.

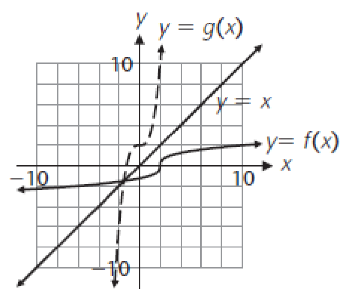


71. $f \circ g(x) = f[g(x)] = f(x^3 + 2) = \sqrt[3]{x^3 + 2 - 2} = \sqrt[3]{x^3} = x$

$g \circ f(x) = g[f(x)] = g(\sqrt[3]{x-2}\,) = (\sqrt[3]{x-2}\,)^3 + 2 = x - 2 + 2 = x$

Graphing f, g, $f \circ g$, and $g \circ f$, we obtain the graph at the right.

The graphs of $f \circ g$ and $g \circ f$ are reflections of each other in the line $y = x$, which is the graph of $f \circ g$ and $g \circ f$.



73. If we let $g(x) = 2x - 7$, then

$h(x) = [g(x)]^4$

Now if we let $f(x) = x^4$, we have

$h(x) = [g(x)]^4 = f[g(x)] = (f \circ g)(x)$

75. If we let $g(x) = 4 + 2x$, then

$h(x) = \sqrt{g(x)}$

Now if we let $f(x) = x^{1/2}$, we have

$h(x) = \sqrt{g(x)} = [g(x)]^{1/2} = f[g(x)] = (f \circ g)(x)$.

77. If we let $f(x) = x^7$, then

$h(x) = 3f(x) - 5$

Now if we let $g(x) = 3x - 5$, we have

$h(x) = 3f(x) - 5 = g[f(x)] = (g \circ f)(x)$

79. If we let $f(x) = x^{-1/2}$, then

$h(x) = 4f(x) + 3$

Now if we let $g(x) = 4x + 3$, we have

$h(x) = 4f(x) + 3 = g[f(x)] = (g \circ f)(x)$

81. fg and gf are identical, since $(fg)(x) = f(x)g(x) = g(x)f(x) = (gf)(x)$ by the commutative law for multiplication of real numbers

83. Yes, the function $g(x) = x$ satisfies these conditions.

$(f \circ g)(x) = f(g(x)) = f(x)$, so $f \circ g = f$

$(g \circ f)(x) = g(f(x)) = f(x)$, so $g \circ f = f$

85. $(f + g)(x) = f(x) + g(x) = x + \dfrac{1}{x} + x - \dfrac{1}{x} = 2x$

$(f - g)(x) = f(x) - g(x) = x + \dfrac{1}{x} - \left(x - \dfrac{1}{x}\right) = \dfrac{2}{x}$

$(fg)(x) = f(x)g(x) = \left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x}\right) = x^2 - \dfrac{1}{x^2}$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x + \frac{1}{x}}{x - \frac{1}{x}} = \dfrac{x^2 + 1}{x^2 - 1}$

Common Error: Domain is not $(-\infty, \infty)$. See below.

The domains of f and g are both $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.

This is therefore the domain of $f + g$, $f - g$, and fg. To find the domain of $\dfrac{f}{g}$, we must exclude from this domain the set of values of x for which $g(x) = 0$.

$$x - \frac{1}{x} = 0$$
$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = -1, 1$$

Hence, the domain of $\frac{f}{g}$ is $\{x \mid x \neq 0, -1, \text{ or } 1\}$ or $(-\infty,-1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$.

87. $$(f+g)(x) = f(x) + g(x) = 1 - \frac{x}{|x|} + 1 + \frac{x}{|x|} = 2$$

$$(f-g)(x) = f(x) - g(x) = 1 - \frac{x}{|x|} - \left(1+\frac{x}{|x|}\right) = 1 - \frac{x}{|x|} - 1 - \frac{x}{|x|} = \frac{-2x}{|x|}$$

$$(fg)(x) = f(x)g(x) = \left(1-\frac{x}{|x|}\right)\left(1+\frac{x}{|x|}\right) = (1)^2 - \left(\frac{x}{|x|}\right)^2 = 1 - \frac{x^2}{|x|^2} = 1 - \frac{x^2}{x^2} = 1 - 1 = 0$$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1-\frac{x}{|x|}}{1+\frac{x}{|x|}} = \frac{|x|-x}{|x|+x}$. This can be further simplified

however, when we examine the domain of $\frac{f}{g}$ below.

The domains of f and g are both $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$

This is therefore the domain of $f+g$, $f-g$, and fg. To find the domain of $\frac{f}{g}$, we must exclude from this domain the set of values of x for which $g(x) = 0$.

$$1 + \frac{x}{|x|} = 0$$
$$|x| + x = 0$$
$$|x| = -x$$

This is true when x is negative. The domain of $\frac{f}{g}$ is the positive numbers, $(0, \infty)$.

On this domain, $|x| = x$, so $\left(\frac{f}{g}\right)(x) = \frac{|x|-x}{|x|+x} = \frac{x-x}{x+x} = \frac{0}{2x} = 0$

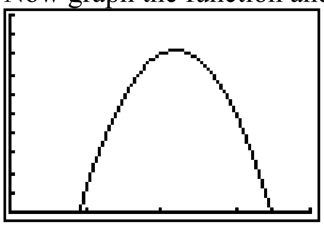
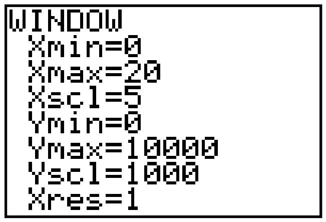
89. Profit is the difference of the amount of money taken in (Revenue) and the amount of money spent (Cost), so

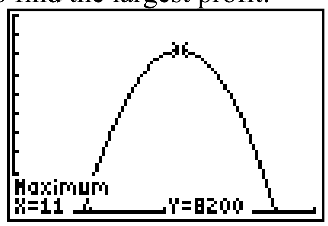
$$P(x) = R(x) - C(x)$$
$$= \left(20x - \frac{1}{200}x^2\right) - (2x + 8{,}000)$$
$$= 20x - \frac{1}{200}x^2 - 2x - 8{,}000 \text{ (Distribute!)}$$
$$= 18x - \frac{1}{200}x^2 - 8{,}000$$

we have a profit function, but it's a function of the demand (x), not the price (p). We were given $x = 4{,}000 - 200p$, so we can substitute $4{,}000 - 200p$ in for x to get desired function:

$$P(x) = P(4{,}000 - 200p) = 18(4{,}000 - 200p) - \frac{1}{200}(4{,}000 - 200p)^2 - 8{,}000$$
$$= 72{,}000 - 3{,}600p - \frac{1}{200}(16{,}000{,}000 - 1{,}600{,}000p + 40{,}000p^2) - 8{,}000$$
$$= 72{,}000 - 3{,}600p - 80{,}000 + 8{,}000p - 200p^2 - 8{,}000$$
$$= -16{,}000 + 4{,}400p - 200p^2$$

Now graph the function and use the maximum feature to find the largest profit.



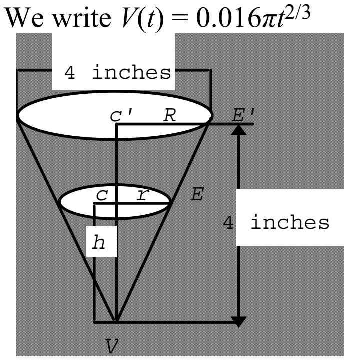
The maximum is (11, 9,200), so the largest profit occurs when the price is \$11.

91. We are given $V(r) = 0.1A(r) = 0.1\pi r^2$ and $r(t) = 0.4t^{1/3}$.
Hence we use composition to express V as a function of the time.

$$(V \circ r)(t) = V[r(t)]$$
$$= 0.1\pi[r(t)]^2$$
$$= 0.1\pi[0.4t^{1/3}]^2$$
$$= 0.1\pi[0.16t^{2/3}]$$
$$= 0.016\pi t^{2/3}$$

We write $V(t) = 0.016\pi t^{2/3}$

93.



(A) We note: In the figure, triangles VCE and $VC'E'$ are similar. Moreover R = radius of cup = $\frac{1}{2}$ diameter of cup = $\frac{1}{2}(4) = 2$ inches. Hence $\frac{r}{2} = \frac{h}{4}$ or $r = \frac{1}{2}h$. We write $r(h) = \frac{1}{2}h$.

(B) Since $V = \frac{1}{3}\pi r^2 h$ and $r = \frac{1}{2}h$, $V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{3}\pi\frac{1}{4}h^2 h = \frac{1}{12}\pi h^3$.

We write $V(h) = \frac{1}{12}\pi h^3$.

(C) Since $V(h) = \frac{1}{12}\pi h^3$ and $h(t) = 4 - 0.5\sqrt{t}$, we use composition to express V as a function of t.

$$(V \circ h)(t) = V[h(t)]$$
$$= \frac{1}{12}\pi\,[h(t)]^3 = \frac{1}{12}\pi\,(4 - 0.5\sqrt{t}\,)^3$$

Section 3-6

1. The function will be one-to-one if and only if each first component of the ordered pairs corresponds to exactly one second component.

3. If a function is not one-to-one, then at least two input elements correspond to one output element. If the correspondence is reversed, then the result cannot be a function. Example: {(1, 3), (2, 3)} when reversed becomes {(3, 1), (3, 2)} which is not a function.

5. The result of composing a function with its inverse is the identity function $f(x) = x$. This makes sense because the function and its inverse undo each other's operations.

7. This is a one-to-one function. All of the first coordinates are distinct, and each first coordinate is paired with a different second coordinate. If all of the ordered pairs are reversed, the situation is the same.

9. This is a function but it is not one-to-one. First coordinates 5 and 2 are both paired with 4, and first coordinates 4 and 3 are both paired with 3. If the ordered pairs are reversed, the result is not a function since first coordinates 3 and 4 will each be paired with two different second coordinates.

11. This is not a function: 1 is a first coordinate that is paired with 2 different second coordinates (as is –3). If the ordered pairs are reversed, the result is also not a function since 4 will be a first coordinate paired with 2 different second coordinates (as will 2).

13. One-to-one

15. The range element 7 corresponds to more than one domain element. Not one-to-one.

17. One-to-one

19. Some range elements (0, for example) correspond to more than one domain element. Not one-to-one.

21. One-to-one

23. One-to-one

25. Assume $F(a) = F(b)$

$$\frac{1}{2}a + 1 = \frac{1}{2}b + 1$$

Then $\frac{1}{2}a = \frac{1}{2}b$

$$a = b$$

Therefore F is one-to-one.

27. $H(x) = 4x - x^2$

Since $H(1) = 4(1) - 1^2 = 3$ and $H(3) = 4(3) - (3)^2 = 3$, both (1, 3) and (3, 3) belong to H. H is not one-to-one.

29. Assume $M(a) = M(b)$

$$\sqrt{a+1} = \sqrt{b+1}$$

Then $a + 1 = b + 1$

$a = b$ $\quad M$ is one-to-one.

31. $$f(g(x)) = f\left(\frac{1}{3}x - \frac{5}{3}\right) = 3\left(\frac{1}{3}x - \frac{5}{3}\right) + 5 = x - 5 + 5 = x$$

$$g(f(x)) = g(3x + 5) = \frac{1}{3}(3x + 5) - \frac{5}{3} = x + \frac{5}{3} - \frac{5}{3} = x$$

f and g are inverses

33. $$f(g(x)) = f(\sqrt[3]{3-x} - 1) = 2 - ((\sqrt[3]{3-x} - 1) + 1)^3 = 2 - (\sqrt[3]{3-x})^3 = 2 - (3 - x) = -1 + x$$

f and g are not inverses since $(f \circ g)(x)$ is not x.

35. $$f(g(x)) = f\left(\frac{3+4x}{2-x}\right) = \frac{2\left(\frac{3+4x}{2-x}\right) - 3}{\frac{3+4x}{2-x} + 4} = \frac{\frac{6+8x}{2-x} - 3}{\frac{3+4x}{2-x} + 4} = \frac{\frac{6+8x}{2-x} - \frac{3(2-x)}{2-x}}{\frac{3+4x}{2-x} + \frac{4(2-x)}{2-x}}$$

$$= \frac{\frac{6+8x-6+3x}{2-x}}{\frac{3+4x+8-4x}{2-x}} = \frac{\frac{11x}{2-x}}{\frac{11}{2-x}} = \frac{11x}{2-x} \cdot \frac{2-x}{11} = x$$

$$g(f(x)) = g\left(\frac{2x-3}{x+4}\right) = \frac{3 + 4\left(\frac{2x-3}{x+4}\right)}{2 - \left(\frac{2x-3}{x+4}\right)} = \frac{3 + \frac{8x-12}{x+4}}{2 - \frac{2x-3}{x+4}} = \frac{\frac{3(x+4)}{x+4} + \frac{8x-12}{x+4}}{\frac{2(x+4)}{x+4} - \frac{2x-3}{x+4}}$$

$$= \frac{\frac{3x+12+8x-12}{x+4}}{\frac{2x+8-2x+3}{x+4}} = \frac{\frac{11x}{x+4}}{\frac{11}{x+4}} = \frac{11x}{x+4} \cdot \frac{x+4}{11} = x$$

f and g are inverses.

37. $$f(g(x)) = f(\sqrt{x-4}) = 4 + (\sqrt{x-4})^2 = 4 + x - 4 = x$$

$$g(f(x)) = g(4 + x^2) = \sqrt{4 + x^2 - 4} = \sqrt{x^2} = x \text{ as long as } x \geq 0.$$

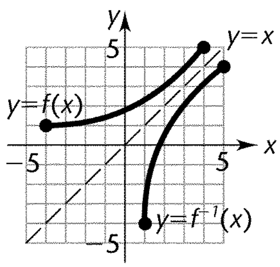
f and g are inverses.

39. $f(g(x)) = f(-\sqrt{1-x}\,) = 1 - (-\sqrt{1-x}\,)^2 = 1 - (1 - x) = x$

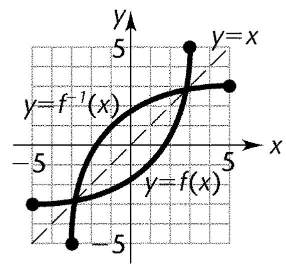
$g(f(x)) = g(1 - x^2) = -\sqrt{1-(1-x^2)} = -\sqrt{x^2} = -x$ as long as $x \ge 0$.

f and g are not inverses since $g \circ f(x)$ is not x.

41. From the graph:
domain of $f = [-4, 4]$
range of $f = [1, 5]$
Therefore:
domain of $f^{-1} = [1, 5]$
range of $f^{-1} = [-4, 4]$



43. From the graph:
domain of $f = [-5, 3]$
range of $f = [-3, 5]$
Therefore:
domain of $f^{-1} = [-3, 5]$
range of $f^{-1} = [-5, 3]$



45. The graph of $f(x)$ is a line; f is one-to-one.
The domain of f is therefore $(-\infty, \infty)$ and the range of f is also $(-\infty, \infty)$.

Write $y = f(x)$

$y = 3x$

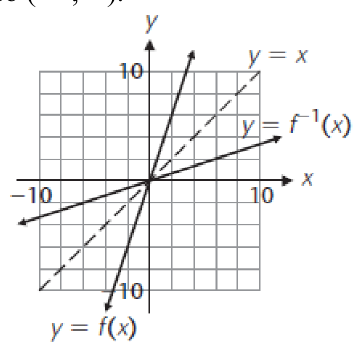
Solve $y = 3x$ for x.

$\frac{1}{3}y = x$

Interchange x and y:

$\frac{1}{3}x = y$

$f^{-1}(x) = \frac{1}{3}x$



The domain of f^{-1} is $(-\infty, \infty)$ and the range of f^{-1} is $(-\infty, \infty)$.

47. The graph of $f(x)$ is a line; f is one-to-one.
The domain and range of f are therefore $(-\infty, \infty)$.

Write $y = f(x)$

$y = 4x - 3$

Solve $y = 4x - 3$ for x.

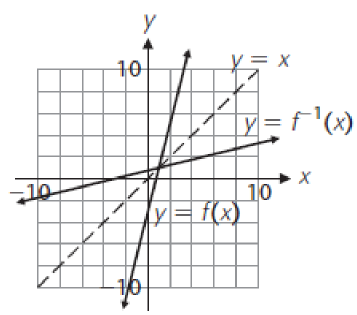
$y + 3 = 4x$

$\frac{y+3}{4} = x$

Interchange x and y:

$\frac{x+3}{4} = y$

$f^{-1}(x) = \frac{x+3}{4}$



The domain and range of f^{-1} are $(-\infty, \infty)$.

49. The graph of $f(x)$ is a line; f is one-to-one. The domain and range of f are therefore $(-\infty, \infty)$.

Write $y = f(x)$

$y = 0.2x + 0.4$

Solve for x.

$y - 0.4 = 0.2x$

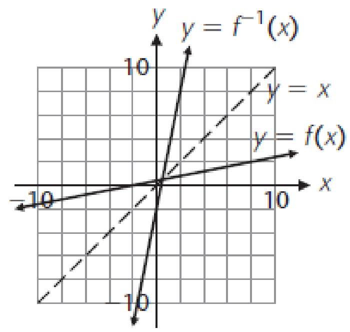
$5y - 2 = x$

Interchange x and y:

$5x - 2 = y$

$f^{-1}(x) = 5x - 2$

The domain and range of f^{-1} are $(-\infty, \infty)$.



51. From the graph as shown, f is an increasing function with domain $[0, \infty)$ and range $[3, \infty)$; f is one-to-one.

Write $y = f(x)$

$y = \sqrt{x} + 3$

Solve for x.

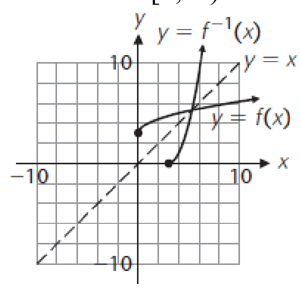
$y - 3 = \sqrt{x}$

$(y - 3)^2 = x$

Interchange x and y.

$(x - 3)^2 = y$

$f^{-1}(x) = (x - 3)^2$ domain $[3, \infty)$ range $[0, \infty)$



Common error: $f^{-1}(x) \neq (x - 3)^2$ for all x. The graph is only the right-hand half of the parabola.

53. From the graph as shown, f is a decreasing function with domain $(-\infty, 16]$ and range $[0, \infty)$; f is one-to-one.

Write $y = f(x)$.

$$y = \frac{1}{2}\sqrt{16 - x}$$

Solve for x.

$2y = \sqrt{16 - x}$

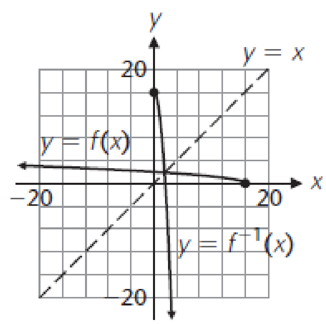
$4y^2 = 16 - x$

$x = 16 - 4y^2$

Interchange x and y.

$y = 16 - 4x^2$

$f^{-1}(x) = 16 - 4x^2$ domain $[0, \infty)$ range $(-\infty, 16]$



55. From the graph as shown, f is a decreasing function with domain $[1, \infty)$ and range $(-\infty, 3]$; f is one-to-one.

Write $y = f(x)$.

$y = 3 - \sqrt{x - 1}$

Solve for x.

$y - 3 = -\sqrt{x - 1}$

$(y - 3)^2 = x - 1$

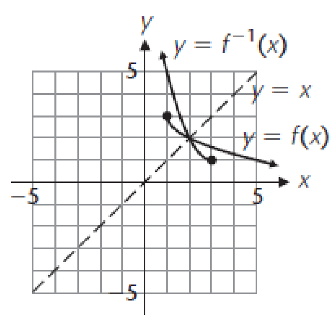
$(y - 3)^2 + 1 = x$

Interchange x and y.

$y = (x - 3)^2 + 1$

$f^{-1}(x) = (x - 3)^2 + 1$ domain $(-\infty, 3]$

range $[1, \infty)$



57. The graph of f is the right-hand half of a parabola; f is one-to-one.
The domain of f is $[0, \infty)$ and the range is $[5, \infty)$.

Write $y = f(x)$.

$$y = x^2 + 5 \qquad x \geq 0$$

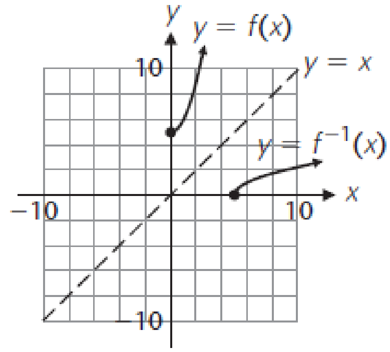
Solve for x.

$$y - 5 = x^2 \qquad x \geq 0$$
$$\sqrt{y-5} = x \qquad x \geq 0$$

(note: only the positive square root is correct)

Interchange x and y.

$$y = \sqrt{x-5} \qquad y \geq 0$$
$$f^{-1}(x) = \sqrt{x-5} \qquad \text{domain } [5, \infty)$$
$$\text{range } [0, \infty)$$



59. The graph of f is the left-hand half of a parabola; f is one-to-one.
The domain of f is $(-\infty, 0]$ and the range is $(-\infty, 4]$.

Write $y = f(x)$.

$$y = 4 - x^2 \qquad x \leq 0$$

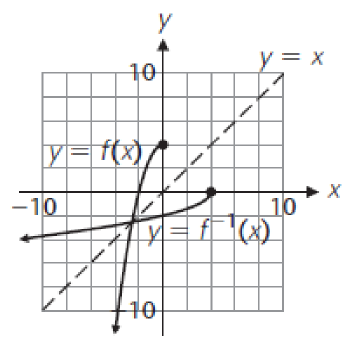
Solve for x.

$$y - 4 = -x^2 \qquad x \leq 0$$
$$4 - y = x^2 \qquad x \leq 0$$
$$-\sqrt{4-y} = x \qquad x \leq 0$$

(note: only the negative square root is correct)

Interchange x and y.

$$y = -\sqrt{4-x} \qquad y \leq 0$$
$$f^{-1}(x) = -\sqrt{4-x} \qquad \text{domain } (-\infty, 4]$$
$$\text{range } (-\infty, 0]$$



61. The graph of f is the right-hand half of a parabola; f is one-to-one.

Write $x^2 + 8x = (x^2 + 8x + 16) - 16 = (x + 4)^2 - 16$

Then the domain of f is given as $[-4, \infty)$ and the range is seen to be $[-16, \infty)$.

Write $y = f(x)$.

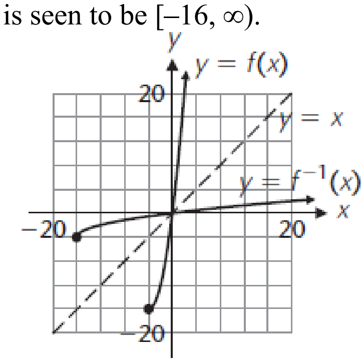
$$y = (x + 4)^2 - 16 \qquad x \geq -4$$

Solve for x.

$$y + 16 = (x + 4)^2 \qquad x \geq -4$$
$$\sqrt{y+16} = x + 4 \qquad x \geq -4$$
$$\sqrt{y+16} - 4 = x \qquad x \geq -4$$

Interchange x and y.

$$y = \sqrt{x+16} - 4 \qquad y \geq -4$$
$$f^{-1}(x) = \sqrt{x+16} - 4 \qquad \text{domain } [-16, \infty)$$
$$\text{range } [-4, \infty)$$



63. The graph of f is the left-hand half of a parabola; f is one-to-one.
The domain of f is $(-\infty, 2]$ and the range is $[0, \infty)$.
Write $y = f(x)$.

$y = (2 - x)^2 \qquad x \le 2$

Solve for x.

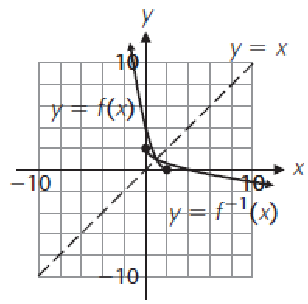
$\sqrt{y} = 2 - x \quad x \le 2$

$x = 2 - \sqrt{y} \qquad x \le 2$

Interchange x and y.

$y = 2 - \sqrt{x} \qquad y \le 2$

$f^{-1}(x) = 2 - \sqrt{x} \qquad$ domain $[0, \infty)$
range $(-\infty, 2]$



65. The graph of f is the right-hand half of a parabola; f is one-to-one.
The domain of f is $[1, \infty)$ and the range is $[2, \infty)$.
Write $y = f(x)$.

$y = (x - 1)^2 + 2 \qquad x \ge 1$

Solve for x.

$y - 2 = (x - 1)^2 \qquad x \ge 1$

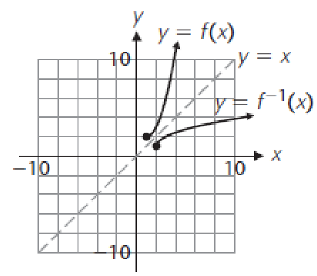
$\sqrt{y-2} = x - 1 \qquad x \ge 1$

$1 + \sqrt{y-2} = x \qquad x \ge 1$

Interchange x and y.

$y = 1 + \sqrt{x-2} \qquad y \ge 1$

$f^{-1}(x) = 1 + \sqrt{x-2} \qquad$ domain $[2, \infty)$
range $[1, \infty)$



67. The graph of f is the left-hand half of a parabola; f is one-to-one.
Write $x^2 + 2x - 2 = (x^2 + 2x + 1) - 1 - 2 = (x + 1)^2 - 3$
Then the domain of f is given as $(-\infty, -1]$ and the range is seen to be $[-3, \infty)$.
Write $y = f(x)$.

$y = (x + 1)^2 - 3 \qquad x \le -1$

Solve for x.

$y + 3 = (x + 1)^2 \qquad x \le -1$

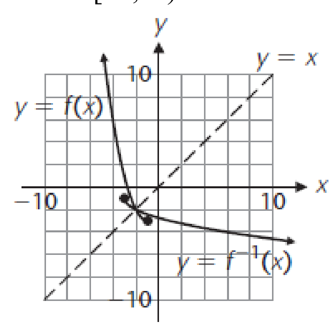
$-\sqrt{y+3} = x + 1 \qquad x \le -1$

$-\sqrt{y+3} - 1 = x \qquad x \le -1$

Interchange x and y.

$y = -\sqrt{x+3} - 1 \qquad y \le -1$

$f^{-1}(x) = -\sqrt{x+3} - 1 \qquad$ domain $[-3, \infty)$
range $(-\infty, -1]$



69. From the graph as shown, f is an increasing function with domain [0, 3] and range [–3, 0]; f is one-to-one. Write $y = f(x)$.

$y = -\sqrt{9-x^2}$ $\quad 0 \le x \le 3$

Solve for x.

$y^2 = 9 - x^2$ $\quad 0 \le x \le 3$

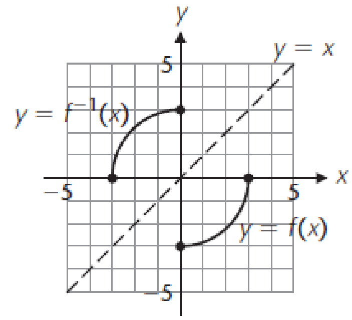
$x^2 = 9 - y^2$ $\quad 0 \le x \le 3$

$x = \sqrt{9-y^2}$ $\quad 0 \le x \le 3$

Interchange x and y.

$y = \sqrt{9-x^2}$ $\quad 0 \le y \le 3$

$f^{-1}(x) = \sqrt{9-x^2}$ $\quad$ domain [–3, 0]
range [0, 3]



71. From the graph as shown, f is an increasing function with domain [–3, 0] and range [0, 3]; f is one-to-one. Write $y = f(x)$.

$y = \sqrt{9-x^2}$ $\quad -3 \le x \le 0$

Solve for x.

$y^2 = 9 - x^2$ $\quad -3 \le x \le 0$

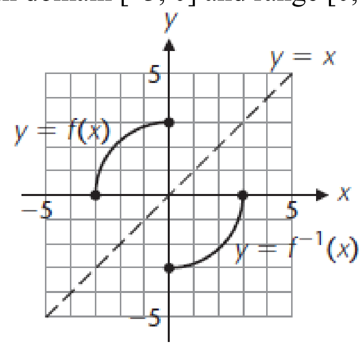
$x^2 = 9 - y^2$ $\quad -3 \le x \le 0$

$x = -\sqrt{9-y^2}$ $\quad -3 \le x \le 0$

Interchange x and y.

$y = -\sqrt{9-x^2}$ $\quad -3 \le y \le 0$

$f^{-1}(x) = -\sqrt{9-x^2}$ domain [0, 3] range [–3, 0]



73. From the graph as shown, f is a decreasing function with domain [–1, 0] and range [0, 1]; f is one-to-one. Write $y = f(x)$.

$y = 1 - \sqrt{1-x^2}$ $\quad -1 \le x \le 0$

Solve for x.

$y - 1 = -\sqrt{1-x^2}$ $\quad -1 \le x \le 0$

$(y-1)^2 = 1 - x^2$ $\quad -1 \le x \le 0$

$y^2 - 2y + 1 = 1 - x^2$ $\quad -1 \le x \le 0$

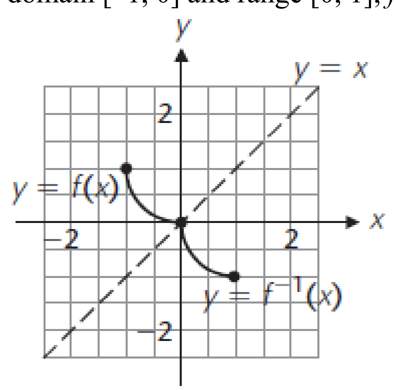
$x^2 = 2y - y^2$ $\quad -1 \le x \le 0$

$x = -\sqrt{2y-y^2}$ $\quad -1 \le x \le 0$

Interchange x and y.

$y = -\sqrt{2x-x^2}$ $\quad -1 \le y \le 0$

$f^{-1}(x) = -\sqrt{2x-x^2}$ domain [0, 1] range [–1, 0]



75. Write $y = f(x)$ $y = 3 - \frac{2}{x}$

Solve for x.

$$y - 3 = -\frac{2}{x}$$
$$x(y-3) = -2$$
$$x = \frac{-2}{y-3}$$
$$x = \frac{2}{3-y}$$

Interchange x and y.

$$y = \frac{2}{3-x}$$
$$f^{-1}(x) = \frac{2}{3-x}$$

(The student should check that $f \circ f^{-1}(x) = x$ and $f^{-1} \circ f(x) = x$.)

77. Write $y = f(x)$. $y = \frac{2}{x-1}$

Solve for x.

$$(x-1)y = 2$$
$$x - 1 = \frac{2}{y}$$
$$x = 1 + \frac{2}{y}$$
$$x = \frac{y}{y} + \frac{2}{y}$$
$$x = \frac{2+y}{y}$$

Interchange x and y.

$$y = \frac{2+x}{x}$$
$$f^{-1}(x) = \frac{2+x}{x}$$

79. Write $y = f(x)$. $y = \frac{2x}{x+1}$

Solve for x.

$$y(x+1) = 2x$$
$$xy + y = 2x$$
$$y = 2x - xy$$
$$y = x(2-y)$$
$$x = \frac{y}{2-y}$$

Interchange x and y.

$$y = \frac{x}{2-x}$$
$$f^{-1}(x) = \frac{x}{2-x}$$

81. Write $y = f(x)$. $y = \frac{2x+5}{3x-4}$

Solve for x.

$$y(3x-4) = 2x + 5$$
$$3xy - 4y = 2x + 5$$
$$3xy - 2x = 4y + 5$$
$$x(3y-2) = 4y + 5$$
$$x = \frac{4y+5}{3y-2}$$

Interchange x and y.

$$y = \frac{4x+5}{3x-2}$$
$$f^{-1}(x) = \frac{4x+5}{3x-2}$$

83. Write $y = f(x)$. $y = 4 - \sqrt[5]{x+2}$

Solve for x.

$$y - 4 = -\sqrt[5]{x+2}$$
$$4 - y = \sqrt[5]{x+2}$$
$$(4-y)^5 = x + 2$$
$$x = (4-y)^5 - 2$$

Interchange x and y.

$$y = (4-x)^5 - 2$$
$$f^{-1}(x) = (4-x)^5 - 2$$

85. Since in passing from a function to its inverse, x and y are interchanged, the x intercept of f is the y intercept of f^{-1} and the y intercept of f is the x intercept of f^{-1}.

87. They are not inverses. The function $f(x) = x^2$ is not one-to-one, so does not have an inverse. If that function is restricted to the domain $[0, \infty)$, then it is one-to-one, and $g(x) = \sqrt{x}$ (which has range $[0, \infty)$) is its inverse.

In problems 89-91, there is more than one possible answer.

89. $f(x) = (2-x)^2$ One possible answer: domain $x \le 2$

Solve $y = f(x)$ for x:

$y = (2-x)^2$ $\quad x \le 2$

$\sqrt{y} = 2 - x$ $\quad$ Positive square root Only since $2 - x \ge 0$

Check:

$$f^{-1}[f(x)] = 2 - \sqrt{(2-x)^2}$$
$= 2 - (2-x)$ since $2 - x \ge 0$ in the domain of f
$$= 2 - 2 + x$$
$$= x$$
$$f[f^{-1}(x)] = [2 - (2 - \sqrt{x})]^2$$

$\sqrt{y} - 2 = -x$

$x = 2 - \sqrt{y} = f^{-1}(y)$

Interchange x and y:

$y = f^{-1}(x) = 2 - \sqrt{x}$ Domain: $x \geq 0$

$= [2 - 2 + \sqrt{x}]^2$

$= [\sqrt{x}]^2$

$= x$

$f^{-1}(x) = 2 - \sqrt{x}$

91. $f(x) = \sqrt{4x - x^2}$

One possible answer: domain $0 \leq x \leq 2$

Solve $y = f(x)$ for x:

$y = \sqrt{4x - x^2}$

$y^2 = 4x - x^2$ $\quad y \geq 0$

$-y^2 = x^2 - 4x$

$4 - y^2 = x^2 - 4x + 4$

$4 - y^2 = (x - 2)^2$

$\underbrace{-\sqrt{4 - y^2}} = x - 2$

negative square root only because $x \leq 2$

$x = 2 - \sqrt{4 - y^2}$ $\quad y \geq 0$ $\quad f^{1}(y) = 2 - \sqrt{4 - y^2}$

Interchange x and y:

$y = f^{1}(x) = 2 - \sqrt{4 - x^2}$ Domain: $0 \leq x \leq 2$

Check: $f^{-1}[f(x)] = 2 - \sqrt{4 - (\sqrt{4x - x^2})^2}$

$= 2 - \sqrt{4 - (4x - x^2)}$

$= 2 - \sqrt{4 - 4x + x^2}$

$= 2 - \sqrt{(2 - x)^2}$

$= 2 - (2 - x)$ since $2 - x \geq 0$

$= x$

$f[f^{-1}(x)] = \sqrt{4(2 - \sqrt{4 - x^2}) - (2 - \sqrt{4 - x^2})^2}$

$= \sqrt{8 - 4\sqrt{4 - x^2} - (4 - 4\sqrt{4 - x^2} + 4 - x^2)}$

$= \sqrt{8 - 4\sqrt{4 - x^2} - 4 + 4\sqrt{4 - x^2} - 4 + x^2}$

$= \sqrt{x^2}$

$= x$ since $0 \leq x$

$f^{-1}(x) = 2 - \sqrt{4 - x^2}$, $0 \leq x \leq 2$

93. $p = 100 + 5h$ domain $[0, \infty)$ range $[100, \infty)$.

Solve for h

$p - 100 = 5h$

$h = \frac{1}{5}(p - 100) = \frac{1}{5}p - 20$ domain $[100, \infty)$

95. (A) From the graph, d is a decreasing function with domain [10, 70], hence its range is from $d(70) = 200$ to $d(10) = 1{,}000$, [200, 1000]

(B) $q = \dfrac{3{,}000}{0.2p + 1}$

Solve for p.

$q(0.2p + 1) = 3{,}000$

$0.2p + 1 = \dfrac{3{,}000}{q}$

$0.2p = \dfrac{3{,}000}{q} - 1$

$p = 5\left(\dfrac{3{,}000}{q} - 1\right)$

$d^{-1}(q) = p = \dfrac{15{,}000}{q} - 5$

domain [200, 1,000], range [10, 70].

97. (A) If r is linearly related to w, then we are looking for a function whose graph passes through

$(w_1, r_1) = (6, 10.50)$ and $(w_2, r_2) = (10, 15.50)$.

We find the slope, then we use the point-slope form to find the equation.

$M = \dfrac{r_2 - r_1}{w_2 - w_1} = \dfrac{15.50 - 10.50}{10 - 6} = 1.25$

$r - r_1 = M(w - w_1)$

$r - 10.50 = 1.25(w - 6)$

$r - 10.50 = 1.25w - 7.5$

$r = m(w) = 1.25w + 3$ domain $[0, \infty)$, range $[3, \infty)$

(B) Solve $r = 1.25w + 3$ for w.

$r - 3 = 1.25w$

$0.8(r - 3) = w$

$m^{-1}(r) = w = 0.8r - 2.4$ domain $[3, \infty)$, range $[0, \infty)$

99. $L = 0.06s^2 - 1.2s + 26$ domain $[10, \infty)$
Complete the square to find the range.
$L = 0.06(s^2 - 20s + ?) + 26$
$L = 0.06(s^2 - 20s + 100) - 6 + 26$
$L = 0.06(s - 10)^2 + 20$ therefore, range $[20, \infty)$
Solve for s.

$$L - 20 = 0.06(s - 10)^2$$

$$\frac{50}{3}(L - 20) = (s - 10)^2$$

$$\sqrt{\frac{50}{3}(L-20)} = s - 10 \quad \text{(positive square root only, since } s \geq 10\text{)}$$

$$f^{-1}(L) = s = 10 + \sqrt{\frac{50}{3}(L-20)} \quad \text{domain } [20, \infty), \text{ range } [10, \infty)$$

CHAPTER 3 REVIEW

1. (A) A function (B) A function
(C) Not a function; two range elements correspond to the domain element 10. *(3-1)*

2. (A) All of the first coordinates are distinct, so this is a function with domain {1, 2, 3}. All of the second coordinates are distinct, so the function is one-to-one. The range is {1, 4, 9}. The inverse function is obtained by reversing the order of the ordered pairs: {(1, 1), (4, 2), (9, 3)}. It has domain {1, 4, 9} and range {1, 2, 3}.
(B) This is not a function: both 1 and 2 are first coordinates that get matched with two different second coordinates.
(C) All of the first coordinates are distinct, so this is a function with domain {Albany, Utica, Akron, Dayton}. The range is {New York, Ohio}. It is not one-to-one since two first components are matched with Ohio, for example.
(D) All of the first components are distinct, so this is a function with domain {Albany, Akron, Tucson, Atlanta, Muncie}. The range is {New York, Ohio, Arizona, Georgia, Indiana}. All of the second components are distinct, so the function is one-to-one. The inverse function is {(New York, Albany), (Ohio, Akron), (Arizona, Tucson), (Georgia, Atlanta), (Indiana, Muncie)}. The domain of the inverse function is {New York, Ohio, Arizona, Georgia, Indiana}. The range of the inverse is {Albany, Akron, Tucson, Atlanta, Muncie}. *(3-1, 3-6)*

3. If there is at least one team that has won more than one Super Bowl, then the correspondence is not a function because one input (team) will correspond with more than one output (year). There are several teams that have won at least two Super Bowls, so this is not a function. *(3-6)*

4. (A) Not a function (fails vertical line test)
(B) A function
(C) A function
(D) Not a function (fails vertical line test) *(3-1)*

5. (A) Function
(B) Not a function—two range elements correspond to some domain elements; for example 2 and –2 correspond to 4.
(C) Function
(D) Not a function—two range elements correspond to some domain elements; for example 2 and –2 correspond to 2. *(3-1)*

6. $f(2) = 3(2) + 5 = 11$
$g(-2) = 4 - (-2)^2 = 0$
$k(0) = 5$
Therefore
$f(2) + g(-2) + k(0)$
$= 11 + 0 + 5 = 16$ *(3-1)*

7. $m(-2) = 2|-2| - 1 = 3$
$g(2) = 4 - (2)^2 = 0$
Therefore $\dfrac{m(-2)+1}{g(2)+4} = \dfrac{3+1}{0+4} = 1$ *(3-1)*

8. $$\frac{f(2+h)-f(2)}{h} = \frac{[3(2+h)+5]-[3(2)+5]}{h} = \frac{6+3h+5-11}{h} = \frac{3h}{h} = 3$$ *(3-1)*

9. $$\frac{g(a+h)-g(a)}{h} = \frac{[4-(a+h)^2]-[4-a^2]}{h} = \frac{4-a^2-2ah-h^2-4+a^2}{h} = \frac{-2ah-h^2}{h} = \frac{h(-2a-h)}{h} = -2a-h$$ *(3-1)*

10. $(f+g)(x) = f(x) + g(x)$
$= 3x + 5 + 4 - x^2 = 9 + 3x - x^2$ *(3-5)*

11. $(f-g)(x) = f(x) - g(x) = 3x + 5 - (4 - x^2)$
$= 3x + 5 - 4 + x^2 = x^2 + 3x + 1$ *(3-5)*

12. $(fg)(x) = f(x)g(x) = (3x + 5)(4 - x^2)$
$= 12x - 3x^3 + 20 - 5x^2$
$= 20 + 12x - 5x^2 - 3x^3$ *(3-5)*

13. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x+5}{4-x^2}$
Domain: $\{x \mid 4 - x^2 \neq 0\}$ or $\{x \mid x \neq \pm 2\}$ *(3-5)*

14. $(f \circ g)(x) = f[g(x)] = f(4 - x^2) = 3(4 - x^2) + 5$
$= 12 - 3x^2 + 5 = 17 - 3x^2$ *(3-5)*

15. $(g \circ f)(x) = g[f(x)] = g(3x + 5) = 4 - (3x + 5)^2$
$= 4 - (9x^2 + 30x + 25) = 4 - 9x^2 - 30x - 25$
$= -21 - 30x - 9x^2$ *(3-5)*

16. (A) $f(1) = (1)^2 - 2(1) = -1$
(B) $f(-4) = (-4)^2 - 2(-4) = 24$
(C) $f(2) \cdot f(-1) = [(2)^2 - 2(2)] \cdot [(-1)^2 - 2(-1)]$
$= 0 \cdot 3 = 0$
(D) $\dfrac{f(0)}{f(3)} = \dfrac{(0)^2-2(0)}{(3)^2-2(3)} = \dfrac{0}{3} = 0$ *(3-5)*

17. When $x = -4$, the corresponding value of $f(x)$ on the graph is 4. $f(-4) = 4$.
When $x = 0$, the corresponding value of $f(x)$ on the graph is -4. $f(0) = -4$.
When $x = 3$, the corresponding value of $f(x)$ on the graph is 0. $f(3) = 0$.
When $x = 5$, there is no corresponding value of $f(x)$ on the graph.
$f(5)$ is not defined. *(3-1, 3-2)*

18. Two values of x correspond to $f(x) = -2$ on the graph. They are $x = -2$ and $x = 1$. *(3-1, 3-2)*

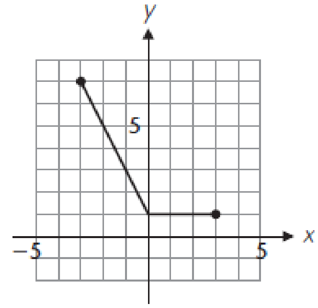
19. Domain: $[-4, 5)$. Range: $[-4, 4]$ *(3-2)*

20. The graph is increasing on $[0, 5)$ and decreasing on $[-4, 0]$. *(3-2)*

21. The graph is discontinuous at $x = 0$. *(3-2)*

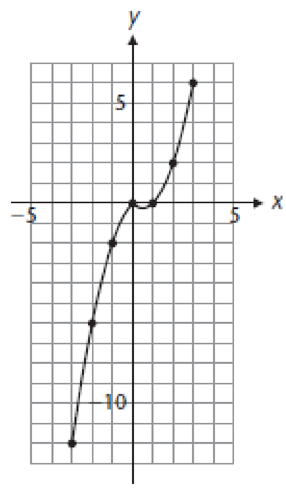
22. Construct a table of values of $f(x)$ and $g(x)$ from the graph, then subtract to obtain $(f-g)(x)$.

x	–3	–2	–1	0	1	2	3
$f(x)$	3	2	1	0	1	2	3
$g(x)$	–4	–3	–2	–1	0	1	2
$(f-g)(x)$	7	5	3	1	1	1	1



(3-5)

23. Use the top 3 rows of the table in problem 22 and multiply to get a table of values for $(fg)(x)$.
(Note that from the graphs of f and g, we can see that $f\left(\frac{1}{2}\right)=\frac{1}{2}$ and $g\left(\frac{1}{2}\right)=-\frac{1}{2}$, so $(fg)\left(\frac{1}{2}\right)=-\frac{1}{4}$.)



(3-5)

24. $(f\circ g)(-1)=f[g(-1)]$.
From the graph of g, $g(-1)=-2$.
From the graph of f, $f[g(-1)]=f(-2)=2$. *(3-5)*

25. $g\circ f(-2)=g[f(-2)]$.
From the graph of f, $f(-2)=2$.
From the graph of g, $g[f(-2)]=g(2)=1$.
(3-5)

26. From the graph of g, $g(1)=0$.
From the graph of f, $f[g(1)]=f(0)=0$. *(3-5)*

27. From the graph of f, $f(-3)=3$.
From the graph of g, $g[f(-3)]=g(3)=2$. *(3-5)*

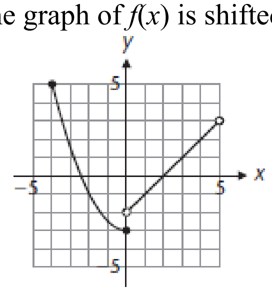
28. Some range elements (1 for example) correspond to more than one domain element. Not one-to-one. *(3-6)*

29. Yes, one-to-one. *(3-6)*

30. (A) $f(-x)=(-x)^5+6(-x)=-x^5-6x$
$=-(x^5+6x)=-f(x)$. Odd
(B) $g(-t)=(-t)^4+3(-t)^2=t^4+3t^2=g(t)$. Even
(C) $h(-z)=(-z)^5+4(-z)^2=-z^5+4z^2$
$-h(z)=-(z^5+4z^2)=-z^5-4z^2$
Therefore, $h(-z)\neq h(z)$; $h(-z)\neq -h(z)$.
h is neither even nor odd.

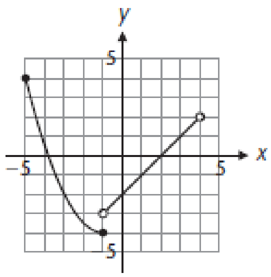
(3-2)

31. The graph of $f(x)$ is shifted up 1 unit.



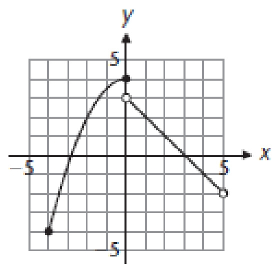
(3-3)

32. The graph of $f(x)$ is shifted left 1 unit



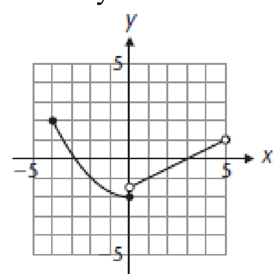
(3-3)

33. The graph of $f(x)$ is reflected in the x axis.



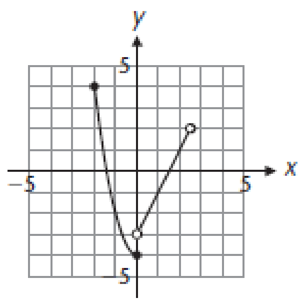
(3-3)

34. The graph of $f(x)$ is vertically shrunk by a factor of 0.5.



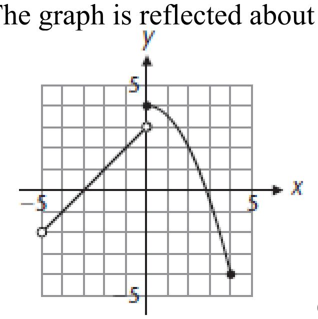
(3-3)

35. The graph is horizontally shrunk by a factor of $\frac{1}{2}$.



(3-3)

36. The graph is reflected about both axes.



(3-3)

37. (A) The graph that is decreasing, then increasing, and has a minimum at $x = 2$ is g.
(B) The graph that is increasing, then decreasing, and has a maximum at $x = -2$ is m.
(C) The graph that is increasing, then decreasing, and has a maximum at $x = 2$ is n.
(D) The graph that is decreasing, then increasing, and has a minimum at $x = -2$ is f. (3-3)

38. The equation corresponding to graph f is $y = (x + 2)^2 - 4$.
(A) y intercept: Set $x = 0$, then $y = (0 + 2)^2 - 4 = 0$
x intercepts: Set $y = 0$, then $0 = (x + 2)^2 - 4$
$$(x + 2)^2 = 4$$
$$x + 2 = \pm 2$$
$$x = 0, -4$$
(B) $(-2, -4)$
(C) The minimum of -4 occurs at the vertex.
(D) Since y is never less than -4, the range is $[-4, \infty)$.
(E) y is increasing on $[-2, \infty)$.
(F) y is decreasing on $(-\infty, -2]$. (3-4)

39. (A) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2-4}{x+3}$ Domain: $\{x \mid x \neq -3\}$, or $(-\infty, -3) \cup (-3, \infty)$
(B) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{x+3}{x^2-4}$ Domain: $\{x \mid x^2 - 4 \neq 0\}$, or $\{x \mid x \neq -2, 2\}$ or $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
(C) $(f \circ g)(x) = f[g(x)] = f(x + 3) = (x + 3)^2 - 4 = x^2 + 6x + 5$ Domain: $(-\infty, \infty)$
(D) $(g \circ f)(x) = g[f(x)] = g(x^2 - 4) = x^2 - 4 + 3 = x^2 - 1$ Domain: $(-\infty, \infty)$ (3-5)

40. (A) Comparing with the vertex N $f(x) = a(x - h)^2 + k$, we see that since $a = -2$ is negative, the function has a maximum $h = -4$ and $k = -10$ at $x = -4$. The maximum value is $f(-4) = -10$.
The vertex is at $(h, k) = (-4, -10)$.
(B) $f(x) = x^2 - 6x + 11$. Complete the square:
$$f(x) = (x^2 - 6x + 9) - 9 + 11$$
$$= (x - 3)^2 + 2$$
Comparing with $f(x) = a(x - h)^2 + k$; $h = 3$ and $k = 2$. Thus, the minimum value is 2 and the vertex is (3, 2). (3-4)

41. $q(x) = 2x^2 - 14x + 3$

$= 2(x^2 - 7x) + 3$

$\frac{1}{2}(-7) = -\frac{7}{2}, \quad \left(-\frac{7}{2}\right)^2 = \frac{49}{4}$

$= 2(x^2 - 7x + \frac{49}{4}) - \frac{49}{2} + 3$

We actually added $2\left(\frac{49}{4}\right)$ so subtract $\frac{49}{2}$

$= 2(x - \frac{7}{2})^2 - \frac{49}{2} + \frac{6}{2}$

$q(x) = 2(x - \frac{7}{2})^2 - \frac{43}{2}$ *(3-4)*

42.
(A) Reflected across x axis
(B) Shifted down 3 units
(C) Shifted left 3 units *(3-5)*

43. (A) When $x = 0$, the corresponding y value on the graph is 0, to the nearest integer.
(B) When $x = 1$, the corresponding y value on the graph is 1, to the nearest integer.
(C) When $x = 2$, the corresponding y value on the graph is 2, to the nearest integer.
(D) When $x = -2$, the corresponding y value on the graph is 0, to the nearest integer. *(3-2)*

44. (A) Two values of x correspond to $y = 0$ on the graph. To the nearest integer, they are –2 and 0.
(B) Two values of x correspond to $y = 1$ on the graph. To the nearest integer, they are –1 and 1.
(C) No value of x corresponds to $y = -3$ on the graph.
(D) To the nearest integer, $x = 3$ corresponds to $y = 3$ on the graph. Also, every value of x such that $x < -2$ corresponds to $y = 3$. *(3-2)*

45. Domain: $(-\infty, \infty)$. Range: $(-3, \infty)$. *(3-2)*

46. Increasing: $[-2, -1]$, $[1, \infty)$. Decreasing: $[-1, 1)$.
Constant: $(-\infty, -2)$ *(3-2)*

47. The graph of q is discontinuous at $x = -2$ and $x = 1$. *(3-2)*

48. $f(x) = 4x^3 - \sqrt{x}$ *(3-1)*

49. The function f multiplies the square of the domain element by 3, adds 4 times the domain element, and then subtracts 6. *(3-1)*

50. This equation defines a function. If you solve the equation for y, you get $y = 5 - 0.5x$. This tells us that for any choice of x, we can calculate a unique y that corresponds to it. *(3-1)*

51. This equation does not define a function since most choices of x will result in two corresponding values of y. For example, the pairs (2, 2) and (2, –2) both make the equation a true statement. *(3-1)*

52. Since $m(x)$ is a polynomial, the domain is the set of all real numbers R, $(-\infty, \infty)$.
To find the x intercepts, solve $m(x) = 0$:

$$x^2 - 4x + 5 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-4}}{2} \quad \text{(no real solutions)}$$

There are no x intercepts. The y intercept is $m(0) = 0^2 - 4(0) + 5 = 5$. *(3-1, 3-2)*

53. The domain is the set of all real numbers for which $\sqrt{x}$ is defined, that is, $[0, \infty)$. Since $2 + 3\sqrt{x}$ is never less than 2, $r(x)$ is never 0 and there are no x intercepts. The y intercept is $r(0) = 2 + 3\sqrt{0} = 2$. *(3-1, 3-2)*

54. The rational expression $\frac{1-x^2}{x^3}$ is defined everywhere except at zeros of the denominator, $x = 0$. The domain is $(-\infty, 0) \cup (0, \infty)$.
A rational expression is 0 if and only if the numerator is zero:

$$1 - x^2 = 0$$
$$1 = x^2$$
$$x = \pm 1$$

The x intercepts of p are –1, 1. The y intercept is $p(0)$, which is not defined, so there is no y intercept. (*3-1, 3-2*)

55. The fractional expression $\frac{x}{\sqrt{3-x}}$ is defined everywhere except at zeros of the denominator, as long as $\sqrt{3-x}$ is defined, thus

$$3 - x > 0$$
$$x < 3$$

The domain is $(-\infty, 3)$. A fractional expression is 0 if and only if the numerator is 0.

Thus $x = 0$ is the x intercept. The y intercept is $f(0) = \frac{0}{\sqrt{3-0}} = 0$. (*3-1, 3-2*)

56. The rational expression $\frac{2x+3}{x^2-4}$ is defined everywhere except at zeros of the denominator.

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.
A rational expression is 0 if and only if the numerator is 0.

$$2x + 3 = 0$$
$$x = -\frac{3}{2}$$

The x intercept is $-\frac{3}{2}$ or –1.5. The y intercept is $g(0) = \frac{2(0)+3}{0^2-4} = -\frac{3}{4}$ or –0.75. (*3-1, 3-2*)

57. The fractional expression $\frac{1}{4-\sqrt{x}}$ is defined everywhere except at zeros of the denominator, as long as $\sqrt{x}$ is defined, thus we must restrict $x \ge 0$ as well as $4 - \sqrt{x}$ to being non-zero.

$$4 - \sqrt{x} = 0$$
$$4 = \sqrt{x}$$
$$16 = x$$

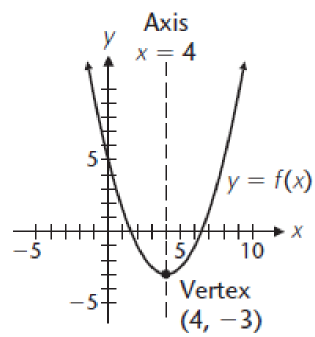
The domain is $[0, 16) \cup (16, \infty)$. Since the numerator is never 0, there are no x intercepts.

The y intercept is $h(0) = \frac{1}{4-\sqrt{0}} = \frac{1}{4}$ or 0.25. (*3-1, 3-2*)

58. (A) $f(x) = 0.5x^2 - 4x + 5$
$= 0.5(x^2 - 8x) + 5$
$= 0.5(x^2 - 8x + 16) + 5 - 0.5(16)$
$= 0.5(x - 4)^2 - 3$

Therefore the line $x = 4$ is the axis of symmetry of the parabola and $(4, -3)$ is its vertex.

(B) The parabola opens upward, since $0.5 = a > 0$. Thus the parabola is decreasing on $(-\infty, 4]$ and increasing on $[4, \infty)$. The range is $[-3, \infty)$.



(3-4)

59. g is a linear function. Its graph passes through $(x_1, y_1) = (-1, 0)$ and $(x_2, y_2) = (1, 4)$. Therefore the slope of the line is

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-0}{1-(-1)} = 2$. The equation of the line is obtained from the point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2[x - (-1)]$$
$$y = 2x + 2$$

The function is given by $g(x) = 2x + 2$.

f is a quadratic function. Its equation must be of the form $f(x) = y = a(x - h)^2 + k$. The vertex of the parabola is at $(1, 2)$. Therefore the equation must have form

$$y = a(x - 1)^2 + 2$$

Since the parabola passes through $(3, 0)$, these coordinates must satisfy the equation.

$$0 = a(3 - 1)^2 + 2$$
$$0 = 4a + 2$$
$$-2 = 4a$$
$$a = -0.5$$

The equation is

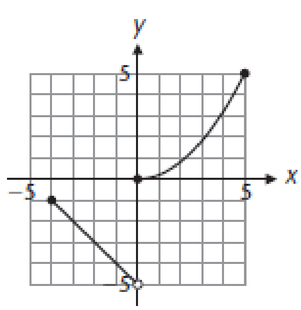
$$y = f(x) = -0.5(x - 1)^2 + 2$$
$$f(x) = -0.5(x^2 - 2x + 1) + 2$$
$$f(x) = -0.5x^2 + x - 0.5 + 2$$
$$f(x) = -0.5x^2 + x + 1.5$$

(3-4)

60. (A) For $-4 \le x < 0$, $f(x) = -x - 5$, so $f(-4) = -(-4) - 5 = -1$ and $f(-2) = -(-2) - 5 = -3$

For $0 \le x \le 5$, $f(x) = 0.2x^2$, so $f(0) = 0.2(0)^2 = 0$, $f(2) = 0.2(2)^2 = 0.8$, and $f(5) = 0.2(5)^2 = 5$.

(B)



(C) Domain: $[-4, 5]$, range: $(-5, -1] \cup [0, 5]$

(D) The graph is discontinuous at $x = 0$.

(E) Decreasing on $[-4, 0)$, increasing on $[0, 5]$.

(3-2, 3-4)

61. $f(x) = \sqrt{x} - 8 \quad g(x) = |x|$

(A) $(f \circ g)(x) = f[g(x)] = f(|x|) = \sqrt{|x|} - 8$

$(g \circ f)(x) = g[f(x)] = g(\sqrt{x} - 8) = |\sqrt{x} - 8|$

(B) The domain of f is $\{x \mid x \geq 0\}$. The domain of g is all real numbers. Hence the domain of $f \circ g$ is the set of those real numbers x for which $g(x)$ is non-negative, that is, all real numbers. The domain of $(g \circ f)$ is the set of all those non-negative numbers x for which $f(x)$ is real, that is all $\{x \mid x \geq 0\}$ or $[0, \infty)$ *(3-5)*

62. (A) $f(x) = x^3$.

Assume $f(a) = f(b)$

$$a^3 = b^3$$
$$a^3 - b^3 = 0$$
$$(a - b)(a^2 + ab + b^2) = 0$$

The only real solutions of this equation are those for which $a - b = 0$, hence $a = b$. Thus $f(x)$ is one-to-one.

(B) $g(x) = (x - 2)^2$. Since $g(3) = g(1) = 1$, g is not one-to-one.

(C) $h(x) = 2x - 3$

Assume $h(a) = h(b)$

$$2a - 3 = 2b - 3$$

Then $2a = 2b$

$$a = b$$

Thus h is one-to-one.

(D) $F(x) = (x + 3)^2 \quad x \geq -3$

Assume $F(a) = F(b) \quad a \geq -3, \; b \geq -3$

$$(a + 3)^2 = (b + 3)^2$$
$$a^2 + 6a + 9 = b^2 + 6b + 9$$
$$a^2 - b^2 + 6a - 6b = 0$$
$$(a - b)(a + b + 6) = 0$$

Either $a + b + 6 = 0$, that is, $a = b = -3$ (since $a \geq -3, \; b \geq -3$) or $a - b = 0$ that is, $a = b$. In either case, $a = b$.

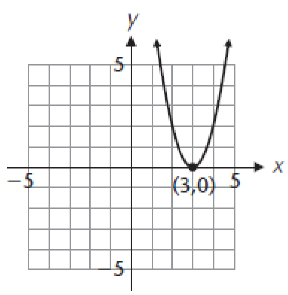
Thus F is one-to-one. *(3-6)*

63. Find the composition of the 2 functions in both orders.

$$(u \circ v)(x) = u(v(x)) = u(0.25x + 2)$$
$$= 4(0.25x + 2) - 8 = x + 8 - 8 = x$$
$$(v \circ u)(x) = v(u(x)) = v(4x - 8)$$
$$= 0.25(4x - 8) + 2 = x - 2 + 2 = x$$

The functions are inverses. *(3-6)*

64. (A) The graph of $y = x^2$ is shifted to the right 3 units, then vertically stretched by a factor of 2.



(B) Once choice is $x \leq 3$.
Another choice is $x \geq 3$.

(C) If $x \leq 3$,

Write $y = f(x)$

$$y = 2(x - 3)^2 \quad x \leq 3$$

Solve for x.

$$\frac{y}{2} = (x-3)^2 \quad x \leq 3$$

$$-\sqrt{\frac{y}{2}} = x - 3 \quad x \leq 3$$

(note: only the negative square root is correct)

$$x = 3 - \sqrt{\frac{y}{2}} \quad y \leq 3$$

Interchange x and y.

$$y = 3 - \sqrt{\frac{x}{2}} \quad x \geq 0$$

If $x \geq 3$,

Write $y = f(x)$

$$y = 2(x - 3)^2 \quad x \geq 3$$

Solve for x.

$$\frac{y}{2} = (x-3)^2 \quad x \geq 3$$

$$\sqrt{\frac{y}{2}} = x - 3 \quad x \geq 3$$

(note: only the positive square root is correct)

$$x = 3 + \sqrt{\frac{y}{2}} \quad y \geq 3$$

Interchange x and y.

$$y = 3 + \sqrt{\frac{x}{2}} \quad x \geq 0$$

(3-6)

65. (A) The graph of $f(x)$ is a line; f is one-to-one.

Write $y = f(x)$.

$y = 3x - 7$

Solve for x.

$y + 7 = 3x$

$\frac{y+7}{3} = x$ Interchange x and y:

$y = \frac{x+7}{3}$

$f^{-1}(x) = \frac{x+7}{3}$

(B) $f^{-1}(5) = \frac{5+7}{3} = 4$

(C) $f^{-1}[f(x)] = f^{-1}(3x - 7) = \frac{3x-7+7}{3} = \frac{3x}{3} = x$

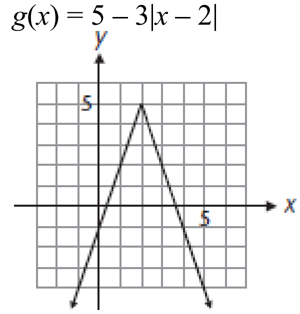
(D) Since $a < b$ implies $3a < 3b$, which implies $3a - 7 < 3b - 7$, or $f(a) < f(b)$, f is increasing.

(3-6)

66. The graph of $y = x^2$ is vertically stretched by a factor of 2, reflected through the x axis and shifted to the left 3 units.
Equation: $y = -2(x + 3)^2$.

(*3-3*)

67. $g(x) = 5 - 3|x - 2|$



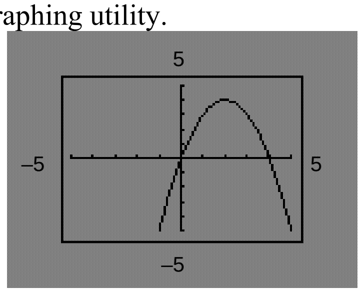
(*3-3*)

68. The graph of $y = x^2$ has been reflected through the x axis, shifted right 4 units and up 3 units so that the parabola has vertex (4, 3).
Equation: $y = -(x - 4)^2 + 3$. (*3-3, 3-4*)

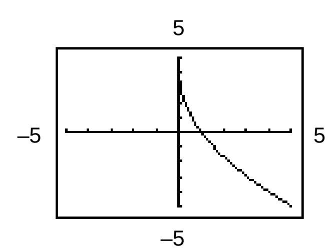
69. The graph of $y = \sqrt[3]{x}$ is vertically stretched by a factor of 2, reflected through the x axis, shifted 1 unit left and 1 unit down.
Equation: $y = -2\sqrt[3]{x+1} - 1$. (*3-3*)

70. It is the same as the graph of g shifted to the right 2 units and down 1 unit, then reflected through the x axis.
(*3-3*)

71. (A) The graph of $y = x^2$ is shifted 2 units to the right, reflected through the x axis, and shifted up 4 units: $y = -(x - 2)^2 + 4$
Check: $y = -(x - 2)^2 + 4$ is graphed on a graphing utility.

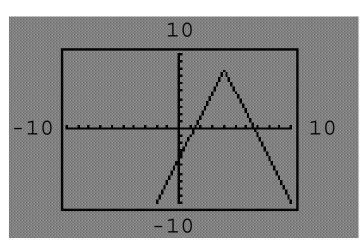


(B)The graph of $y = \sqrt{x}$ is reflected through the x axis, stretched by a factor of 4, and shifted up 4 units: $y = 4 - 4\sqrt{x}$
Check: $y = 4 - 4\sqrt{x}$ is graphed on a graphing utility.



(*3-3*)

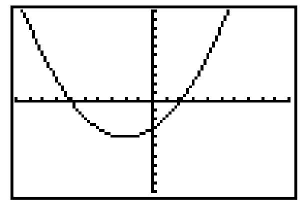
72. $g(x) = 8 - 3|x - 4|$
The –4 shifts 4 units right, the 3 stretches vertically by a factor of 3, and the 8 shifts 8 units up.



(*3-3*)

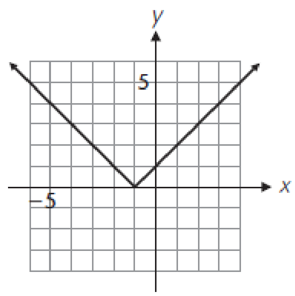
73. The equation is $t(x) = \left(\frac{1}{2}(x+2)\right)^2 - 4$. The $\frac{1}{2}$ stretches horizontally by 2, the +2 shifts 2 units left, and the –4 shifts 4 units down. This function can be simplified:

$$\left(\frac{1}{2}(x+2)\right)^2 - 4 = \left(\frac{1}{2}x+1\right)^2 - 4 = \frac{1}{4}x^2 + x + 1 - 4 = \frac{1}{4}x^2 + x - 3.$$



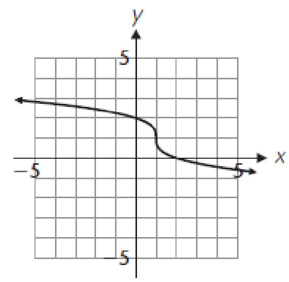
(3-3)

74. The graph of $y = |x|$ is shifted 1 unit left.



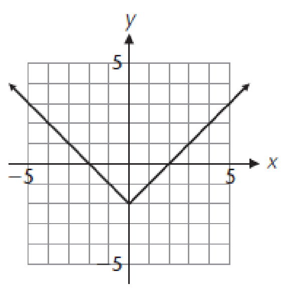
(3-3)

75. The graph of $y = \sqrt[3]{x}$ is shifted 1 unit right, reflected through the x axis, and shifted 1 unit up.



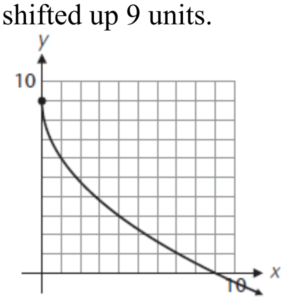
(3-3)

76. The graph of $y = |x|$ is shifted 2 units down.



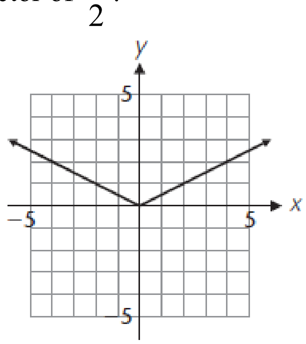
(3-3)

77. The graph of $y = \sqrt{x}$ is reflected through the x axis, stretched vertically by a factor of 3, and shifted up 9 units.



(3-3)

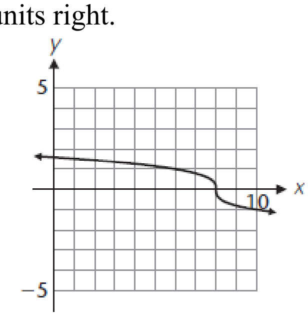
78. The graph of $y = |x|$ is shrunk vertically by a factor of $\frac{1}{2}$.



(3-3)

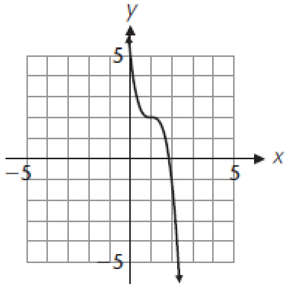
79. $\sqrt[3]{4-0.5x} = \sqrt[3]{0.5(8-x)}$

The graph of $y = \sqrt[3]{x}$ is stretched horizontally by a factor of 2, reflected across the y axis, and shifted 8 units right.



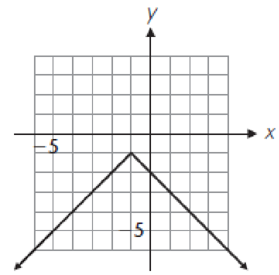
(3-3)

80. The graph of $y = x^3$ is shifted right 1 unit, stretched vertically by a factor of 3, reflected through the x axis, and shifted up 2 units.



(*3-3*)

81. The graph of $y = |x|$ is shifted left 1 unit, reflected through the x axis, and shifted down 1 unit.



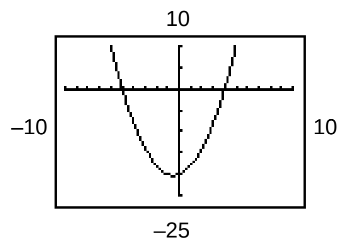
(*3-3*)

82. $x^2 + x < 20$

$f(x) = x^2 + x - 20 < 0$

$f(x) = (x + 5)(x - 4) < 0$

The zeros of f are –5 and 4. Plotting the graph of $f(x)$, we see that $f(x) < 0$ for $-5 < x < 4$, or (–5, 4).



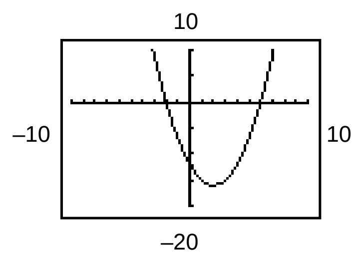
(*3-4*)

83. $x^2 > 4x + 12$

$f(x) = x^2 - 4x - 12 > 0$

$f(x) = (x + 2)(x - 6) > 0$

The zeros of f are –2 and 6. Plotting the graph of $f(x)$, we see that $f(x) > 0$ for $x < -2$ and $x > 6$ or $(-\infty, -2) \cup (6, \infty)$.



(*3-4*)

84. The domain is the set of all real numbers x such that $\sqrt{25 - x^2}$ is a real number, that is, such that $25 - x^2 \geq 0$ or $-5 \leq x \leq 5$, [–5, 5]. (*3-1*)

85. (A) $(fg)(x) = f(x)g(x) = x^2\sqrt{1-x}$

The domain of f is $(-\infty, \infty)$. The domain of g is $(-\infty, 1]$. Hence the domain of fg is the intersection of these sets, that is, $(-\infty, 1]$.

(B) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{\sqrt{1-x}}$

To find the domain of $\frac{f}{g}$, we exclude from $(-\infty, 1]$ the set of values of x for which $g(x) = 0$

$$\sqrt{1-x} = 0$$
$$1 - x = 0$$
$$x = 1$$

Thus the domain of $\frac{f}{g}$ is $(-\infty, 1)$

(C) $(f \circ g)(x) = f[g(x)] = f(\sqrt{1-x}) = [\sqrt{1-x}]^2 = 1 - x$.

The domain of $f \circ g$ is the set of those numbers in $(-\infty, 1]$ for which $g(x)$ is real, that is $(-\infty, 1]$.

(D) $(g \circ f)(x) = g[f(x)] = g(x^2) = \sqrt{1-x^2}$

The domain of $g \circ f$ is the set of those real numbers for which $f(x)$ is in $(-\infty, 1]$, that is, $x^2 \leq 1$, or $-1 \leq x \leq 1$. [–1, 1].

(*3-5*)

86. (A) Write $y = f(x)$.

$$y = \frac{x+2}{x-3}$$

Solve for x.

$$y(x-3) = x+2$$
$$xy - 3y = x+2$$
$$xy - x = 3y+2$$
$$x(y-1) = 3y+2$$
$$x = \frac{3y+2}{y-1}$$

Interchange x and y:

$$f^{-1}(x) = y = \frac{3x+2}{x-1}$$

(B) $f^{-1}(3) = \frac{3(3)+2}{3-1} = \frac{11}{2}$

(C) $f^{-1}[f(x)] = f^{-1}\left(\frac{x+2}{x-3}\right)$

$$= \frac{3\left(\frac{x+2}{x-3}\right)+2}{\frac{x+2}{x-3}-1}$$
$$= \frac{3(x+2)+2(x-3)}{x+2-(x-3)}$$
$$= \frac{3x+6+2x-6}{x+2-x+3}$$
$$= \frac{5x}{5} = x$$

(3-6)

87. (A) Write $y = f(x)$.

$$y = \sqrt{x-1}$$

Domain of f: $[1, \infty)$, Range: $[0, \infty)$.

Solve for x.

$$y^2 = x-1$$
$$y^2 + 1 = x$$

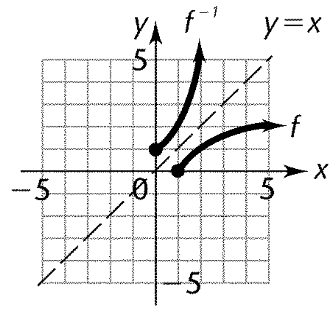
Interchange x and y.

$$x^2 + 1 = y$$

$f^{-1}(x) = x^2 + 1$ Domain of f^{-1}: $[0, \infty)$, Range: $[1, \infty)$

(B) See part (A)

(C)



(3-6)

88. (A) Domain of $f = [0, \infty)$ = Range of f^{-1}

Since $x^2 \geq 0$, $x^2 - 1 \geq -1$, so

Range of $f = [-1, \infty)$ = Domain of f^{-1}

(B) $f(x) = x^2 - 1 \quad x \geq 0$

f is one-to-one on its domain (steps omitted)

Solve $y = f(x)$ for x:

$$y = x^2 - 1 \quad x \geq 0$$
$$x^2 = y + 1 \quad x \geq 0$$
$$x = \underbrace{\sqrt{y+1}} = f^{-1}(y)$$

positive square root since $x \geq 0$

Interchange x and y:

$y = f^{-1}(x) = \sqrt{x+1}$ Domain: $[-1, \infty)$

Check:

$$f^{-1}[f(x)] = \sqrt{x^2 - 1 + 1} \quad x \geq 0$$
$$= \sqrt{x^2} \quad x \geq 0$$
$$= x \text{ since } x \geq 0$$
$$f[f^{-1}(x)] = (\sqrt{x+1})^2 - 1$$
$$= x + 1 - 1$$
$$= x$$

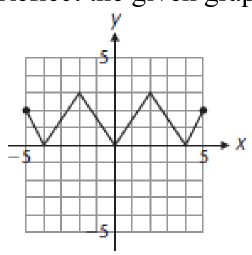
(C) $f^{-1}(3) = \sqrt{3+1} = 2$

(D) $f^{-1}[f(4)] = 4$

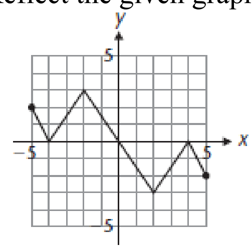
(E) $f^{-1}[f(x)] = x$

(3-6)

89. (A) Reflect the given graph across the y axis:

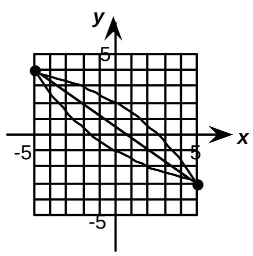


(B) Reflect the given graph across the origin:

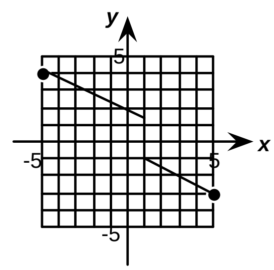


(*3-3*)

90. (A) The graph must cross the x axis exactly once. Some possible graphs are shown:



(B) The graph may cross the x axis once, but it may fail to cross the x axis at all. A possible graph of the latter type is shown:



(*3-2*)

91. (A) If $0 \le x \le 2{,}000$, $E(x) = 6 \cdot 20 = 120$

If $2{,}000 < x \le 5{,}000$,

$$\begin{aligned} E(x) &= \text{Base salary} + \text{Commission on sales over } \$2{,}000 \\ &= 120 + 0.1(x - 2{,}000) \\ &= 120 + 0.1x - 200 \\ &= 0.1x - 80 \end{aligned}$$

If $x > 5{,}000$,

$$\begin{aligned} E(x) &= \text{Salary} + \text{Bonus} \\ &= (0.1x - 80) + 250 \\ &= 0.1x + 170 \end{aligned}$$

Summarizing,

$$E(x) = \begin{cases} 120 & \text{if } 0 \le x \le 2{,}000 \\ 0.1x - 80 & \text{if } 2{,}000 < x \le 5{,}000 \\ 0.1x + 170 & \text{if } x > 5{,}000 \end{cases}$$

(B) If $0 \le x \le 2{,}000$, $E(x) = 120$, so $E(2{,}000) = \$120$.
If $2{,}000 < x \le 5{,}000$, $E(x) = 0.1x - 80$, so $E(4{,}000) = 0.1(4{,}000) - 80 = \$320$.
If $x > 5{,}000$, $E(x) = 0.1x + 170$, so $E(6{,}000) = 0.1(6{,}000) + 170 = \$770$.

(C) In order to average more than \$400 a week, she must sell more than \$2,000. Can she earn \$400 a week if she sells between \$2,000 and \$5,000?
Solve

$$\begin{aligned} 0.1x - 80 &= 400 \\ 0.1x &= 480 \\ x &= \$4{,}800 \end{aligned}$$

This is between \$2,000 and \$5,000, so, yes, she can average \$400 a week if she can sell \$4,800 on average each week. (*3-2*)

92. The function describing the height of the stuntman is $h(t) = 120 - 16t^2$, since the initial height is 120 feet. Substitute 0 for $h(t)$ and solve.

$$\begin{aligned} 0 &= 120 - 16t^2 \\ 16t^2 &= 120 \\ t^2 &= 7.5 \\ t &= 2.7 \text{ seconds after jumping} \end{aligned}$$

(*3-4*)

93. (A) If r is linearly related to c, then we are looking for a function whose graph passes through $(c_1, r_1) = (30, 48)$ and $(c_2, r_2) = (20, 32)$. We find the slope, then we use the point-slope form to find the equation.

$$m = \frac{r_2 - r_1}{c_2 - c_1} = \frac{32 - 48}{20 - 30} = 1.6$$

$$r - r_1 = m(c - c_1)$$
$$r - 48 = 1.6(c - 30)$$
$$r - 48 = 1.6c - 48$$
$$f(c) = r = 1.6c \quad \text{domain: } [10, \infty) \text{ range: } [16, \infty)$$

(B) $f(105) = 1.6(105) = \$168$

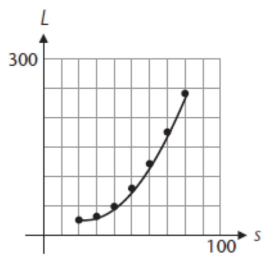
(C) Solve $r = 1.6c$ for c.

$0.625r = c = f^{-1}(r)$

Domain: $[16, \infty)$, range: $[10, \infty)$

(D) $f^{-1}(39.99) = 0.625(39.99) = \$24.99$ *(3-6)*

94. (A)



(B) $L = 0.06s^2 - 2.4s + 50$ domain $[20, \infty)$

Complete the square to find the range.

$L = 0.06(s^2 - 40s + ?) + 50$

$L = 0.06(s^2 - 40s + 400) - 0.06(400) + 50$

$L = 0.06(s - 20)^2 + 26$ therefore, range $[26, \infty)$

Solve for s.

$$L - 26 = 0.06(s - 20)^2$$
$$\frac{50}{3}(L - 26) = (s - 20)^2$$
$$\sqrt{\frac{50}{3}(L-26)} = s - 20 \quad \text{(positive square root only, since } s \ge 20)$$
$$f^{-1}(L) = s = 20 + \sqrt{\frac{50}{3}(L-26)} \quad \text{domain } [26,\infty), \text{ range } [20,\infty).$$

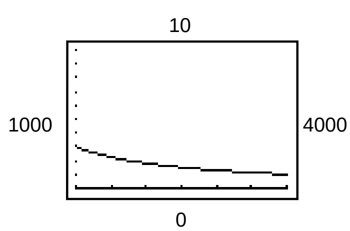
(C)

Substitute $L = 200$ into $f^{-1}(L)$ to obtain

$$S = 20 + \sqrt{\frac{50}{3}(200-26)} = 74 \text{ mph}$$

(3-6)

95. (A) Examining the graph of p, we obtain



Clearly, d is a decreasing function, passing the horizontal line test, so d is one-to-one.

When $q = 1000$, $p = d(1000) = \dfrac{9}{1+0.002(1000)} = 3$

When $q = 4000$, $p = d(4000) = \dfrac{9}{1+0.002(4000)} = 1$

Thus the range of d is $1 \le p \le 3$ or $[1, 3]$.

(B) Solve $p = d(q)$ for q.

$$p = \frac{9}{1+0.002q}$$
$$p(1 + 0.002q) = 9$$
$$1 + 0.002q = \frac{9}{p}$$
$$0.002q = \frac{9}{p} - 1$$
$$q = \frac{4,500}{p} - 500 = d^{-1}(p)$$

Domain d^{-1} = range $d = [1, 3]$

Range d^{-1} = domain $d = [1000, 4000]$.

(C) Revenue is price times number sold, so we need to multiply the variable p (price) by the function q (number sold) that we obtained in part B:

$$p \cdot q = p \cdot \left(\frac{4{,}500}{p} - 500\right) = 4{,}500 - 500p$$

(D) Revenue is price times number sold, so we need to multiply the function p (price) times the variable q (numbers sold):

$$p \cdot q = \frac{9}{1+0.002q} \cdot q = \frac{9q}{1+0.002q}$$ *(3-6)*

96. Profit is revenue minus cost so we start by subtracting the revenue and cost functions:

$$P(x) = R(x) - C(x) = (50x - 0.1x^2) - (10x + 1{,}500)$$
$$= 50x - 0.1x^2 - 10x - 1{,}500 = -0.1x^2 + 40x - 1{,}500$$

The variable is x but we're asked to find profit in terms of price (p) so we need to substitute $500 - 10p$ in for x.

$$p(500 - 10p) = -0.1(500 - 10p)^2 + 40(500 - 10p) - 1{,}500$$
$$= -0.1(250{,}000 - 10{,}000p + 100p^2) + 20{,}000 - 400p - 1{,}500$$
$$= -25{,}000 + 1{,}000p - 10p^2 + 20{,}000 - 400p - 1{,}500$$
$$= -10p^2 + 600p - 6{,}500$$

so $P(p) = -10p^2 + 600p - 6{,}500$

This is a quadratic function, so its maximum occurs at the vertex of its parabola graph which is found at

$$p = -\frac{b}{2a} = -\frac{600}{2(-10)} = 30$$

Then $f(p) = f\left(-\frac{b}{2a}\right) = f(30) = -10(30)^2 + 600(30) - 6{,}500 = 2{,}500$

Thus the price that produces the largest profit is \$30 and this profit is \$2,500. *(3-4)*

97. (A) From the figure, we see that $A = x(y + y) = 2xy$. Since the fence consists of four pieces of length y and three pieces of length x, we have $3x + 4y = 120$. Hence $4y = 120 - 3x$

$$y = 30 - \frac{3}{4}x$$

$$A = 2x\left(30 - \frac{3}{4}x\right)$$

$$A(x) = 60x - \frac{3}{2}x^2$$

(B) Since both x and y must be positive, we have $x > 0$

$$30 - \frac{3}{4}x > 0 \text{ or } -\frac{3}{4}x > -30 \text{ or } x < 40$$

Hence $0 < x < 40$ is the domain of A.

(C) The function A is a quadratic function. Completing the square yields:

$$A(x) = -\frac{3}{2}x^2 + 60x$$
$$= -\frac{3}{2}(x^2 - 40x)$$
$$= -\frac{3}{2}(x^2 - 40x + 400) + \frac{3}{2} \cdot 400$$
$$= -\frac{3}{2}(x - 20)^2 + 600$$

Comparing with $f(x) = a(x - h)^2 + k$, the total area will be maximum when $x = 20$.

Then $y = 30 - \frac{3}{4}x = 30 - \frac{3}{4}(20) = 15$ *(3-4)*

98. (A) $f(1) = 1 - ([\![\sqrt{1}]\!])^2 = 1 - 1 = 0$
$f(2) = 2 - ([\![\sqrt{2}]\!])^2 = 2 - 1 = 1$
$f(3) = 3 - ([\![\sqrt{3}]\!])^2 = 3 - 1 = 2$
$f(4) = 4 - ([\![\sqrt{4}]\!])^2 = 4 - 4 = 0$
$f(5) = 5 - ([\![\sqrt{5}]\!])^2 = 5 - 4 = 1$
$f(6) = 6 - ([\![\sqrt{6}]\!])^2 = 6 - 4 = 2$
$f(7) = 7 - ([\![7]\!])^2 = 7 - 4 = 3$
$f(8) = 8 - ([\![8]\!])^2 = 8 - 4 = 4$
$f(9) = 9 - ([\![9]\!])^2 = 9 - 9 = 0$
$f(10) = 10 - ([\![10]\!])^2 = 10 - 9 = 1$
$f(11) = 11 - ([\![11]\!])^2 = 11 - 9 = 2$
$f(12) = 12 - ([\![12]\!])^2 = 12 - 9 = 3$
$f(13) = 13 - ([\![13]\!])^2 = 13 - 9 = 4$
$f(14) = 14 - ([\![14]\!])^2 = 14 - 9 = 5$
$f(15) = 15 - ([\![15]\!])^2 = 15 - 9 = 6$
$f(16) = 16 - ([\![16]\!])^2 = 16 - 16 = 0$

(B) $f(n^2) = n^2 - ([\![\sqrt{n^2}]\!])^2$
$= n^2 - ([\![n]\!])^2$ since $\sqrt{n^2} = n$ if n is positive
$= n^2 - (n)^2$ since $[\![n]\!] = n$ if n is a (positive) integer.
$= 0$

(C) It determines if an integer is a perfect square integer. If $f(x) = 0$, then x is a perfect square and if $f(x) \neq 0$, x is not a perfect square integer.

(*3-2*)

99. Amounts less than \$3,000: Tax is 2% of income (x) = $0.02x$
Amounts between \$3,000 and \$5,000: Tax is \$60 + 3% of income over 3,000 ($x - 3{,}000$) so tax = $60 + 0.03(x - 3{,}000) = 0.03x - 30$
Amounts between \$5,000 and \$17,000: Tax is \$120 + 5% of income over 5,000 ($x - 5{,}000$) so tax = $120 + 0.05(x - 5{,}000) = 0.05x - 130$
Amounts over \$17,000: Tax is \$720 + 5.75% of income over 17,000 ($x - 17{,}000$) so tax = $720 + 0.0575(x - 17{,}000) = 0.0575x - 257.5$

Combined, we get $t(x) = \begin{cases} 0.02x & 0 \le x \le 3{,}000 \\ 0.03x - 30 & 3{,}000 < x \le 5{,}000 \\ 0.05x - 130 & 5{,}000 < x \le 17{,}000 \\ 0.0575x - 257.5 & x > 17{,}000 \end{cases}$

$t(2{,}000) = 0.02(2{,}000) = 40$. The tax on \$2,000 is \$40.
$t(4{,}000) = 0.03(4{,}000) - 30 = 120 - 30 = 90$. The tax on \$4,000 is \$90.
$t(10{,}000) = 0.05(10{,}000) - 130 = 500 - 130 = 370$. The tax on \$10,000 is \$130.
$t(30{,}000) = 0.0575(30{,}000) - 257.5 = 1{,}725 - 257.5 = 1{,}467.5$. The tax on \$30,000 is \$1,467.50.