CHAPTER 5

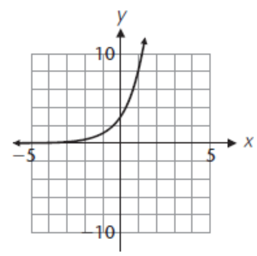
Section 5-1

1. An exponential function is a function where the variable appears in an exponent.
3. If $b > 1$, the function is an increasing function. If $0 < b < 1$, the function is a decreasing function.
5. A positive number raised to any real power will give a positive result.
7. (A) The graph of $y = (0.2)^x$ is decreasing and passes through the point $(-1, 0.2^{-1}) = (-1, 5)$. This corresponds to graph g.
(B) The graph of $y = 2^x$ is increasing and passes through the point (1, 2). This corresponds to graph n.
(C) The graph of $y = \left(\frac{1}{3}\right)^x$ is decreasing and passes through the point (−1, 3). This corresponds to graph f.
(D) The graph of $y = 4^x$ is increasing and passes through the point (1, 4). This corresponds to graph m.

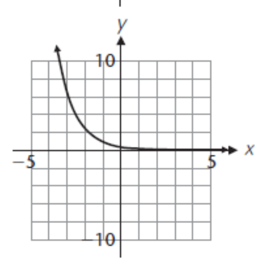
9. 16.24 **11.** 7.524 **13.** 1.649 **15.** 4.469 **17.** $10^{3x-1}10^{4-x} = 10^{3x-1+4-x} = 10^{2x+3}$

19. $\frac{3^x}{3^{1-x}} = 3^{x-(1-x)} = 3^{x-1+x} = 3^{2x-1}$ **21.** $\left(\frac{4^x}{5^y}\right)^{3z} = \frac{4^{3xz}}{5^{3yz}}$ **23.** $\frac{e^{5x}}{e^{2x+1}} = e^{5x-(2x+1)} = e^{5x-2x-1} = e^{3x-1}$

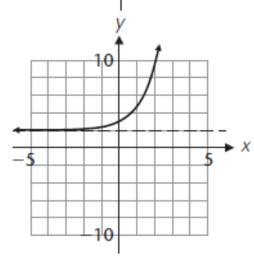
25. The graph of g is the same as the graph of f stretched vertically by a factor of 3. Therefore g is increasing and the graph has horizontal asymptote $y = 0$.



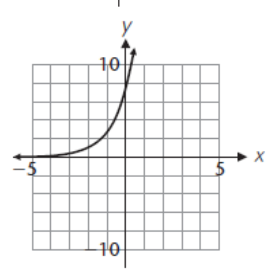
27. The graph of g is the same as the graph of f reflected through the y axis and shrunk vertically by a factor of $\frac{1}{3}$. Therefore g is decreasing and the graph has horizontal asymptote $y = 0$.



29. The graph of g is the same as the graph of f shifted upward 2 units. Therefore g is increasing and the graph has horizontal asymptote $y = 2$.



31. The graph of g is the same as the graph of f shifted 2 units to the left. Therefore g is increasing and the graph has horizontal asymptote $y = 0$.



33. $5^{3x} = 5^{4x-2}$ if and only if
$3x = 4x - 2$
$-x = -2$
$x = 2$

35. $7^{x^2} = 7^{2x+3}$ if and only if
$x^2 = 2x + 3$
$x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$x = -1, 3$

37. $\left(\frac{4}{5}\right)^{6x+1} = \frac{5}{4}$
$\left(\frac{4}{5}\right)^{6x+1} = \left(\frac{4}{5}\right)^{-1}$ if and only if
$6x + 1 = -1$
$6x = -2$
$x = -\frac{1}{3}$

39. $(1-x)^5 = (2x-1)^5$ if and only if
$1 - x = 2x - 1$
$-3x = -2$
$x = \frac{2}{3}$

41. $2xe^{-x} = 0$ if $2x = 0$ or $e^{-x} = 0$. Since e^{-x} is never 0, the only solution is $x = 0$.

43. $x^2e^x - 5xe^x = 0$
$xe^x(x-5) = 0$
$x = 0$ or $e^x = 0$ or $x - 5 = 0$
never $\quad x = 5$
$x = 0, 5$

45. $9^{x^2} = 3^{3x-1}$
$(3^2)^{x^2} = 3^{3x-1}$
$3^{2x^2} = 3^{3x-1}$ if and only if
$2x^2 = 3x - 1$
$2x^2 - 3x + 1 = 0$
$(2x-1)(x-1) = 0$
$x = \frac{1}{2}, 1$

47. $25^{x+3} = 125^x$
$(5^2)^{x+3} = (5^3)^x$
$5^{2x+6} = 5^{3x}$ if and only if
$2x + 6 = 3x$
$x = 6$

49. $4^{2x+7} = 8^{x+2}$
$(2^2)^{2x+7} = (2^3)^{x+2}$
$2^{4x+14} = 2^{3x+6}$ if and only if
$4x + 14 = 3x + 6$
$x = -8$

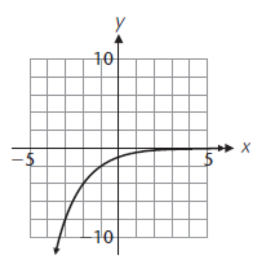
51. $a^2 = a^{-2}$
$a^2 = \frac{1}{a^2}$
$a^4 = 1 \quad (a \neq 0)$
$a^4 - 1 = 0$
$(a-1)(a+1)(a^2+1) = 0$
$a = 1$ or $a = -1$

This does not violate the exponential property mentioned because $a = 1$ and a negative are excluded from consideration in the statement of the property.

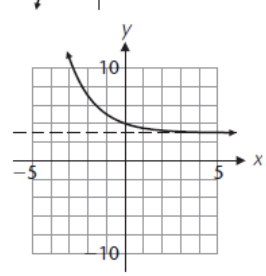
53. $1^{-3} = \frac{1}{1^3} = 1, 1^{-2} = \frac{1}{1^2} = 1, 1^{-1} = \frac{1}{1^1} = 1,$
$1^0 = 1, 1^2 = 1, 1^3 = 1.$

$1^x = 1$ for all real x; the function $f(x) = 1^x$ is neither increasing nor decreasing and is equal to $f(x) = 1$, thus the variable is effectively not in the exponent at all.

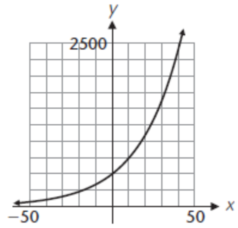
55. The graph of g is the same as the graph of f reflected through the x axis; g is increasing; horizontal asymptote: $y = 0$.



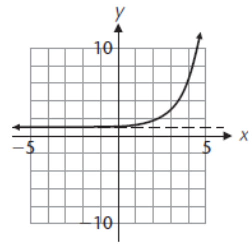
57. The graph of g is the same as the graph of f stretched horizontally by a factor of 2 and shifted upward 3 units; g is decreasing; horizontal asymptote: $y = 3$.



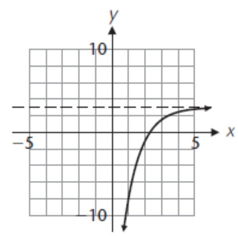
59. The graph of g is the same as the graph of f stretched vertically by a factor of 500; g is increasing; horizontal asymptote: $y = 0$.



61. The graph of g is the same as the graph of f shifted 3 units to the right, stretched vertically by a factor of 2, and shifted upward 1 unit; g is increasing; horizontal asymptote: $y = 1$.



63. The graph of g is the same as the graph of f shifted 2 units to the right, reflected in the origin, stretched vertically by a factor of 4, and shifted upward 3 units; g is increasing; horizontal asymptote: $y = 3$.



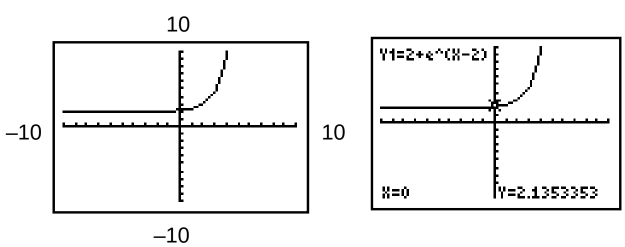
65. $$\frac{-2x^3e^{-2x} - 3x^2e^{-2x}}{x^6} = \frac{x^2e^{-2x}(-2x-3)}{x^6} = \frac{e^{-2x}(-2x-3)}{x^4}$$

67. $(e^x + e^{-x})^2 + (e^x - e^{-x})^2 = (e^x)^2 + 2(e^x)(e^{-x}) + (e^{-x})^2 + (e^x)^2 - 2(e^x)(e^{-x}) + (e^{-x})^2$

$= e^{2x} + 2 + e^{-2x} + e^{2x} - 2 + e^{-2x}$
$= 2e^{2x} + 2e^{-2x}$

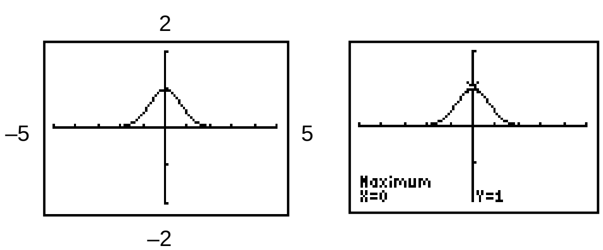
> **Common Errors:**
> $(e^x)^2 \neq e^{x^2}$
> $e^{2x} + e^{2x} \neq e^{4x}$

69. Examining the graph of $y = f(x)$, we obtain



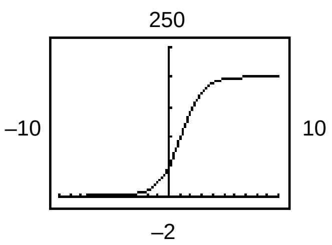
There are no local extrema and no x intercepts. The y intercept is 2.14. As $x \to -\infty$, $y \to 2$, so the line $y = 2$ is a horizontal asymptote.

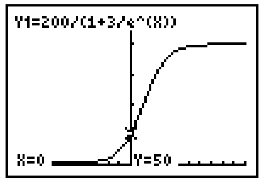
71. Examining the graph of $y = s(x)$, we obtain



There is a local maximum at $s(0) = 1$, and 1 is the y intercept. There is no x intercept. As $x \to \infty$ or $x \to -\infty$, $y \to 0$, so the line $y = 0$ (the x axis) is a horizontal asymptote

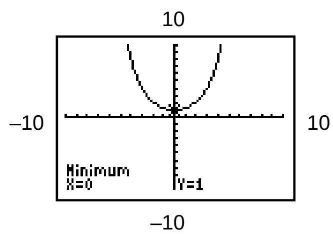
73. Examining the graph of $y = F(x)$, we obtain





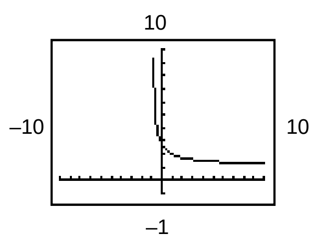
There are no local extrema and no x intercepts.
When $x = 0$, $F(0) = \dfrac{200}{1+3e^{-0}} = 50$ is the y intercept.
As $x \to -\infty$, $y \to 0$, so the line $y = 0$ (the x axis) is a horizontal asymptote. As $x \to \infty$, $y \to 200$, so the line $y = 200$ is also a horizontal asymptote.

75.

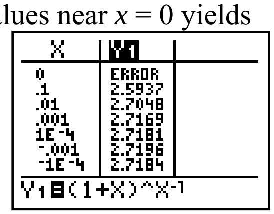


The local minimum is $f(0) = 1$, so zero is the y intercept. There are no x intercepts or horizontal asymptotes; $f(x) \to \infty$ as $x \to \infty$ and $x \to -\infty$.

77. Examining the graph of $y = f(x)$, we obtain



As $x \to 0$, $f(x) = (1 + x)^{1/x}$ seems to approach a value near 3. A table of values near $x = 0$ yields



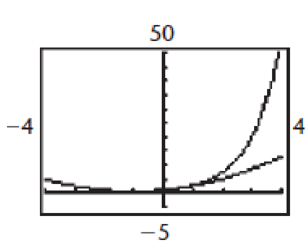
Although $f(0)$ is not defined, as $x \to 0$, $f(x)$ seems to approach a number near 2.718. In fact, it approaches e, since as $x \to 0$, $u = \dfrac{1}{x} \to \infty$, and $f(x) = \left(1+\dfrac{1}{u}\right)^u$ must approach e as $u \to \infty$.

79. Make a table of values, substituting in each requested x value:

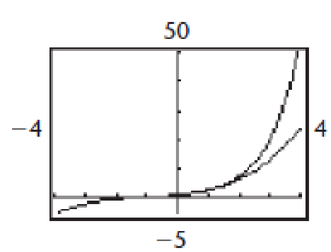
x	1.4	1.41	1.414	1.4142	1.41421	1.414214
2^x	2.639016	2.657372	2.664750	2.665119	2.665138	2.665145

The approximate value of $2^{\sqrt{2}}$ is 2.665145 to six decimal places. Using a calculator to compute directly, we get 2.665144.

81.

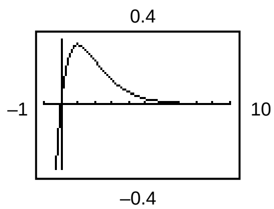


83.

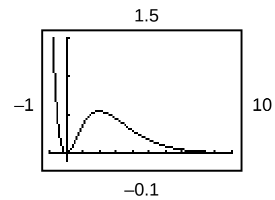


85. Here are graphs of $f_1(x) = \frac{x}{e^x}$, $f_2(x) = \frac{x^2}{e^x}$, and $f_3(x) = \frac{x^3}{e^x}$. In each case as $x \to \infty$, $f_n(x) \to 0$. The line $y = 0$ is a horizontal asymptote. As $x \to -\infty$, $f_1(x) \to -\infty$ and $f_3(x) \to -\infty$, while $f_2(x) \to \infty$. It appears that as $x \to -\infty$, $f_n(x) \to \infty$ if n is even and $f_n(x) \to -\infty$ if n is odd.

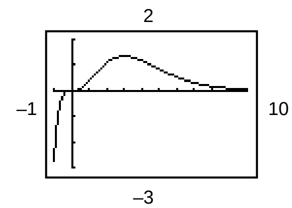
f_1:



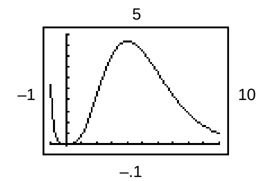
f_2:



f_3:



As confirmation of these observations, we show the graph of $f_4 = \frac{x^4}{e^x}$ (not required).



87. We use the compound interest formula

$A = P\left(1 + \frac{r}{m}\right)^n$ to find P: $P = \frac{A}{\left(1+\frac{r}{m}\right)^n}$

$m = 365$ $r = 0.0625$ $A = 100{,}000$ $n = 365\cdot 17$

$P = \frac{100{,}000}{\left(1+\frac{0.0625}{365}\right)^{365\cdot 17}}$ = \$34,562.00 to the nearest dollar

89. We use the Continuous Compound Interest Formula

$A = Pe^{rt}$

$P = 5{,}250$ $r = 0.0638$

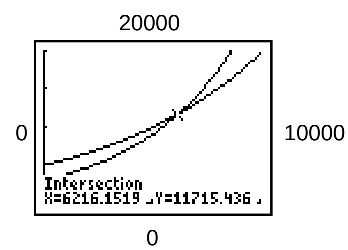
(A) $t = 6.25$ $A = 5{,}250e^{(0.0638)(6.25)} = \$7822.30$

(B) $t = 17$ $A = 5{,}250e^{(0.0638)(17)} = \$15{,}530.85$

91. We use the compound interest formula $A = P\left(1 + \frac{r}{m}\right)^n$

For the first account, $P = 3000$, $r = 0.08$, $m = 365$. Let $y_1 = A$, then $y_1 = 3000(1 + 0.08/365)^x$ where x is the number of compounding periods (days). For the second account, $P = 5000$, $r = 0.05$, $m = 365$. Let $y_2 = A$, then $y_2 = 5000(1 + 0.05/365)^x$

Examining the graphs of y_1 and y_2, we obtain the graphs at the right. The graphs intersect at $x = 6216.15$ days. Comparing the amounts in the accounts, we see that the first account is worth more than the second for $x \geq 6217$ days.

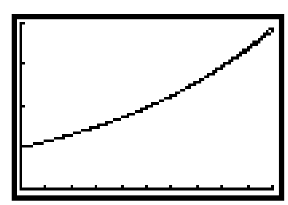


X	Y1	Y2
6215	11712	11714
6216	11715	11715
6217	11718	11717
6218	11720	11718
6219	11723	11720
6220	11725	11722
6221	11728	11723

X=6218

93. We use the compound interest formula $A = P\left(1 + \frac{r}{m}\right)^n$

For the first account, $P = 10{,}000$, $r = 0.049$, $m = 365$. Let $y_1 = A$, then $y_1 = 10000(1 + 0.049/365)^x$ where x is the number of compounding periods (days). For the second account, $P = 10{,}000$, $r = 0.05$, $m = 4$. Let $y_2 = A$, then $y_2 = 10000(1 + 0.05/4)^{4x/365}$ where x is the number of days. Examining the graphs of y_1 and y_2, we obtain the graph at the right. The two graphs are just about indistinguishable from one another. Examining a table of values, we obtain:



X	Y_1	Y_2
0	10000	10000
91.25	10123	10125
182.5	10248	10252
273.75	10374	10380
365	10502	10509
456.25	10632	10641
547.5	10763	10774

X=0

X	Y_1	Y_2
638.75	10895	10909
730	11030	11045
821.25	11165	11183
912.5	11303	11323
1003.8	11442	11464
1095	11583	11608
1186.3	11726	11753

X=1186.25

The two accounts are extremely close in value, but the second account is always larger than the first. The first will never be larger than the second.

95. We use the Continuous Compound Interest Formula

$A = Pe^{rt}$

$P = \dfrac{A}{e^{rt}}$ or $P = Ae^{-rt}$

$A = 30{,}000 \quad r = 0.06 \quad t = 10$

$P = 30{,}000e^{(-0.06)(10)}$

$P = \$16{,}464.35$

97. We use the compound interest formula $A = P\left(1+\dfrac{r}{m}\right)^n$

Flagstar Bank: $P = 5{,}000 \quad r = 0.0312 \quad m = 4 \quad n = (4)(3)$

$$A = 5{,}000\left(1+\frac{0.0312}{4}\right)^{(4)(3)} = \$5{,}488.61$$

UmbrellaBank.com: $P = 5{,}000 \quad r = 0.03 \quad m = 365 \quad n = (365)(3)$

$$A = 5{,}000\left(1+\frac{0.03}{365}\right)^{(365)(3)} = \$5{,}470.85$$

Allied First Bank: $P = 5{,}000 \quad r = 0.0296 \quad m = 12 \quad n = (12)(3)$

$$A = 5{,}000\left(1+\frac{0.0296}{12}\right)^{(12)(3)} = \$5{,}463.71$$

99. We use the compound interest formula

$A = P\left(1+\dfrac{r}{m}\right)^n \quad m = 52$ [Note: If $m = 365/7$ is used the answers will differ very slightly.]

$P = 4{,}000 \quad r = 0.06$

$$A = 4{,}000\left(1+\frac{0.06}{52}\right)^n$$

(A) $n = (52)(0.5)$, hence

$$A = 4{,}000\left(1+\frac{0.06}{52}\right)^{(52)(0.5)}$$

$= \$4{,}121.75$

(B) $n = (52)(10) = 520$, hence

$$A = 4{,}000\left(1+\frac{0.06}{52}\right)^{520}$$

$= \$7{,}285.95$

Section 5-2

1. Doubling time is the time it takes a population to double. Half-life is the time it takes for half of an initial quantity of a radioactive substance to decay.

3. Exponential growth is the simple model $A = A_0e^{kt}$, i.e. unlimited growth. Limited growth models more realistically incorporate the fact that there is a reasonable maximum value for A.

5. Use the doubling time model $A = A_0(2)^{t/d}$ with $A_0 = 200, d = 5$. $A = 200(2)^{t/5}$

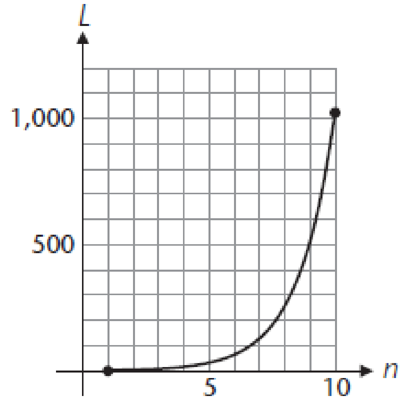
7. Use the continuous growth model $A = A_0e^{rt}$ with $A_0 = 2{,}000, r = 0.02$. $A = 2{,}000e^{0.02t}$

9. Use the half-life model $A = A_0\left(\dfrac{1}{2}\right)^{t/h}$ with $A_0 = 100, h = 6$. $A = 100\left(\dfrac{1}{2}\right)^{t/6}$

11. Use the exponential decay model $A = A_0e^{-kt}$ with $A_0 = 4, k = 0.124$. $A = 4e^{-0.124t}$

13.

n	L
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024



15. Use the doubling time model: $P = P_0 2^{t/d}$
Substituting $P_0 = 10$ and $d = 2.4$, we have
$P = 10(2^{t/2.4})$

(A) $t = 7$, hence $P = 10(2^{7/2.4})$
$= 75.5$ 76 flies

(B) $t = 14$, hence $P = 10(2^{14/2.4})$
$= 570.2$ 570 flies

17. Use the doubling time model $A = A_0\left(\frac{1}{2}\right)^{t/h}$

with $A_0 = 2{,}200$, $d = 2$. $A = 2{,}200(2)^{t/2}$ where t is years after 1970.

(A) For $t = 20$: $A = 2{,}200(2)^{20/2} = 2{,}252{,}800$

(B) For $t = 35$: $A = 2{,}200(2)^{35/2} = 407{,}800{,}360$

19. Use the half-life model $A = A_0\left(\frac{1}{2}\right)^{t/h} = A_0 2^{-t/h}$

Substituting $A_0 = 25$ and $h = 12$, we have
$A = 25(2^{-t/12})$

(A) $t = 5$, hence $A = 25(2^{-5/12}) = 19$ pounds

(B) $t = 20$, hence $A = 25(2^{-20/12}) = 7.9$ pounds

21. Use the continuous growth model $A = A_0 e^{rt}$ with $A_0 = 6.8$, $r = 0.01188$, $t = 2020 - 2008 = 12$
$A = 6.8e^{0.01188(12)} = 7.8$ billion

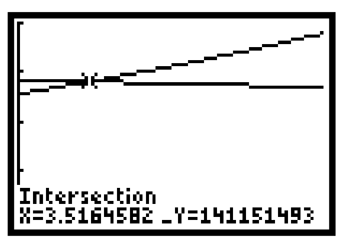
23. Use the continuous growth model $A = A_0 e^{rt}$.

Let A_1 = the population of Russia and
A_2 = the population of Nigeria.

For Russia, $A_0 = 1.43 \times 10^8$, $r = -0.0037$
$A_1 = 1.43 \times 10^8 e^{-0.0037t}$

For Nigeria, $A_0 = 1.29 \times 10^8$, $r = 0.0256$
$A_2 = 1.29 \times 10^8 e^{0.0256t}$

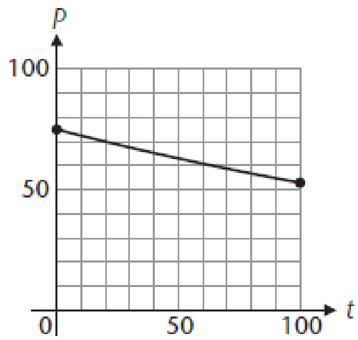
Below is a graph of A_1 and A_2.



From the graph, assuming $t = 0$ in 2005, it appears that the two populations became equal when t was approximately 3.5, in 2008. After that the population of Nigeria will be greater than that of Russia.

25. A table of values can be generated by a graphing calculator and yields

t	P
0	75
10	72
20	70
30	68
40	65
50	63
60	61
70	59
80	57
90	55
100	53



27. $I = I_0 e^{-0.00942d}$

(A) $d = 50$ $I = I_0 e^{-0.00942(50)} = 0.62 I_0$ 62%

(B) $d = 100$ $I = I_0 e^{-0.00942(100)} = 0.39 I_0$ 39%

29. Use the continuous growth model $A = A_0 e^{rt}$ with $A_0 = 33.2$ million, $r = 0.0237$

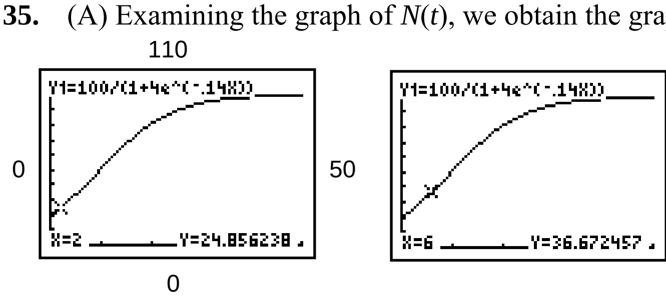
(A) In 2014, assuming $t = 0$ in 2007, substitute $t = 7$.
$A = 33.2e^{0.0237(7)} = 39.2$ million

(B) In 2020, substitute $t = 13$.
$A = 33.2e^{0.0237(13)} = 45.2$ million

31. $T = T_m + (T_0 - T_m)e^{-kt}$
$T_m = 40°$ $T_0 = 72°$ $k = 0.4$ $t = 3$
$T = 40 + (72 - 40)e^{-0.4(3)}$
$T = 50°$

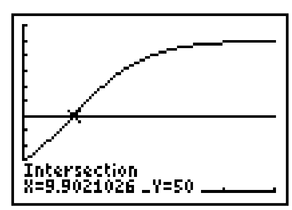
33. As t increases without bound, $e^{-0.2t}$ approaches 0, hence $q = 0.0009(1 - e^{-0.2t})$ approaches 0.0009. Hence 0.0009 coulomb is the maximum charge on the capacitor.

35. (A) Examining the graph of $N(t)$, we obtain the graphs below.



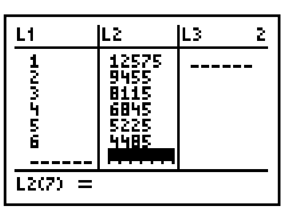
After 2 years, 25 deer will be present. After 6 years, 37 deer will be present.

(B) Applying a built-in routine, we obtain the graph at the right. It will take 10 years for the herd to grow to 50 deer.

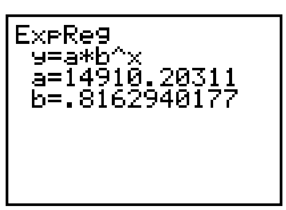


(C) As t increases without bound, $e^{-0.14t}$ approaches 0, hence $N = \frac{100}{1+4e^{-0.14t}}$ approaches 100. Hence 100 is the number of deer the island can support.

37.

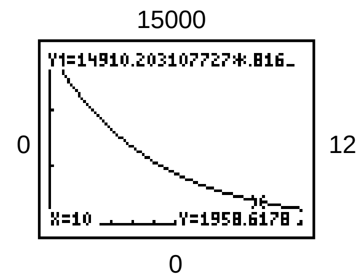


Enter the data.

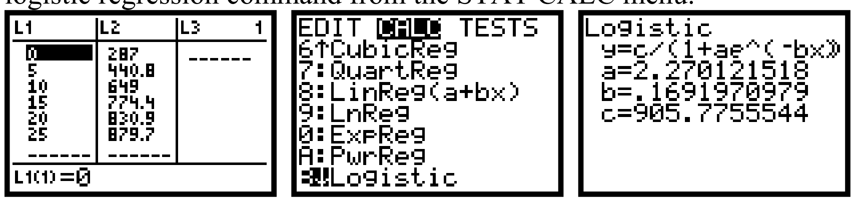


Compute the regression equation.

The model gives $y = 14910.20311(0.8162940177)^x$.
Clearly, when $x = 0$, $y = \$14,910$ is the estimated purchase price.
Applying a built-in routine, we obtain the graph at the right.
When $x = 10$, the estimated value of the van is \$1,959.



39. (A) The independent variable is years since 1980, so enter 0, 5, 10, 15, 20, and 25 as L_1. The dependent variable is power generation in North America, so enter the North America column as L_2. Then use the logistic regression command from the STAT CALC menu.



The model is

$$y = \frac{906}{1+2.27e^{-0.169x}}$$

(B) Since $x = 0$ corresponds to 1980, use $x = 30$ to predict power generation in 2010.

$$y = \frac{906}{1+2.27e^{-0.169(30)}} = 893.3 \text{ billion kilowatt hours}$$

Use $x = 40$ to predict power generation in 2020.

$$y = \frac{906}{1+2.27e^{-0.169(40)}} = 903.6 \text{ billion kilowatt hours}$$

Section 5-3

1. The exponential function $f(x) = b^x$ for $b > 0$, $b \neq 1$ and the logarithmic function $g(x) = \log_b x$ are inverse functions for each other.

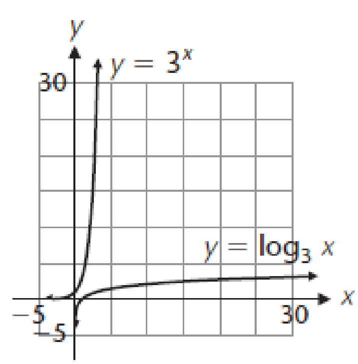
3. The range of the exponential function is the positive real numbers, hence the domain of the logarithmic function must also be the positive real numbers.

5. $\log_5 3 = \log_e 3/\log_e 5$ or $\log_{10}3/\log_{10}5$.

7. $81 = 3^4$ **9.** $0.001 = 10^{-3}$ **11.** $\frac{1}{36} = 6^{-2}$ **13.** $\log_4 8 = \frac{3}{2}$ **15.** $\log_{32}\frac{1}{2} = -\frac{1}{5}$ **17.** $\log_{2/3}\frac{8}{27} = 3$

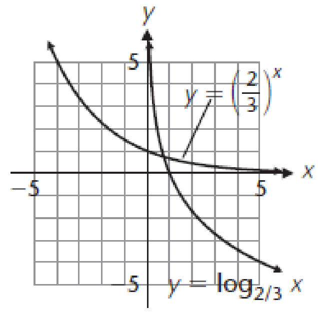
19. Make a table of values for each function:

x	$f(x)=3^x$	x	$f^{-1}(x)=\log_3 x$
–3	1/27	1/27	–3
–2	1/9	1/9	–2
–1	1/3	1/3	–1
0	1	1	0
1	3	3	1
2	9	9	2
3	27	27	3



21. Make a table of values for each function:

x	$f(x)=(2/3)^x$	x	$f^{-1}(x)=\log_{2/3} x$
–3	27 / 8	27 / 8	–3
–2	9 / 4	9 / 4	–2
–1	3 / 2	3 / 2	–1
0	1	1	0
1	2 / 3	2 / 3	1
2	4 / 9	4 / 9	2
3	8 / 27	8 / 27	3



23. 0 **25.** 1 **27.** 4 **29.** $\log_{10} 0.01 = \log_{10} 10^{-2} = -2$ **31.** $\log_3 27 = \log_3 3^3 = 3$

33. $\log_{1/2} 2 = \log_{1/2}\left(\frac{1}{2}\right)^{-1} = -1$ **35.** 5 **37.** $\log_5 \sqrt[3]{5} = \log_5 5^{1/3} = \frac{1}{3}$ **39.** 4.6923 **41.** 3.9905

43. $\log_7 13 = \frac{\ln 13}{\ln 7} = 1.3181$
using the change of base formula

45. $\log_5 120.24 = \frac{\ln 120.24}{\ln 5} = 2.9759$
using the change of base formula

47. $x = 10^{5.3027} = 200{,}800$

49. $x = 10^{-3.1773} = 6.648 \times 10^{-4} = 0.0006648$

51. $x = e^{3.8655} = 47.73$

53. $x = e^{-0.3916} = 0.6760$

55. Write $\log_2 x = 2$ in equivalent exponential form.
$x = 2^2 = 4$

57. $\log_4 16 = \log_4 4^2 = 2$
$y = 2$

59. Write $\log_b 16 = 2$ in equivalent exponential form.
$16 = b^2$
$b^2 = 16$
$b = 4$
since bases are required to be positive

61. Write $\log_b 1 = 0$ in equivalent exponential form.
$1 = b^0$
This statement is true if b is any real number except 0. However, bases are required to be positive and 1 is not allowed, so the original statement is true if b is any positive real number except 1.

63. Write $\log_4 x = \frac{1}{2}$ in equivalent exponential form.
$x = 4^{1/2} = 2$

65. $\log_{1/3} 9 = \log_{1/3} 3^2 = \log_{1/3} \frac{1}{\left(\frac{1}{3}\right)^2} = \log_{1/3}\left(\frac{1}{3}\right)^{-2} = -2$

67. Write $\log_b 1000 = \frac{3}{2}$ in equivalent exponential form
$1000 = b^{3/2}$
$10^3 = b^{3/2}$
$(10^3)^{2/3} = (b^{3/2})^{2/3}$
(If two numbers are equal the results are equal if they are raised to the same exponent.)
$10^{3(2/3)} = b^{3/2(2/3)}$
$10^2 = b$
$b = 100$

69. Write $\log_8 x = -\frac{4}{3}$ in equivalent exponential form.
$8^{-4/3} = x$
$x = (8^{1/3})^{-4} = 2^{-4} = \frac{1}{16}$

71. Write $\log_{16} 8 = y$ in equivalent exponential form.
$16^y = 8$
$(2^4)^y = 2^3$
$2^{4y} = 2^3$ if and only if
$4y = 3$
$y = \frac{3}{4}$

73. 4.959 **75.** 7.861 **77.** 2.280 **79.** $\log x - \log y$

81. $\log(x^4y^3) = \log x^4 + \log y^3 = 4\log x + 3\log y$

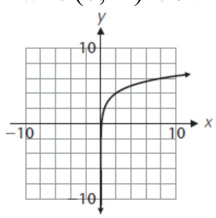
83. $\ln\left(\frac{x}{y}\right)$

85. $2\ln x + 5\ln y - \ln z = \ln x^2 + \ln y^5 - \ln z = \ln(x^2y^5) - \ln z$
$= \ln\left(\frac{x^2y^5}{z}\right)$

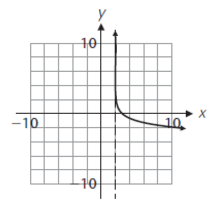
87. $\log(xy) = \log x + \log y = -2 + 3 = 1$

89. $\log\left(\frac{\sqrt{x}}{y^3}\right) = \log\sqrt{x} - \log y^3 = \frac{1}{2}\log x - 3\log y = \frac{1}{2}(-2) - 3\cdot 3 = -10$

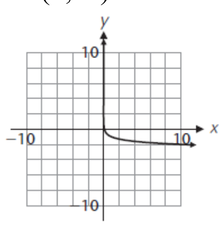
91. The graph of g is the same as the graph of f shifted upward 3 units; g is increasing.
Domain: $(0, \infty)$ Vertical asymptote: $x = 0$



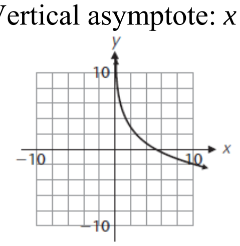
93. The graph of g is the same as the graph of f shifted 2 units to the right; g is decreasing.
Domain: $(2, \infty)$ Vertical asymptote: $x = 2$



95. The graph of g is the same as the graph of f reflected through the x axis and shifted downward 1 unit; g is decreasing.
Domain: $(0, \infty)$ Vertical asymptote: $x = 0$



97. The graph of g is the same as the graph of f reflected through the x axis, stretched vertically by a factor of 3, and shifted upward 5 units. g is decreasing. Domain: $(0, \infty)$
Vertical asymptote: $x = 0$

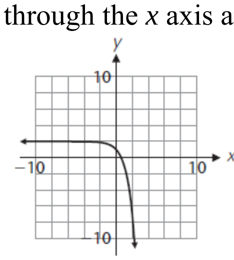


99. Write $y = \log_5 x$
In exponential form:
$5^y = x$
Interchange x and y:
$5^x = y$
Therefore $f^{-1}(x) = 5^x$.

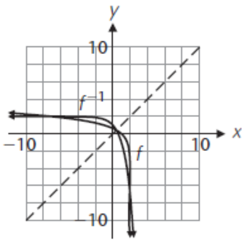
101. Write $y = 4\log_3(x + 3)$
$\frac{y}{4} = \log_3(x + 3)$
In exponential form:
$3^{y/4} = x + 3$
$x = 3^{y/4} - 3$
Interchange x and y:
$y = 3^{x/4} - 3$
Therefore $f^{-1}(x) = 3^{x/4} - 3$

103. (A) Write $y = \log_3(2 - x)$
In exponential form:
$3^y = 2 - x$
$x = 2 - 3^y$
Interchange x and y:
$y = 2 - 3^x$
Therefore, $f^{-1}(x) = 2 - 3^x$

(B) The graph is the same as the graph of $y = 3^x$ reflected through the x axis and shifted 2 units upward.

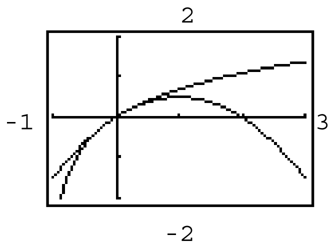


(C)

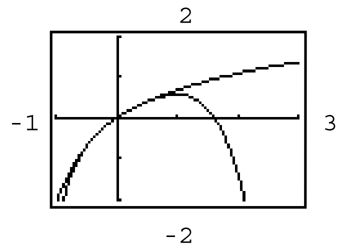


105. The inequality sign in the last step reverses because $\log \frac{1}{3}$ is negative.

107.



109.



111. Let $u = \log_b M$ and $v = \log_b N$. Changing each equation to exponential form, $b^u = M$ and $b^v = N$.

Then we can write M/N as $\frac{M}{N} = \frac{b^u}{b^v} = b^{u-v}$ using a familiar property of exponents. Now change this equation to logarithmic form: $\log_b \left(\frac{M}{N}\right) = u - v$

Finally, recall the way we defined u and v in the first line of our proof: $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$

Section 5-4

1. Answers will vary.

3. The intensity of a sound and the energy released by an earthquake can vary from extremely small to extremely large. A logarithmic scale can condense this variation into a range that can be easily comprehended.

5. We use the decibel formula $D = 10 \log \frac{I}{I_0}$

(A) $I = I_0$

$D = 10 \log \frac{I_0}{I_0}$

$D = 10 \log 1$

$D = 0$ decibels

(B) $I_0 = 1.0 \times 10^{-12}$ $I = 1.0$

$D = 10 \log \frac{1.0}{1.0 \times 10^{-12}}$

$D = 120$ decibels

7. We use the decibel formula

$D = 10 \log \frac{I}{I_0}$

$I_2 = 1000 I_1$

$D_1 = 10 \log \frac{I_1}{I_0}$ $D_2 = 10 \log \frac{I_2}{I_0}$

$$D_2 - D_1 = 10 \log \frac{I_2}{I_0} - 10 \log \frac{I_1}{I_0}$$

$$= 10 \log \left(\frac{I_2}{I_0} \div \frac{I_1}{I_0}\right) = 10 \log \frac{I_2}{I_1}$$

$$= 10 \log \frac{1000 I_1}{I_1} = 10 \log 1000 = 30 \text{ decibels}$$

9. We use the magnitude formula

$M = \frac{2}{3} \log \frac{E}{E_0}$ with $E = 1.99 \times 10^{-17}$, $E_0 = 10\ 4.40$

$M = \frac{2}{3} \log \frac{1.99 \times 10^{-17}}{10^{4.40}} = 8.6$

11. We use the magnitude formula $M = \frac{2}{3} \log \frac{E}{E_0}$

For the Long Beach earthquake,

$$6.3 = \frac{2}{3} \log \frac{E_1}{E_0}$$

$$9.45 = \log \frac{E_1}{E_0}$$

(Change to exponential form)

$$\frac{E_1}{E_0} = 10^{9.45}$$

$$E_1 = E_0 \cdot 10^{9.45}$$

For the Anchorage earthquake,

$$8.3 = \frac{2}{3} \log \frac{E_2}{E_0}$$

$$12.45 = \log \frac{E_2}{E_0}$$

(Change to exponential form)

$$\frac{E_2}{E_0} = 10^{12.45}$$

$$E_2 = E_0 \cdot 10^{12.45}$$

Now we can compare the energy levels by dividing the more powerful (Anchorage) by the less (Long Beach):

$$\frac{E_2}{E_1} = \frac{E_0 \cdot 10^{12.45}}{E_0 \cdot 10^{9.45}} = 10^3$$

$E_2 = 10^3 E_1$, or 1000 times as powerful

13. Use the magnitude formula $M = \frac{2}{3}\log\frac{E}{E_0}$ with $E = 1.34\times10^{14}, E_0 = 10^{4.40}$: $M = \frac{2}{3}\log\frac{1.34\times10^{14}}{10^{4.40}} = 6.5$

15. Use the magnitude formula $M = \frac{2}{3}\log\frac{E}{E_0}$ with $E = 2.38\times10^{21}, E_0 = 10^{4.40}$: $M = \frac{2}{3}\log\frac{2.38\times10^{21}}{10^{4.40}} = 11.3$

17. We use the rocket equation.

$$v = c \ln \frac{W_t}{W_b}$$

$v = 2.57 \ln (19.8)$

$v = 7.67$ km/s

19. (A) $pH = -\log[H^+] = -\log(4.63 \times 10^{-9}) = 8.3$.
Since this is greater than 7, the substance is basic.

(B) $pH = -\log[H^+] = -\log(9.32 \times 10^{-4}) = 3.0$
Since this is less than 7, the substance is acidic.

21. Since $pH = -\log[H^+]$, we have

$5.2 = -\log[H^+]$, or

$[H^+] = 10^{-5.2} = 6.3 \times 10^{-6}$ moles per liter

23. $m = 6 - 2.5 \log \frac{L}{L_0}$

(A) We find m when $L = L_0$

$$m = 6 - 2.5 \log \frac{L_0}{L_0}$$

$m = 6 - 2.5 \log 1$

$m = 6$

(B) We compare L_1 for $m = 1$ with L_2 for $m = 6$

$$1 = 6 - 2.5 \log \frac{L_1}{L_0} \qquad 6 = 6 - 2.5 \log \frac{L_2}{L_0}$$

$$-5 = -2.5 \log \frac{L_1}{L_0} \qquad 0 = -2.5 \log \frac{L_2}{L_0}$$

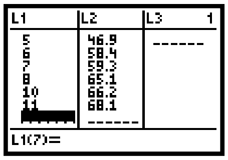
$$2 = \log \frac{L_1}{L_0} \qquad 0 = \log \frac{L_2}{L_0}$$

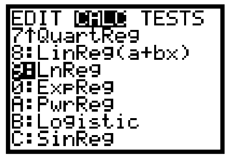
$$\frac{L_1}{L_0} = 10^2 \qquad \frac{L_2}{L_0} = 1$$

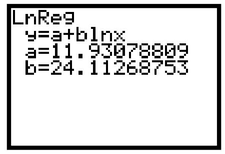
$$L_1 = 100L_0 \qquad L_2 = L_0$$

Hence $\frac{L_1}{L_2} = \frac{100L_0}{L_0} = 100$. The star of magnitude 1 is 100 times brighter.

25. (A) Enter the years since 1995 as L_1. Enter the values shown in the column headed “% with home access” as L_2. Use the logarithmic regression model from the STAT CALC menu.







The model is $y = 11.9 + 24.1 \ln x$. Evaluating this for $x = 13$ (year 2008) yields 73.7%. Evaluating for $x = 20$ (year 2015) yields 84.1%.
(B) No; the predicted percentage goes over 100 sometime around 2034.

Section 5-5

1. The logarithm function is the inverse of the exponential function, moreover, $\log_b M^p = p\log_b M$. This property of logarithms can often be used to get a variable out of an exponent in solving an equation.

3. If $\log_b u = \log_b v$, then $u = v$ because the logarithm is a one-to-one function.

5. $(\ln x)^2$ means to take the logarithm of x, then square the result.

$\ln x^2$ means to square x, then take the logarithm of the result.

7.
$$10^{-x} = 0.0347$$
$$-x = \log_{10} 0.0347$$
$$x = -\log_{10} 0.0347$$
$$x = 1.46$$

9.
$$10^{3x+1} = 92$$
$$3x + 1 = \log_{10} 92$$
$$3x = \log_{10} 92 - 1$$
$$x = \frac{\log_{10} 92 - 1}{3}$$
$$x = 0.321$$

11.
$$e^x = 3.65$$
$$x = \ln 3.65$$
$$x = 1.29$$

13.
$$e^{2x-1} + 68 = 207$$
$$e^{2x-1} = 139$$
$$2x - 1 = \ln 139$$
$$x = \frac{1 + \ln 139}{2}$$
$$x = 2.97$$

15.
$$2^3 2^{-x} = 0.426$$
$$2^{-x} = \frac{0.426}{2^3}$$
$$\ln 2^{-x} = \ln \frac{0.426}{8}$$
$$-x \ln 2 = \ln \frac{0.426}{8}$$
$$x = \frac{\ln \frac{0.426}{8}}{-\ln 2}$$
$$x = 4.23$$

17.
$$\log_5 x = 2$$
$$5^2 = x$$
$$x = 25$$

19.
$$\log(t - 4) = -1$$
$$10^{-1} = t - 4$$
$$t = 4 + 10^{-1}$$
$$t = 4 + \frac{1}{10}$$
$$t = \frac{41}{10}$$

21.
$$\log 5 + \log x = 2$$
$$\log(5x) = 2$$
$$5x = 10^2$$
$$5x = 100$$
$$x = 20$$

23.
$$\log x + \log(x - 3) = 1$$
$$\log[x(x - 3)] = 1$$
$$x(x - 3) = 10^1$$
$$x^2 - 3x = 10$$
$$x^2 - 3x - 10 = 0$$
$$(x - 5)(x + 2) = 0$$
$$x = 5 \text{ or } -2$$

> **Common Error:**
> $\log(x - 3) \neq \log x - \log 3$

Check:
$\log 5 + \log(5 - 3) \overset{\surd}{=} 1$
$\log(-2) + \log(-2 - 3)$ is not defined.
$x = 5$

25.
$$\log(x + 1) - \log(x - 1) = 1$$
$$\log \frac{x+1}{x-1} = 1$$
$$\frac{x+1}{x-1} = 10^1$$
$$\frac{x+1}{x-1} = 10$$
$$x + 1 = 10(x - 1)$$
$$x + 1 = 10x - 10$$
$$11 = 9x$$
$$x = \frac{11}{9}$$

> **Common Error:**
> $\frac{x+1}{x-1} \neq \log 1$

Check:
$$\log\left(\frac{11}{9} + 1\right) - \log\left(\frac{11}{9} - 1\right) \overset{?}{=} 1$$
$$\log \frac{20}{9} - \log \frac{2}{9} \overset{?}{=} 1$$
$$\log 10 \overset{\surd}{=} 1$$

27.

$$2 = 1.05^x$$
$$\ln 2 = x \ln 1.05$$
$$\frac{\ln 2}{\ln 1.05} = x$$
$$x = 14.2$$

29. $e^{-1.4x} + 5 = 0$

No solution. Both terms on the left side are always positive, so they can never add to 0.

31.

$$123 = 500e^{-0.12x}$$
$$\frac{123}{500} = e^{-0.12x}$$
$$\ln\left(\frac{123}{500}\right) = -0.12x$$
$$\frac{\ln\left(\frac{123}{500}\right)}{-0.12} = x$$
$$x = 11.7$$

33.

$$e^{-x^2} = 0.23$$
$$-x^2 = \ln 0.23$$
$$x^2 = -\ln 0.23$$
$$x = \pm\sqrt{-\ln 0.23}$$
$$x = \pm 1.21$$

35.

$$\log(5 - 2x) = \log(3x + 1)$$
$$5 - 2x = 3x + 1$$
$$4 = 5x$$
$$x = \frac{4}{5}$$

37.

$$\log x - \log 5 = \log 2 - \log(x - 3)$$
$$\log \frac{x}{5} = \log \frac{2}{x-3}$$
$$\frac{x}{5} = \frac{2}{x-3}$$

Excluded value: $x \neq 3$

$$5(x-3)\,\frac{x}{5} = 5(x-3)\,\frac{2}{x-3}$$
$$(x - 3)x = 10$$
$$x^2 - 3x = 10$$
$$x^2 - 3x - 10 = 0$$
$$(x - 5)(x + 2) = 0$$
$$x = 5, -2$$

Solution: 5

Check:

$\log 5 - \log 5 \overset{\surd}{=} \log 2 - \log 2$

$\log(-2)$ is not defined

39.

$$\ln x = \ln(2x - 1) - \ln(x - 2)$$
$$\ln x = \ln \frac{2x-1}{x-2}$$
$$x = \frac{2x-1}{x-2}$$

Excluded value: $x \neq 2$

$$x(x - 2) = (x - 2)\,\frac{2x-1}{x-2}$$
$$x(x - 2) = 2x - 1$$
$$x^2 - 2x = 2x - 1$$
$$x^2 - 4x + 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$a = 1,\ b = -4,\ c = 1$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

Check:

$$\ln(2 + \sqrt{3}) \overset{?}{=} \ln[2(2 + \sqrt{3}) - 1] - \ln[(2 + \sqrt{3}) - 2]$$
$$\ln(2 + \sqrt{3}) \overset{?}{=} \ln(3 + 2\sqrt{3}) - \ln\sqrt{3}$$
$$\ln(2 + \sqrt{3}) \overset{?}{=} \ln\left(\frac{3 + 2\sqrt{3}}{\sqrt{3}}\right)$$
$$\ln(2 + \sqrt{3}) \overset{\surd}{=} \ln(\sqrt{3} + 2)$$

$\ln(x - 2)$ is not defined if $x = 2 - \sqrt{3}$

Solution: $2 + \sqrt{3}$

41.

$$\begin{aligned}\log(2x+1) &= 1 - \log(x-1)\\ \log(2x+1) + \log(x-1) &= 1\\ \log[(2x+1)(x-1)] &= 1\\ (2x+1)(x-1) &= 10\\ 2x^2 - x - 1 &= 10\\ 2x^2 - x - 11 &= 0\\ x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a}\\ a = 2,\ b = -1,\ c &= -11\\ x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-11)}}{2(2)}\\ x &= \frac{1 \pm \sqrt{89}}{4}\end{aligned}$$

Check:
$$\begin{aligned}\log\left(2\frac{1+\sqrt{89}}{4}+1\right) &\stackrel{?}{=} 1 - \log\left(\frac{1+\sqrt{89}}{4}-1\right)\\ \log\left(\frac{1+\sqrt{89}+2}{2}\right) &\stackrel{?}{=} 1 - \log\left(\frac{1+\sqrt{89}-4}{4}\right)\\ \log\left(\frac{3+\sqrt{89}}{2}\right) &\stackrel{?}{=} 1 - \log\left(\frac{\sqrt{89}-3}{4}\right)\\ \log\left(\frac{3+\sqrt{89}}{2}\right) &\stackrel{?}{=} \log 10 - \log\left(\frac{\sqrt{89}-3}{4}\right)\\ &\stackrel{?}{=} \log\left(\frac{40}{\sqrt{89}-3}\right)\\ &\stackrel{?}{=} \log\left[\frac{40\left(\sqrt{89}+3\right)}{89-9}\right]\\ &\stackrel{\surd}{=} \log\left(\frac{\sqrt{89}+3}{2}\right)\end{aligned}$$

$\log(x-1)$ is not defined if $x = \frac{1-\sqrt{89}}{4}$. Solution: $x = \frac{1+\sqrt{89}}{4}$

43.

$$\begin{aligned}\ln(x+1) &= \ln(3x+3)\\ x+1 &= 3x+3\\ -2x &= 2\\ x &= -1\end{aligned}$$

Check: $\ln(-1+1)$ is not defined

No solution.

45.

$$\begin{aligned}(\ln x)^3 &= \ln x^4\\ (\ln x)^3 &= 4 \ln x\\ (\ln x)^3 - 4\ln x &= 0\\ \ln x[(\ln x)^2 - 4] &= 0\\ \ln x(\ln x - 2)(\ln x + 2) &= 0\end{aligned}$$

$\ln x = 0$, $x = 1$ $\qquad$ $\ln x - 2 = 0$, $\ln x = 2$, $x = e^2$ $\qquad$ $\ln x + 2 = 0$, $\ln x = -2$, $x = e^{-2}$

Check:

$(\ln 1)^3 \stackrel{?}{=} \ln 1^4$, $0 \stackrel{\surd}{=} 0$ $\qquad$ $(\ln e^2)^3 \stackrel{?}{=} \ln(e^2)^4$, $8 \stackrel{\surd}{=} 8$ $\qquad$ $(\ln e^{-2})^3 \stackrel{?}{=} \ln(e^{-2})^4$, $-8 \stackrel{\surd}{=} -8$

Solution: $1, e^2, e^{-2}$

47.

$$\begin{aligned}\ln(\ln x) &= 1\\ \ln x &= e^1\\ \ln x &= e\\ x &= e^e\end{aligned}$$

49.

$$\begin{aligned}A &= Pe^{rt}\\ \frac{A}{P} &= e^{rt}\\ \ln\frac{A}{P} &= rt\\ \frac{1}{t}\ln\frac{A}{P} &= r\\ r &= \frac{1}{t}\ln\frac{A}{P}\end{aligned}$$

51.

$$\begin{aligned}D &= 10\log\frac{I}{I_0}\\ \frac{D}{10} &= \log\frac{I}{I_0}\\ \frac{I}{I_0} &= 10^{D/10}\\ I &= I_0(10^{D/10})\end{aligned}$$

53.

$$\begin{aligned}M &= 6 - 2.5\log\frac{I}{I_0}\\ 6 - M &= 2.5\log\frac{I}{I_0}\\ \frac{6-M}{2.5} &= \log\frac{I}{I_0}\\ \frac{I}{I_0} &= 10^{(6-M)/2.5}\\ I &= I_0[10^{(6-M)/2.5}]\end{aligned}$$

55.
$$
\begin{aligned}
I &= \frac{E}{R}(1 - e^{-Rt/L}) \\
RI &= E(1 - e^{-Rt/L}) \\
\frac{RI}{E} &= 1 - e^{-Rt/L} \\
\frac{RI}{E} - 1 &= -e^{-Rt/L} \\
-\left(\frac{RI}{E} - 1\right) &= e^{-Rt/L} \\
-\frac{RI}{E} + 1 &= e^{-Rt/L} \\
1 - \frac{RI}{E} &= e^{-Rt/L} \\
\ln\left(1 - \frac{RI}{E}\right) &= -\frac{Rt}{L} \\
-\frac{L}{R}\ln\left(1 - \frac{RI}{E}\right) &= t \\
t &= -\frac{L}{R}\ln\left(1 - \frac{RI}{E}\right)
\end{aligned}
$$

57.
$$
\begin{aligned}
y &= \frac{e^x + e^{-x}}{2} \\
2y &= e^x + e^{-x} \\
2y &= e^x + \frac{1}{e^x} \\
2ye^x &= (e^x)^2 + 1 \\
0 &= (e^x)^2 - 2ye^x + 1
\end{aligned}
$$

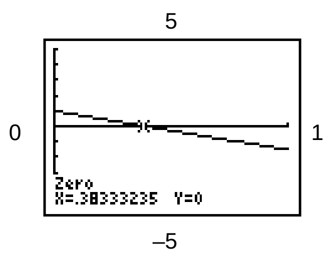
This equation is quadratic in e^x

$$
e^x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1,\ b = -2y,\ c = 1
$$

$$
\begin{aligned}
e^x &= \frac{-(-2y) \pm \sqrt{(-2y)^2 - 4(1)(1)}}{2(1)} \\
e^x &= \frac{2y \pm \sqrt{4y^2 - 4}}{2} \\
e^x &= \frac{2(y \pm \sqrt{y^2 - 1})}{2} \\
e^x &= y \pm \sqrt{y^2 - 1} \\
x &= \ln(y \pm \sqrt{y^2 - 1})
\end{aligned}
$$

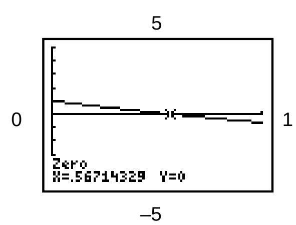
59.
$$
\begin{aligned}
y &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
y &= \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} \\
y &= \frac{e^x e^x - \frac{1}{e^x} e^x}{e^x e^x + \frac{1}{e^x} e^x} \\
y &= \frac{e^{2x} - 1}{e^{2x} + 1} \\
y(e^{2x} + 1) &= e^{2x} - 1 \\
ye^{2x} + y &= e^{2x} - 1 \\
1 + y &= e^{2x} - ye^{2x} \\
1 + y &= (1 - y)e^{2x} \\
e^{2x} &= \frac{1 + y}{1 - y} \\
2x &= \ln\frac{1 + y}{1 - y} \\
x &= \frac{1}{2}\ln\frac{1 + y}{1 - y}
\end{aligned}
$$

61. Graphing $y = 2^{-x} - 2x$ and applying a built-in routine, we obtain



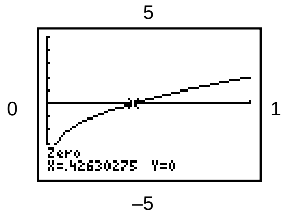
The required solution of $2^{-x} - 2x = 0$, $0 \le x \le 1$, is 0.38.

63. Graphing $y = e^{-x} - x$ and applying a built-in routine, we obtain



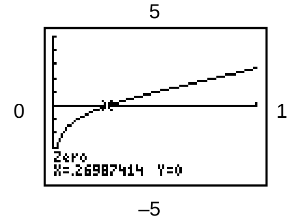
The required solution of $e^{-x} - x = 0$, $0 \le x \le 1$, is 0.57.

65. Graphing $y = \ln x + 2x$ and applying a built-in routine, we obtain



The required solution of $\ln x + 2x = 0$, $0 \le x \le 1$, is 0.43.

67. Graphing $y = \ln x + e^x$ and applying a built-in routine, we obtain



The required solution of $\ln x + e^x = 0$, $0 \le x \le 1$, is 0.27.

69. To find the doubling time we replace A in $A = P(1 + 0.07)^n$ with $2P$ and solve for n.

$$2P = P(1.07)^n$$
$$2 = (1.07)^n$$
$$\ln 2 = n \ln 1.07$$
$$n = \frac{\ln 2}{\ln 1.07}$$
$$n = 10 \text{ years to the nearest year}$$

71. We solve $A = Pe^{rt}$ for r, with $A = 2{,}500$, $P = 1{,}000$, $t = 10$

$$2{,}500 = 1{,}000e^{r(10)}$$
$$2.5 = e^{10r}$$
$$10r = \ln(2.5)$$
$$r = \frac{1}{10}\ln 2.5 = 0.0916 \text{ or } 9.16\%$$

73. (A) We're given $P_0 = 10.5$ (we could use 10.5 million, but if you look carefully at the calculations below, you'll see that the millions will cancel out anyhow), 11.3 for P, and 2 for t (since May 2007 is two years after May 2005).

$$11.3 = 10.5e^{r\cdot 2}$$
$$\frac{11.3}{10.5} = e^{2r}$$
$$\ln\left(\frac{11.3}{10.5}\right) = 2r$$
$$r = \frac{\ln\left(\frac{11.3}{10.5}\right)}{2} \approx 0.0367$$

The annual growth rate is 3.67%.

(B) $P = 10.5e^{0.0367t}$; plug in 20 for P and solve for t.

$$20 = 10.5e^{0.0367t}$$
$$\frac{20}{10.5} = e^{0.0367t}$$
$$\ln\left(\frac{20}{10.5}\right) = 0.0367t$$
$$t = \frac{\ln\left(\frac{20}{10.5}\right)}{0.0367} \approx 17.6$$

The illegal immigrant population is predicted to reach 20 million near the end of 2022, which is 17.6 years after May 2005.

75. We solve $P = P_0e^{rt}$ for t with $P = 2P_0$, $r = 0.0114$.

$$2P_0 = P_0e^{0.0114t}$$
$$2 = e^{0.0114t}$$
$$\ln 2 = 0.0114t$$
$$t = \frac{\ln 2}{0.0114}$$
$$t = 61 \text{ years to the nearest year}$$

77. We're given $A_0 = 5, A = 1, t = 6$:

$$A = A_0\left(\frac{1}{2}\right)^{t/h}$$
$$1 = 5\left(\frac{1}{2}\right)^{6/h}$$
$$\frac{1}{5} = \left(\frac{1}{2}\right)^{6/h}$$
$$\ln\frac{1}{5} = \ln\left(\frac{1}{2}\right)^{6/h}$$
$$\ln\frac{1}{5} = \frac{6}{h}\ln\left(\frac{1}{2}\right)$$
$$h \cdot \ln\frac{1}{5} = 6\ln\left(\frac{1}{2}\right)$$
$$h = \frac{6\ln(1/2)}{\ln(1/5)} \approx 2.58$$

The half-life is about 2.58 hours.

79. Let A_0 represent the amount of Carbon-14 originally present. Then the amount left in 2003 was $0.289A_0$. Plug this in for *A*, and solve for *t*:

$$0.289A_0 = A_0e^{-0.000124t}$$
$$0.289 = e^{-0.000124t}$$
$$\ln 0.289 = \ln e^{-0.000124t}$$
$$\ln 0.289 = -0.000124t$$
$$t = \frac{\ln 0.289}{-0.000124} \approx 10,010$$

The sample was about 10,010 years old.

81. Let A_0 represent the amount of Carbon-14 originally present. Then the amount left in 2004 was $0.883A_0$. Plug this in for *A*, and solve for *t*:

$$0.883A_0 = A_0e^{-0.000124t}$$
$$0.883 = e^{-0.000124t}$$
$$\ln 0.883 = \ln e^{-0.000124t}$$
$$\ln 0.883 = -0.000124t$$
$$t = \frac{\ln 0.883}{-0.000124} \approx 1,003$$

It was 1,003 years old in 2004, so it was made in 1001.

83. We solve $q = 0.0009(1 - e^{-0.2t})$ for *t* with $q = 0.0007$

$$0.0007 = 0.0009(1 - e^{-0.2t})$$
$$\frac{0.0007}{0.0009} = 1 - e^{-0.2t}$$
$$\frac{7}{9} = 1 - e^{-0.2t}$$
$$-\frac{2}{9} = -e^{-0.2t}$$
$$\frac{2}{9} = e^{-0.2t}$$
$$\ln\frac{2}{9} = -0.2t$$
$$t = \frac{\ln\frac{2}{9}}{-0.2}$$
$$t = 7.52 \text{ seconds}$$

85. First, we solve $T = T_m + (T_0 - T_m)e^{-kt}$ for *k*, with $T = 61.5°$, $T_m = 40°$, $T_0 = 72°$, $t = 1$

$$61.5 = 40 + (72 - 40)e^{-k(1)}$$
$$21.5 = 32e^{-k}$$
$$\frac{21.5}{32} = e^{-k}$$
$$\ln\frac{21.5}{32} = -k$$
$$k = -\ln\frac{21.5}{32}$$
$$k = 0.40$$

Now we solve $T = T_m + (T_0 - T_m)e^{-0.40t}$ for *t*, with $T = 50°$, $T_m = 40°$, $T_0 = 72°$

$$50 = 40 + (72 - 40)e^{-0.40t}$$
$$10 = 32e^{-0.40t}$$
$$\frac{10}{32} = e^{-0.40t}$$
$$\ln\frac{10}{32} = -0.40t$$
$$t = \frac{\ln 10/32}{-0.40}$$
$$t = 2.9 \text{ hours}$$

87. (A) Plug in $M = 7.0$ and solve for *E*:

$$7 = \frac{2}{3}\log\frac{E}{10^{4.40}}$$
$$\frac{21}{2} = \log\frac{E}{10^{4.4}}$$
$$10^{21/2} = \frac{E}{10^{4.4}}$$
$$E = 10^{21/2}\cdot 10^{4.4} \approx 7.94\times 10^{14} \text{ joules}$$

(B) $\dfrac{7.94\times 10^{14} \text{ joules}}{2.88\times 10^{14} \text{ joules/day}} = 2.76$ days

89. First, find the energy released by one magnitude 7.5 earthquake:

$$7.5 = \frac{2}{3}\log\frac{E}{10^{4.40}}$$
$$11.25 = \log\frac{E}{10^{4.4}}$$

Finally, divide by the energy consumption per year:

$$\frac{5.364\times 10^{16} \text{ joules}}{1.05\times 10^{17} \text{ joules/year}} = 0.510$$

So this energy could power the U.S. for 0.510 years, or about 186 days.

$10^{11.25} = \frac{E}{10^{4.4}}$

$E = 10^{11.25} \cdot 10^{4.4} \approx 4.47 \times 10^{15}$ joules

Now multiply by twelve to get the energy released by twelve such earthquakes:

$12 \cdot 4.47 \times 10^{15} = 5.364 \times 10^{16}$

CHAPTER 5 REVIEW

1. (A) The graph of $y = \log_2 x$ passes through (1, 0) and (2, 1). This corresponds to graph m.

(B) The graph of $y = 0.5^x$ passes through (0, 1) and (1, 0.5). This corresponds to graph f.

(C) The graph of $y = \log_{0.5} x$ passes through (1, 0) and (0.5, 1). This corresponds to graph n.

(D) The graph of $y = 2^x$ passes through (0, 1) and (1, 2). This corresponds to graph g. *(5-1, 5-3)*

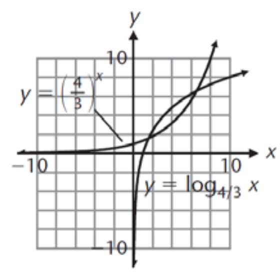
2. $\log m = n$ *(5-3)* **3.** $\ln x = y$ *(5-3)* **4.** $x = 10^y$ *(5-3)* **5.** $y = e^x$ *(5-3)*

6. (A) Make a table of values:

x	−2	−1	0	1	2	3
$\left(\frac{4}{3}\right)^x$	$\frac{9}{16}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\frac{16}{9}$	$\frac{64}{27}$

(5-1)

(B) The function in part (B) is the inverse of the one graphed in part (A), so its graph is a reflection about the line $y = x$ of the graph to the left. To plot points, just switch the x and y coordinates of the points from the table in part (A).



(5-3)

7. $\frac{7^{x+2}}{7^{2-x}} = 7^{(x+2)-(2-x)}$

$= 7^{x+2-2+x}$

$= 7^{2x}$ *(5-1)*

8. $\left(\frac{e^x}{e^{-x}}\right)^x = [e^{x-(-x)}]^x$

$= (e^{2x})^x = e^{2x \cdot x}$

$= e^{2x^2}$ *(5-1)*

9. $\log_2 x = 3$

$x = 2^3$

$x = 8$ *(5-3)*

10. $\log_x 25 = 2$

$25 = x^2$

$x = 5$

since bases are restricted positive *(5-3)*

11. $\log_3 27 = x$

$\log_3 3^3 = x$

$x = 3$ *(5-3)*

12. $10^x = 17.5$

$x = \log_{10} 17.5$

$x = 1.24$ *(5-5)*

13. $e^x = 143{,}000$

$x = \ln 143{,}000$

$x = 11.9$ *(5-5)*

14. $\ln x = -0.01573$

$x = e^{-0.01573}$

$x = 0.984$ *(5-3)*

15. $\log x = 2.013$

$x = 10^{2.013}$

$x = 103$ *(5-3)*

16. 1.145 *(5-3)* **17.** Not defined. ($-e$ is not in the domain of the logarithm function.) *(5-3)* **18.** 2.211 *(5-3)* **19.** 11.59 *(5-1)*

20. $2\log a - \frac{1}{3}\log b + \log c = \log a^2 + \log c - \log b^{\frac{1}{3}}$

$= \log(a^2 c) - \log b^{\frac{1}{3}}$

$= \log\left(\frac{a^2 c}{\sqrt[3]{b}}\right)$ *(5-3)*

21. $\ln \frac{a^5}{\sqrt{b}} = \ln a^5 - \ln \sqrt{b}$

$= 5\ln a - \ln b^{\frac{1}{2}}$

$= 5\ln a - \frac{1}{2}\ln b$ *(5-3)*

22. $3^x = 120$

$\log_3 3^x = \log_3 120$

$x = \log_3 120$

23. $10^{2x} = 500$

$\log 10^{2x} = \log 500$

$2x = \log 500$

24. $\log_2(4x-5) = 5$

$2^5 = 4x - 5$

$32 = 4x - 5$

or, using the change-of-base formula $\frac{\ln 120}{\ln 3}$ (5-5)

$x = \frac{\log 500}{2}$ (5-5)

$37 = 4x$

$x = \frac{37}{4}$ (5-5)

25. $\ln(x-5) = 0$

$x - 5 = e^0$

$x - 5 = 1$

$x = 6$

(5-5)

26. $\ln(2x-1) = \ln(x+3)$

$2x - 1 = x + 3$

$x = 4$

Check:

$\ln(2\cdot 4 - 1) \overset{?}{=} \ln(4+3)$

$\ln 7 \overset{\surd}{=} \ln 7$

(5-5)

27. $\log(x^2 - 3) = 2\log(x-1)$

$\log(x^2 - 3) = \log(x-1)^2$

$x^2 - 3 = (x-1)^2$

$x^2 - 3 = x^2 - 2x + 1$

$-3 = -2x + 1$

$-4 = -2x$

$x = 2$

Check:

$\log(2^2 - 3) \overset{?}{=} 2\log(2-1)$ $\log 1 \overset{?}{=} 2\log 1$

$0 \overset{\surd}{=} 0$

(5-5)

28. $e^{x^2-3} = e^{2x}$

$x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

(5-5)

29. $4^{x-1} = 2^{1-x}$

$(2^2)^{x-1} = 2^{1-x}$

$2^{2(x-1)} = 2^{1-x}$

$2(x-1) = 1 - x$

$2x - 2 = 1 - x$

$3x = 3$

$x = 1$ (5-5)

30. $2x^2e^{-x} = 18e^{-x}$

$2x^2e^{-x} - 18e^{-x} = 0$

$2e^{-x}(x^2 - 9) = 0$

$2e^{-x}(x-3)(x+3) = 0$

$2e^{-x} = 0$ $x - 3 = 0$ $x + 3 = 0$

never $x = 3$ $x = -3$

Solution: 3, –3 (5-5)

31. $\log_{1/4} 16 = x$

$\log_{1/4} 4^2 = x$

$\log_{1/4}\left(\frac{1}{4}\right)^{-2} = x$

$x = -2$

(5-5)

32. $\log_x 9 = -2$

$x^{-2} = 9$

$\frac{1}{x^2} = 9$

$1 = 9x^2$

$\frac{1}{9} = x^2$

$x = \pm\sqrt{\frac{1}{9}}$

$x = \frac{1}{3}$ since bases are restricted positive

(5-5)

33. $\log_{16} x = \frac{3}{2}$

$16^{3/2} = x$

$64 = x$

$x = 64$

(5-5)

34. $\log_x e^5 = 5$

$e^5 = x^5$

$x = e$ (5-5)

35. $10^{\log_{10} x} = 33$

$\log_{10} x = \log_{10} 33$

$x = 33$ (5-5)

36. $x = 2(10^{1.32})$

$x = 41.8$ (5-1)

37. $x = \log_5 23$

$x = \frac{\log 23}{\log 5}$ or $\frac{\ln 23}{\ln 5}$

$x = 1.95$ (5-3)

38. $\ln x = -3.218$

$x = e^{-3.218}$

$x = 0.0400$ (5-3)

39. $x = \log(2.156 \times 10^{-7})$

$x = -6.67$

(5-3)

40. $x = \frac{\ln 4}{\ln 2.31}$

$x = 1.66$ (5-3)

41. $25 = 5(2)^x$

$\frac{25}{5} = 2^x$

$5 = 2^x$

$\ln 5 = x\ln 2$

$\frac{\ln 5}{\ln 2} = x$

$x = 2.32$

(5-5)

42. $4{,}000 = 2{,}500e^{0.12x}$

$\frac{4{,}000}{2{,}500} = e^{0.12x}$

$0.12x = \ln\frac{4{,}000}{2{,}500}$

$x = \frac{1}{0.12}\ln\frac{4{,}000}{2{,}500}$

$x = 3.92$ (5-5)

43. $0.01 = e^{-0.05x}$

$-0.05x = \ln 0.01$

$x = \frac{\ln 0.01}{-0.05}$

$x = 92.1$ (5-5)

44. $5^{2x-3} = 7.08$

$(2x-3)\log 5 = \log 7.08$

$2x - 3 = \frac{\log 7.08}{\log 5}$

$x = \frac{1}{2}\left[3 + \frac{\log 7.08}{\log 5}\right] = 2.11$ (5-5)

45.

$$\frac{e^x - e^{-x}}{2} = 1$$
$$e^x - e^{-x} = 2$$
$$e^x - \frac{1}{e^x} = 2$$
$$e^x e^x - e^x\left(\frac{1}{e^x}\right) = 2e^x$$
$$(e^x)^2 - 1 = 2e^x$$
$$(e^x)^2 - 2e^x - 1 = 0$$

This equation is quadratic in e^x: $e^x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 1,$ $b = -2,$ $c = -1$

$$e^x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2}$$
$$e^x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$x = \ln(1 \pm \sqrt{2})$ $1 - \sqrt{2}$ is negative, hence not in the domain of the logarithm function. $x = \ln(1 + \sqrt{2})$

$x = 0.881$ (5-5)

46.

$$\log 3x^2 - \log 9x = 2$$
$$\log \frac{3x^2}{9x} = 2$$
$$\frac{3x^2}{9x} = 10^2$$
$$\frac{x}{3} = 100$$
$$x = 300$$

Check:

$$\log(3\cdot300^2) - \log(9\cdot300) \stackrel{?}{=} 2$$
$$\log(270{,}000) - \log(2{,}700) \stackrel{?}{=} 2$$
$$\log \frac{270{,}000}{2{,}700} \stackrel{?}{=} 2$$
$$\log 100 \stackrel{\surd}{=} 2$$

(5-5)

47.

$$\log x - \log 3 = \log 4 - \log(x + 4)$$
$$\log \frac{x}{3} = \log \frac{4}{x+4}$$
$$\frac{x}{3} = \frac{4}{x+4}$$ excluded value: $x \neq -4$
$$3(x + 4)\frac{x}{3} = 3(x + 4)\frac{4}{x+4}$$
$$(x + 4)x = 12$$
$$x^2 + 4x = 12$$
$$x^2 + 4x - 12 = 0$$
$$(x + 6)(x - 2) = 0$$
$$x = -6 \quad x = 2$$

Check: log(–6) is not defined

$$\log 2 - \log 3 \stackrel{?}{=} \log 4 - \log(2 + 4)$$
$$\log \frac{2}{3} \stackrel{?}{=} \log \frac{4}{6}$$
$$\log \frac{2}{3} \stackrel{\surd}{=} \log \frac{2}{3}$$

Solution: 2 (5-5)

48.

$$\ln(x + 3) - \ln x = 2 \ln 2$$
$$\ln \frac{x+3}{x} = \ln 2^2$$
$$\frac{x+3}{x} = 2^2$$
$$\frac{x+3}{x} = 4$$
$$x + 3 = 4x$$
$$3 = 3x$$
$$x = 1$$

Check:

$$\ln(1 + 3) - \ln 1 \stackrel{?}{=} 2 \ln 2$$
$$\ln 4 - 0 \stackrel{?}{=} 2 \ln 2$$
$$\ln 4 \stackrel{\surd}{=} \ln 4$$

(5-5)

49. $\ln(2x + 1) - \ln(x - 1) = \ln x$

$$\ln \frac{2x+1}{x-1} = \ln x$$
$$\frac{2x+1}{x-1} = x$$ Excluded value: $x \neq 1$
$$(x - 1)\frac{2x+1}{x-1} = x(x - 1)$$
$$2x + 1 = x^2 - x$$
$$0 = x^2 - 3x - 1$$

Check: $\ln\left(\frac{3-\sqrt{13}}{2}\right)$ is not defined

$$\ln\left(2\cdot\frac{3+\sqrt{13}}{2} + 1\right) - \ln\left(\frac{3+\sqrt{13}}{2} - 1\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$
$$\ln(3 + \sqrt{13} + 1) - \ln\left(\frac{3+\sqrt{13}-2}{2}\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 1, b = -3, c = -1$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)} = \frac{3 \pm \sqrt{13}}{2}$$

$$\ln(4 + \sqrt{13}) - \ln\left(\frac{1+\sqrt{13}}{2}\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

$$\ln\left(\frac{4+\sqrt{13}}{1} \cdot \frac{2}{1+\sqrt{13}}\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

$$\ln\left(\frac{\left(4+\sqrt{13}\right)2}{1+\sqrt{13}}\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

$$\ln\left(\frac{\left(4+\sqrt{13}\right)2\left(1-\sqrt{13}\right)}{\left(1+\sqrt{13}\right)\left(1-\sqrt{13}\right)}\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

$$\ln\left(\frac{2\left(4-3\sqrt{13}-13\right)}{1-13}\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

$$\ln\left(\frac{-18-6\sqrt{13}}{-12}\right) \stackrel{?}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

$$\ln\left(\frac{3+\sqrt{13}}{2}\right) \stackrel{\checkmark}{=} \ln\left(\frac{3+\sqrt{13}}{2}\right)$$

Solution: $\frac{3+\sqrt{13}}{2}$ (5-5)

50.

$$(\log x)^3 = \log x^9$$
$$(\log x)^3 = 9 \log x$$
$$(\log x)^3 - 9 \log x = 0$$
$$\log x[(\log x)^2 - 9] = 0$$
$$\log x(\log x - 3)(\log x + 3) = 0$$
$$\log x = 0 \quad \log x - 3 = 0 \quad \log x + 3 = 0$$
$$x = 1 \quad \log x = 3 \quad \log x = -3$$
$$x = 10^3 \quad x = 10^{-3}$$

Check: $(\log 1)^3 \stackrel{?}{=} \log 1^9$

$$0 \stackrel{\checkmark}{=} 0$$
$$(\log 10^3)^3 \stackrel{?}{=} \log(10^3)^9$$
$$27 \stackrel{\checkmark}{=} 27$$
$$(\log 10^{-3})^3 \stackrel{?}{=} \log(10^{-3})^9$$
$$-27 \stackrel{\checkmark}{=} -27$$

Solution: $1, 10^3, 10^{-3}$ (5-5)

51. $\ln(\log x) = 1$

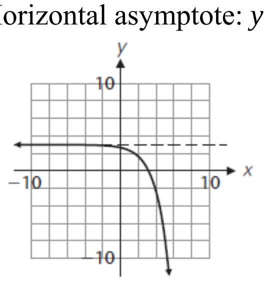
$$\log x = e$$
$$x = 10^e$$

(5-5)

52. $(e^x + 1)(e^{-x} - 1) - e^x(e^{-x} - 1) = e^xe^{-x} - e^x + e^{-x} - 1 - e^xe^{-x} + e^x = 1 - e^x + e^{-x} - 1 - 1 + e^x = e^{-x} - 1$ (5-1)

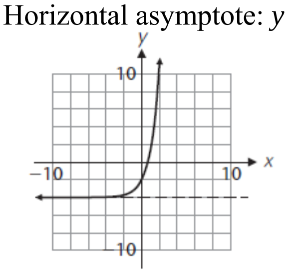
53. $(e^x + e^{-x})(e^x - e^{-x}) - (e^x - e^{-x})^2 = (e^x)^2 - (e^{-x})^2 - [(e^x)^2 - 2e^xe^{-x} + (e^{-x})^2] = e^{2x} - e^{-2x} - [e^{2x} - 2 + e^{-2x}]$
$= e^{2x} - e^{-2x} - e^{2x} + 2 - e^{-2x} = 2 - 2e^{-2x}$ (5-1)

54. The graph of g is the same as the graph of f reflected through the x axis, shrunk vertically by a factor of $\frac{1}{3}$, and shifted upward 3 units; g is decreasing. Domain: all real numbers
Horizontal asymptote: $y = 3$



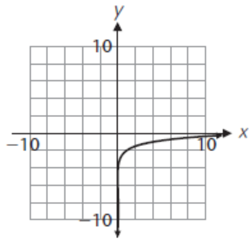
(5-1)

55. The graph of g is the same as the graph of f stretched vertically by a factor of 2 and shifted downward 4 units; g is increasing.
Domain: all real numbers
Horizontal asymptote: $y = -4$



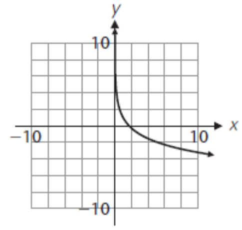
(5-1)

56. The graph of g is the same as the graph of f shifted downward 2 units; g is increasing.
Domain: $(0, \infty)$ Vertical asymptote: $x = 0$



(5-3)

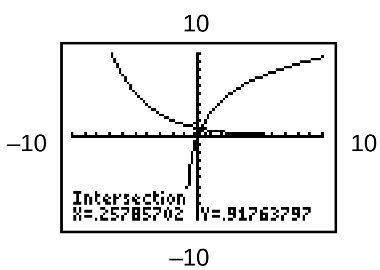
57. The graph of g is the same as the graph of f stretched vertically by a factor of 2 and shifted upward 1 unit; g is decreasing.
Domain: $(0, \infty)$ Vertical asymptote: $x = 0$



(5-3)

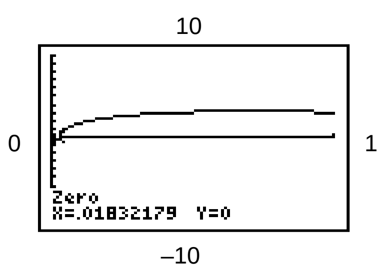
58. If the graph of $y = e^x$ is reflected in the x axis, y is replaced by $-y$ and the graph becomes the graph of $-y = e^x$ or $y = -e^x$.
If the graph of $y = e^x$ is reflected in the y axis, x is replaced by $-x$ and the graph becomes the graph of $y = e^{-x}$ or $y = \frac{1}{e^x}$ or $y = \left(\frac{1}{e}\right)^x$. (5-1)

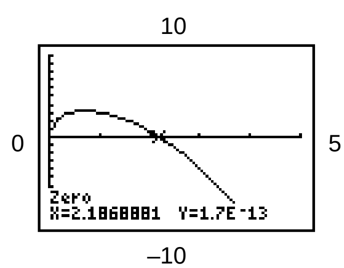
59. (A) For $x > -1$, $y = e^{-x/3}$ decreases from $e^{1/3}$ to 0 while $\ln(x + 1)$ increases from $-\infty$ to ∞. Consequently, the graphs can intersect at exactly one point.
(B) Graphing $y_1 = e^{-x/3}$ and $y_2 = 4\ln(x + 1)$ we obtain



The solution of $e^{-x/3} = 4\ln(x + 1)$ is $x = 0.258$. (5-5)

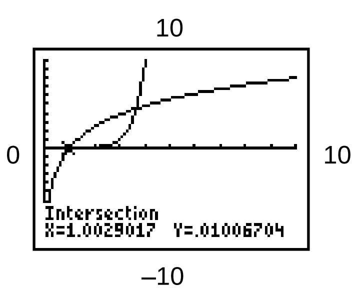
60. Examining the graph of $f(x) = 4 - x^2 + \ln x$, we obtain

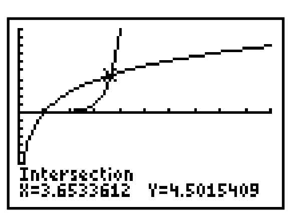




The zeros are at 0.018 and 2.187. (5-5)

61. Graphing $y_1 = 10^{x-3}$ and $y_2 = 8\log x$, we obtain





The graphs intersect at (1.003, 0.010) and (3.653, 4.502). (5-5)

62.

$$D = 10 \log \frac{I}{I_0}$$
$$\frac{D}{10} = \log \frac{I}{I_0}$$
$$10^{D/10} = \frac{I}{I_0}$$
$$I_0 10^{D/10} = I$$
$$I = I_0(10^{D/10})$$

(5-5)

63.

$$y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$
$$\sqrt{2\pi}\, y = e^{-x^2/2}$$
$$-\frac{x^2}{2} = \ln(\sqrt{2\pi}\, y)$$
$$x^2 = -2 \ln(\sqrt{2\pi}\, y)$$
$$x = \pm\sqrt{-2\ln(\sqrt{2\pi}\, y)}$$

(5-5)

64.

$$x = -\frac{1}{k} \ln \frac{I}{I_0}$$
$$-kx = \ln \frac{I}{I_0}$$
$$\frac{I}{I_0} = e^{-kx}$$
$$I = I_0(e^{-kx})$$ (5-5)

65.

$$r = P \frac{i}{1-(1+i)^{-n}}$$
$$\frac{r}{P} = \frac{i}{1-(1+i)^{-n}}$$
$$\frac{P}{r} = \frac{1-(1+i)^{-n}}{i}$$
$$\frac{Pi}{r} = 1 - (1+i)^{-n}$$
$$\frac{Pi}{r} - 1 = -(1+i)^{-n}$$
$$1 - \frac{Pi}{r} = (1+i)^{-n}$$
$$\ln\left(1-\frac{Pi}{r}\right) = -n \ln(1+i)$$
$$\frac{\ln\left(1-\frac{Pi}{r}\right)}{-\ln(1+i)} = n$$
$$n = -\frac{\ln\left(1-\frac{Pi}{r}\right)}{\ln(1+i)}$$

(5-5)

66.

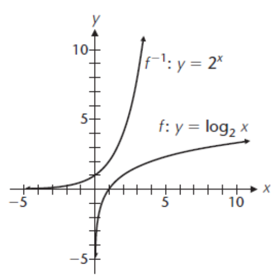
$$\ln y = -5t + \ln c$$
$$\ln y - \ln c = -5t$$
$$\ln\left(\frac{y}{c}\right) = -5t$$
$$\frac{y}{c} = e^{-5t}$$
$$y = ce^{-5t}$$

(5-5)

67.

x	$y = \log_2 x$	$x = \log_2 y$	y
1	0	0	1
2	1	1	2
4	2	2	4
8	3	3	8

Domain $f = (0, \infty) = $ Range f^{-1}
Range $f = (-\infty, \infty) = $ Domain f^{-1}



(5-3)

68. If $\log_1 x = y$, then we would have to have $1^y = x$; that is, $1 = x$ for arbitrary positive x, which is impossible. (5-3)

69. Let $u = \log_b M$ and $v = \log_b N$; then $M = b^u$ and $N = b^v$.
Thus, $\log(MN) = \log_b(b^u b^v) = \log_b b^{u+v} = u + v = \log_b M + \log_b N$. (5-3)

70. We solve $P = P_0(1.03)^t$ for t, using $P = 2P_0$.

$$2P_0 = P_0(1.03)^t$$
$$2 = (1.03)^t$$
$$\ln 2 = t \ln 1.03$$
$$\frac{\ln 2}{\ln 1.03} = t$$
$$t = 23.4 \text{ years}$$

(5-2)

71. We solve $P = P_0 e^{0.03t}$ for t using $P = 2P_0$.

$$2P_0 = P_0 e^{0.03t}$$
$$2 = e^{0.03t}$$
$$\ln 2 = 0.03t$$
$$\frac{\ln 2}{0.03} = t$$
$$t = 23.1 \text{ years}$$

(5-2)

72. A_0 = original amount

$0.01A_0$ = 1 percent of original amount

We solve $A = A_0e^{-0.000124t}$ for t, using $A = 0.01A_0$.

$$0.01A_0 = A_0e^{-0.000124t}$$
$$0.01 = e^{-0.000124t}$$
$$\ln 0.01 = -0.000124t$$
$$\frac{\ln 0.01}{-0.000124} = t$$
$$t = 37{,}100 \text{ years} \qquad (5\text{-}2)$$

73. (A) When $t = 0$, $N = 1$. As t increases by 1/2, N doubles. Hence $N = 1\cdot(2)^{t \div 1/2}$

$$N = 2^{2t} \text{ (or } N = 4^t\text{)}$$

(B) We solve $N = 4^t$ for t, using

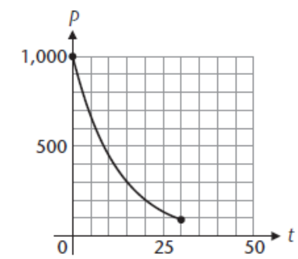
$$N = 10^9$$
$$10^9 = 4^t$$
$$9 = t \log 4$$
$$t = \frac{9}{\log 4}$$
$$t = 15 \text{ days} \qquad (5\text{-}2)$$

74. We use $A = Pe^{rt}$ with $P = 1$, $r = 0.03$, and $t = 2011 - 1 = 2010$.

$$A = 1e^{0.03(2010)}$$
$$A = 1.5 \times 10^{26} \text{ dollars } (5\text{-}1)$$

75.(A)

t	p
0	1,000
5	670
10	449
15	301
20	202
25	135
30	91



(B)As t tends to infinity, P appears to tend to 0. (5-1)

76. $M = \frac{2}{3}\log\frac{E}{E_0}$ $\qquad E_0 = 10^{4.40}$

We use $E = 1.99 \times 10^{14}$

$$M = \frac{2}{3}\log\frac{1.99\times10^{14}}{10^{4.40}}$$
$$M = \frac{2}{3}\log(1.99 \times 10^{9.6})$$
$$M = \frac{2}{3}(\log 1.99 + 9.6)$$
$$M = \frac{2}{3}(0.299 + 9.6)$$
$$M = 6.6 \qquad (5\text{-}4)$$

77. We solve $M = \frac{2}{3}\log\frac{E}{E_0}$ for E, using

$E_0 = 10^{4.40}$, $M = 8.3$

$$8.3 = \frac{2}{3}\log\frac{E}{10^{4.40}}$$
$$\frac{3}{2}(8.3) = \log\frac{E}{10^{4.40}}$$
$$12.45 = \log\frac{E}{10^{4.40}}$$
$$\frac{E}{10^{4.40}} = 10^{12.45}$$
$$E = 10^{4.40}\cdot10^{12.45}$$
$$E = 10^{16.85} \text{ or } 7.08 \times 10^{16} \text{ joules} \qquad (5\text{-}4)$$

78. We use the given formula twice, with

$$I_2 = 100{,}000I_1$$
$$D_1 = 10\log\frac{I_1}{I_0} \qquad D_2 = 10\log\frac{I_2}{I_0}$$
$$D_2 - D_1 = 10\log\frac{I_2}{I_0} - 10\log\frac{I_1}{I_0} = 10\log\left(\frac{I_2}{I_0}\div\frac{I_1}{I_0}\right) = 10\log\frac{I_2}{I_1}$$
$$= 10\log\frac{100{,}000I_1}{I_1} = 10\log 100{,}000 = 50 \text{ decibels}$$

The level of the louder sound is 50 decibels more. (5-2)

79. $I = I_0e^{-kd}$

To find k, we solve for k using $I = \frac{1}{2}I_0$ and $d = 73.6$

$$\frac{1}{2}I_0 = I_0e^{-k(73.6)}$$
$$\frac{1}{2} = e^{-73.6k}$$
$$-73.6k = \ln\frac{1}{2}$$
$$k = \frac{\ln\frac{1}{2}}{-73.6}$$
$$k = 0.00942$$

We now find the depth at which 1% of the surface light remains. We solve $I = I_0e^{-0.00942d}$ for d with $I = 0.01I_0$

$$0.01I_0 = I_0e^{-0.00942d}$$
$$0.01 = e^{-0.00942d}$$
$$-0.00942d = \ln 0.01$$
$$d = \frac{\ln 0.01}{-0.00942}$$
$$d = 489 \text{ feet}$$

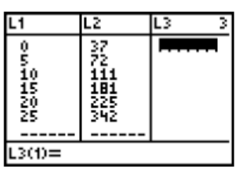
(*5-2*)

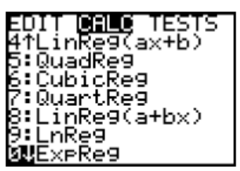
80. We solve $N = \dfrac{30}{1+29e^{-1.35t}}$ for t with $N = 20$.

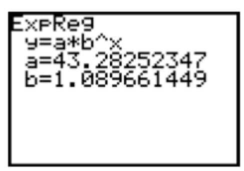
$$20 = \frac{30}{1+29e^{-1.35t}}$$
$$\frac{1}{20} = \frac{1+29e^{-1.35t}}{30}$$
$$1.5 = 1 + 29e^{-1.35t}$$
$$0.5 = 29e^{-1.35t}$$
$$\frac{0.5}{29} = e^{-1.35t}$$
$$-1.35t = \ln\frac{0.5}{29}$$
$$t = \frac{\ln\frac{0.5}{29}}{-1.35}$$
$$t = 3 \text{ years}$$

(*5-2*)

81. (A) The independent variable is years since 1980, so enter 0, 5, 10, 15, 20, and 25 as L_1. The dependent variable is Medicare expenditures, so enter that column as L_2. Then use the exponential regression command on the STAT CALC menu.







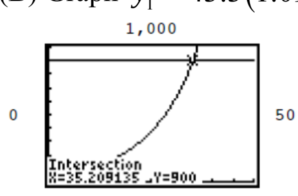
The exponential model is $y = 43.3(1.09)^x$.

To find total expenditures in 2010 and 2020, we plug in 30 and 40 for x:

$y(30) = 43.3(1.09)^{30} = 574;\ \ y(40) = 43.3(1.09)^{40} = 1{,}360$

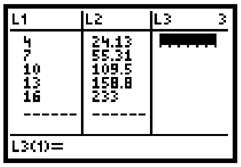
Expenditures are predicted to be $574 billion in 2010 and $1,360 billion in 2020.

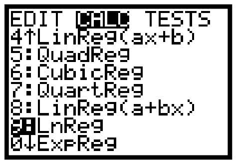
(B) Graph $y_1 = 43.3(1.09)^x$ and $y_2 = 900$ and use the INTERSECT command:

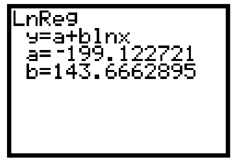


Expenditures are predicted to reach $900 billion in 2015. (*5-2*)

82. (A) The independent variable is years since 1990, so enter 4, 7, 10, 13, 16 as L_1. The dependent variable is the number of subscribers, so enter the subscribers' column as L_2. Then use the logarithmic regression command from the STAT CALC menu.

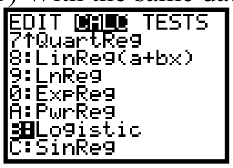


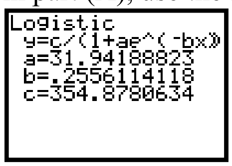




The model is $y = -199.1 + 143.7 \ln x$. Evaluating this at $x = 25$ (year 2015) gives 263.5 million subscribers.

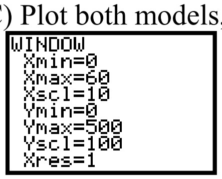
(B) With the same data as in part (A), use the logistic regression command from the STAT CALC menu.

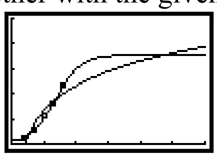




The model is $y = \dfrac{354.9}{1+31.94e^{-0.2556x}}$. Evaluating this at $x = 25$ (year 2015) gives 336.8 million subscribers.

(C) Plot both models, together with the given data points, on the same screen.





Clearly, the logistic model fits the data better. Moreover, the logarithmic model predicts that the number of subscribers becomes infinite, eventually, which is absurd. The logistic model predicts eventual leveling off near 354.9 million, which still seems high compared with the US population, but is more reasonable. The logistic model wins on both criteria specified in the problem.

(*5-2*, *5-4*)