CHAPTER 2

Section 2-1

1. To each point P in the plane there corresponds a single ordered pair of numbers (a, b) called the coordinates of the point. To each ordered pair of numbers (a, b) there corresponds a single point, called the graph of the pair.

3. This can be done by imagining a mirror placed along the y axis; draw a graph so that its reflection in this mirror would be the graph already present; each coordinate given as (a, b) is reflected as $(-a, b)$.

5. The set of all points for which the x coordinate is 0 is the y axis.

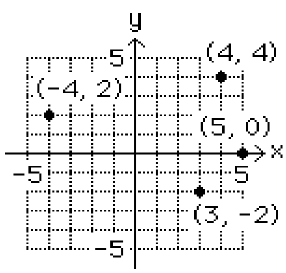
7. The set of all points for which the x and y coordinates are negative is quadrant III.

9. The set of all points for which the x coordinate is positive and the y coordinate is negative is quadrant IV.

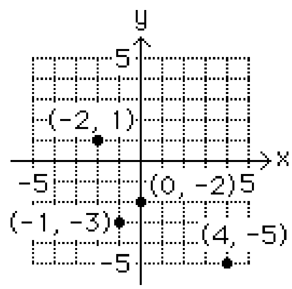
11. The set of all points for which x is positive, excluding those points for which $y = 0$ (positive x axis), includes quadrants I and IV.

13. The set of all points for which $xy < 0$ includes those points for which the x coordinate is positive and the y coordinate is negative (quadrant IV) and also those points for which the x coordinate is negative and the y coordinate is positive (quadrant II).

15.



17.



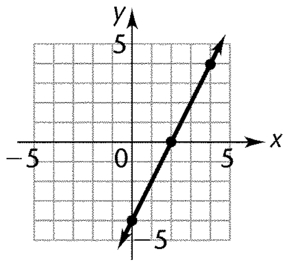
19. Point A has coordinates $(2, 4)$. Its reflection through the y axis is $A'(-2, 4)$.
Point B has coordinates $(3, -1)$. Its reflection through the y axis is $B'(-3, -1)$.
Point C has coordinates $(-4, 0)$. Its reflection through the y axis is $C'(4, 0)$.
Point D has coordinates $(-5, 2)$. Its reflection through the y axis is $D'(5, 2)$.

21. Point A has coordinates $(-3, -3)$. Its reflection through the origin is $A'(3, 3)$.
Point B has coordinates $(0, 4)$. Its reflection through the origin is $B'(0, -4)$.
Point C has coordinates $(-3, 2)$. Its reflection through the origin is $C'(3, -2)$.
Point D has coordinates $(5, -1)$. Its reflection through the origin is $D'(-5, 1)$.

23. $y = 2x - 4$

Test y axis	Test x axis	Test origin
Replace x with $-x$:	Replace y with $-y$:	Replace x with $-x$ and y with $-y$:
$y = 2(-x) - 4$	$-y = 2x - 4$	$-y = 2(-x) - 4$
$y = -2x - 4$	$y = -2x + 4$	$y = 2x + 4$

x	y
0	-4
2	0
4	4



The graph has none of these symmetries.

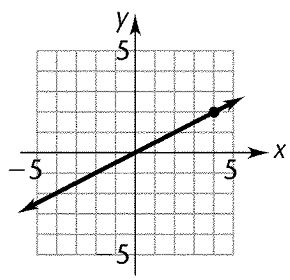
25. $y = \frac{1}{2}x$

Test y axis
Replace x with $-x$:
$y = \frac{1}{2}(-x)$
$y = -\frac{1}{2}x$

Test x axis
Replace y with $-y$:
$-y = \frac{1}{2}x$
$y = -\frac{1}{2}x$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = \frac{1}{2}(-x)$
$y = \frac{1}{2}x$

x	y
0	0
4	2



The graph has symmetry with respect to the origin.
We reflect the portion of the graph in quadrant I through the origin, using the origin symmetry.

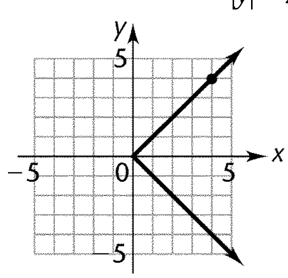
27. $|y| = x$

Test y axis
Replace x with $-x$:
$|y| = -x$

Test x axis
Replace y with $-y$:
$|-y| = x$
$|y| = x$

Test origin
Replace x with $-x$ and y with $-y$:
$|-y| = -x$
$|y| = -x$

x	y
0	0
4	4



The graph has symmetry with respect to the x axis.
We reflect the portion of the graph where $y \geq 0$ through the x axis, using the x axis symmetry.

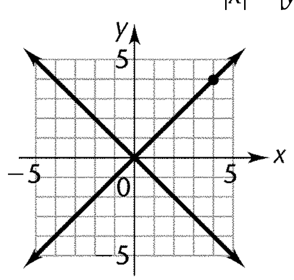
29. $|x| = |y|$

Test y axis
Replace x with $-x$:
$|-x| = |y|$
$|x| = |y|$

Test x axis
Replace y with $-y$:
$|x| = |-y|$
$|x| = |y|$

Origin symmetry
follows automatically.

x	y
0	0
4	4

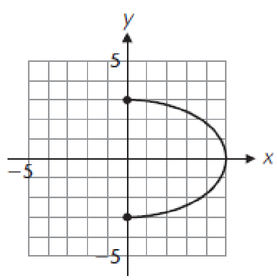


The graph has all three symmetries.
We reflect the portion of the graph in quadrant I through the y axis, and the x axis, and the origin, using all three symmetries.

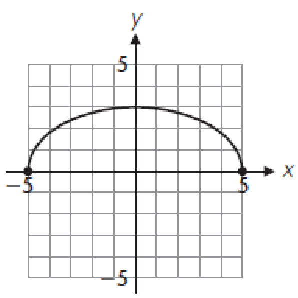
31. (A) When $x = 8$, the corresponding y value on the graph is 6, to the nearest integer.
(B) When $x = –5$, the corresponding y value on the graph is –5, to the nearest integer.
(C) When $x = 0$, the corresponding y value on the graph is –1, to the nearest integer.
(D) When $y = 6$, the corresponding x value on the graph is 8, to the nearest integer.
(E) When $y = –5$, the corresponding x value on the graph is –5, to the nearest integer.
(F) When $y = 0$, the corresponding x value on the graph is 5, to the nearest integer.

33. (A) When $x = 1$, the corresponding y value on the graph is 6, to the nearest integer.
(B) When $x = –8$, the corresponding y value on the graph is 4, to the nearest integer.
(C) When $x = 0$, the corresponding y value on the graph is 4, to the nearest integer.
(D) When $y = –6$, the corresponding x value on the graph is 8, to the nearest integer.
(E) Three values of x correspond to $y = 4$ on the graph. To the nearest integer they are –8, 0, and 6.
(F) Three values of x correspond to $y = 0$ on the graph. To the nearest integer they are –7, –2, and 7.

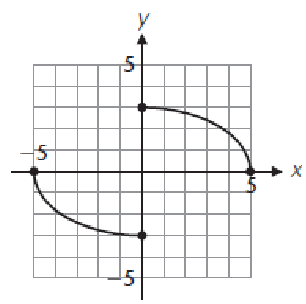
35. (A) Reflect the given graph across the x axis.



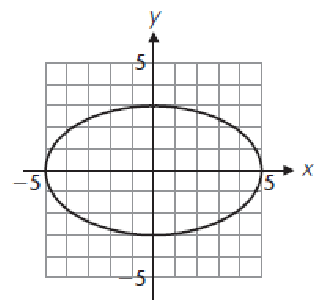
(B) Reflect the given graph across the y axis.



(C) Reflect the given graph through the origin.



(D) Reflect the given graph across the y axis, then reflect the resulting curve across the x axis.



37. $2x + 7y = 0$

Test y axis
Replace x with $–x$:
$2(–x) + 7y = 0$
$–2x + 7y = 0$

Test x axis
Replace y with $–y$:
$2x + 7(–y) = 0$
$2x – 7y = 0$

Test origin
Replace x with $–x$ and y with $–y$:
$2(–x) + 7(–y) = 0$
$–2x – 7y = 0$
$2x + 7y = 0$

The graph has symmetry with respect to the origin.

39. $x^2 – 4xy^2 = 3$

Test y axis
Replace x with $–x$:
$(–x)^2 – 4(–x)y^2 = 3$
$x^2 + 4xy^2 = 3$

Test x axis
Replace y with $–y$:
$x^2 – 4x(–y)^2 = 3$
$x^2 – 4xy^2 = 3$

Test origin
Replace x with $–x$ and y with $–y$:
$(–x)^2 – 4(–x)(–y)^2 = 3$
$x^2 + 4xy^2 = 3$

The graph has symmetry with respect to the x axis.

41. $x^4 – 5x^2y + y^4 = 1$

Test y axis
Replace x with $–x$:
$(–x)^4 – 5(–x)^2y + y^4 = 1$
$x^4 – 5x^2y + y^4 = 1$

Test x axis
Replace y with $–y$:
$x^4 – 5x^2(–y) + (–y)^4 = 1$
$x^4 + 5x^2y + y^4 = 1$

Test origin
Replace x with $–x$ and y with $–y$:
$(–x)^4 – 5(–x)^2(–y) + (–y)^4 = 1$
$x^4 + 5x^2y + y^4 = 1$

The graph has symmetry with respect to the y axis.

43. $x^3 - y^3 = 8$

Test y axis
Replace x with $-x$:
$(-x)^3 - y^3 = 8$
$-x^3 - y^3 = 8$

Test x axis
Replace y with $-y$:
$x^3 - (-y)^3 = 8$
$x^3 + y^3 = 8$

Test origin
Replace x with $-x$ and y with $-y$:
$(-x)^3 - (-y)^3 = 8$
$-x^3 + y^3 = 8$

The graph has none of these symmetries.

45. $x^4 - 4x^2y^2 + y^4 = 81$

Test y axis
Replace x with $-x$:
$(-x)^4 - 4(-x)^2y^2 + y^4 = 81$
$x^4 - 4x^2y^2 + y^4 = 81$

Test x axis
Replace y with $-y$:
$x^4 - 4x^2(-y)^2 + (-y)^4 = 81$
$x^4 - 4x^2y^2 + y^4 = 81$

Origin symmetry follows automatically

The graph has symmetry with respect to the x axis, the y axis, and the origin.

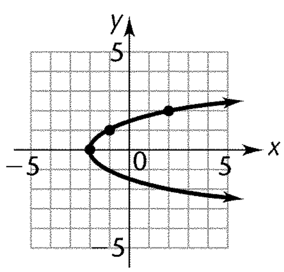
47. $y^2 = x + 2$

Test y axis
Replace x with $-x$:
$y^2 = -x + 2$

Test x axis
Replace y with $-y$:
$(-y)^2 = x + 2$
$y^2 = x + 2$

Test origin
Replace x with $-x$ and y with $-y$:
$(-y)^2 = (-x) + 2$
$y^2 = -x + 2$

x	y
-2	0
-1	1
2	2



The graph has symmetry with respect to the x axis. To obtain the portion of the graph for $y \ge 0$, we sketch $y = \sqrt{x+2}$, $x \ge -2$. We reflect the portion of the graph for $y \ge 0$ across the x axis, using the x axis symmetry.

49. $y = x^2 + 1$

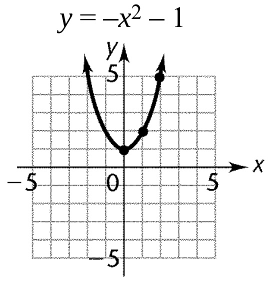
Test y axis
Replace x with $-x$:
$y = (-x)^2 + 1$
$y = x^2 + 1$

Test x axis
Replace y with $-y$:
$(-y) = x^2 + 1$
$y = -x^2 - 1$

Test origin
Replace x with $-x$ and y with $-y$:
$(-y) = (-x)^2 + 1$
$y = -x^2 - 1$

The graph has symmetry with respect to the y axis. We reflect the portion of the graph for $x \ge 0$ across the y axis, using the y axis symmetry.

x	y
0	1
1	2
2	5



51. $4y^2 - x^2 = 1$

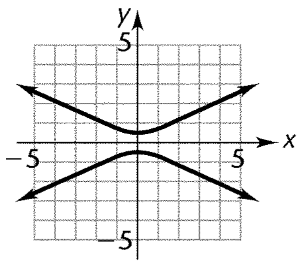
Test y axis
Replace x with $-x$:
$4y^2 - (-x)^2 = 1$
$4y^2 - x^2 = 1$

Test x axis
Replace y with $-y$:
$4(-y)^2 - x^2 = 1$
$4y^2 - x^2 = 1$

Origin symmetry follows automatically.

The graph has all three symmetries. $y = \pm\frac{1}{2}\sqrt{x^2+1}$.

x	y
0	$\frac{1}{2}$
2	$\frac{1}{2}\sqrt{5} \approx 1.1$
4	$\frac{1}{2}\sqrt{17} \approx 2.0$



The graph has all three symmetries.

To obtain the quadrant I portion of the graph, we sketch $y = \frac{1}{2}\sqrt{x^2 + 1}$, $x \geq 0$. We reflect this graph across the y axis, then reflect everything across the x axis.

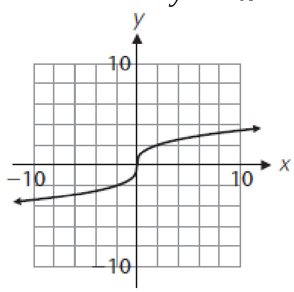
53. $y^3 = x$

Test y axis
Replace x with $-x$:
$y^3 = -x$

Test x axis
Replace y with $-y$:
$(-y)^3 = x$
$y^3 = -x$

Test origin
Replace x with $-x$ and y with $-y$:
$(-y)^3 = -x$
$y^3 = x$

x	y
0	0
1	1
8	2



The graph has symmetry with respect to the origin. We reflect the portion of the graph in quadrant I through the origin, using the origin symmetry.

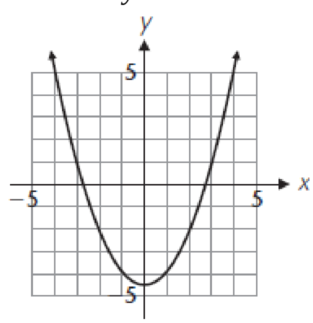
55. $y = 0.6x^2 - 4.5$

Test y axis
Replace x with $-x$:
$y = 0.6(-x)^2 - 4.5$
$y = 0.6x^2 - 4.5$

Test x axis
Replace y with $-y$:
$-y = 0.6x^2 - 4.5$
$y = -0.6x^2 + 4.5$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = 0.6(-x)^2 - 4.5$
$y = -0.6x^2 + 4.5$

x	y
0	−4.5
1	−3.9
2	−2.1
3	0.9
4	5.1



The graph has symmetry with respect to the y axis. We reflect the portion of the graph in quadrant I through the y axis, using the y axis symmetry.

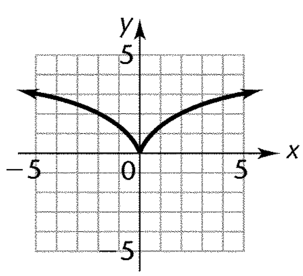
57. $y = x^{2/3}$

Test y axis
Replace x with $-x$:
$y = (-x)^{2/3}$
$y = x^{2/3}$

Test x axis
Replace y with $-y$:
$-y = x^{2/3}$
$y = -x^{2/3}$

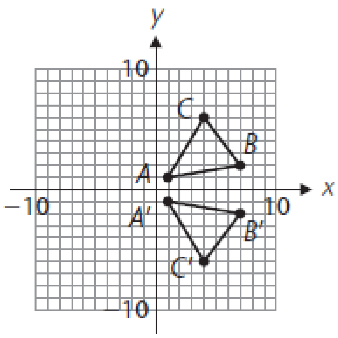
Test origin
Replace x with $-x$ and y with $-y$:
$-y = (-x)^{2/3}$
$y = -x^{2/3}$

x	y
0	0
1	1
2	$2^{2/3} \approx 1.6$
3	$3^{2/3} \approx 2.1$



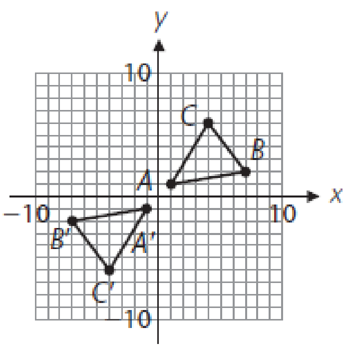
The graph has symmetry with respect to the y axis. We reflect the portion of the graph for $x \geq 0$ across the y axis, using the y axis symmetry.

59. (A) and (B)



(C) The triangles are mirror images of each other, reflected across the x axis. Changing the sign of the y coordinate reflects the graph across the x axis.

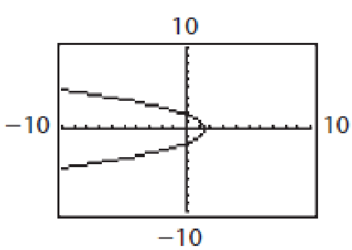
61. (A) and (B)



(C) The triangles are mirror images of each other, reflected across the origin. Changing the signs of both coordinates reflects the graph through the origin.

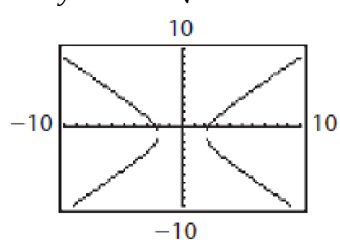
63.

$$\begin{aligned} 2x + y^2 &= 3 \\ y^2 &= 3 - 2x \\ y &= \pm\sqrt{3-2x} \end{aligned}$$



65.

$$\begin{aligned} x^2 - (y+1)^2 &= 4 \\ -(y+1)^2 &= 4 - x^2 \\ (y+1)^2 &= x^2 - 4 \\ y + 1 &= \pm\sqrt{x^2-4} \\ y &= -1 \pm \sqrt{x^2-4} \end{aligned}$$



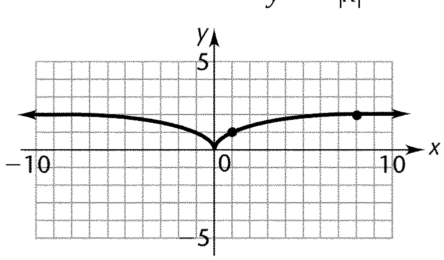
67. $y^3 = |x|$

Test y axis
Replace x with $-x$:
$y^3 = |-x|$
$y^3 = |x|$

Test x axis
Replace y with $-y$:
$(-y)^3 = |x|$
$y^3 = -|x|$

Test origin
Replace x with $-x$ and y with $-y$:
$(-y)^3 = |-x|$
$y^3 = -|x|$

x	y
0	0
1	1
8	2



The graph has symmetry with respect to the y axis. We reflect the portion of the graph for $x \geq 0$ across the y axis using the y axis symmetry.

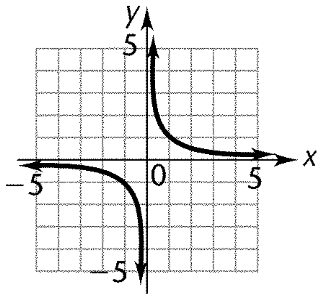
69. $xy = 1$

Test y axis
Replace x with $-x$:
$(-x)y = 1$
$xy = -1$

Test x axis
Replace y with $-y$:
$x(-y) = 1$
$xy = -1$

Test origin
Replace x with $-x$ and y with $-y$:
$(-x)(-y) = 1$
$xy = 1$

x	y
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
$\frac{1}{2}$	2
$\frac{1}{3}$	3



The graph has symmetry with respect to the origin. We reflect the portion of the graph in quadrant I through the origin, using the origin symmetry.

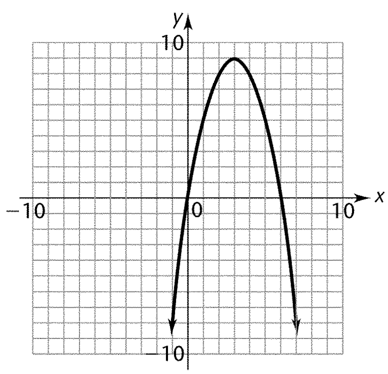
71. $y = 6x - x^2$

Test y axis
Replace x with $-x$:
$y = 6(-x) - (-x)^2$
$y = -6x - x^2$

Test x axis
Replace y with $-y$:
$-y = 6x - x^2$
$y = -6x + x^2$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = 6(-x) - (-x)^2$
$y = 6x + x^2$

x	y
–1	–7
0	0
1	5
2	8
3	9
4	8
5	5
6	0
7	–7



The graph has none of these three symmetries. A larger table of values is needed since we have no symmetry information.

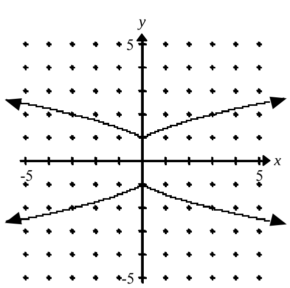
73. $y^2 = |x| + 1$

Test y axis
Replace x with $-x$:
$y^2 = |-x| + 1$
$y^2 = |x| + 1$

Test x axis
Replace y with $-y$:
$(-y^2) = |x| + 1$
$y^2 = |x| + 1$

Origin symmetry follows automatically.

x	y
0	1
1	$\sqrt{2} \approx 1.4$
3	2



The graph has symmetry with respect to the x axis, the y axis, and the origin. $y = \pm\sqrt{|x|+1}$ To obtain the quadrant I portion of this graph, we sketch $y = \sqrt{|x|+1}$ $x \geq 0$. We reflect this graph across the y axis, then reflect everything across the x axis.

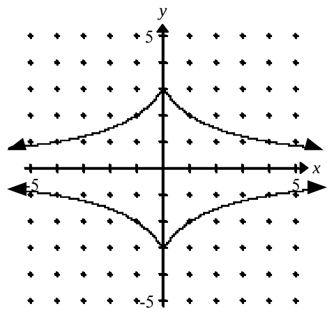
75. $|xy| + 2|y| = 6$

Test y axis
Replace x with $-x$:
$|(-x)y| + 2|y| = 6$
$|xy| + 2|y| = 6$

Test x axis
Replace y with $-y$:
$|x(-y)| + 2|-y| = 6$
$|xy| + 2|y| = 6$

Origin symmetry follows automatically.

x	y
0	3
1	2
4	1



The graph has symmetry with respect to the x axis, the y axis, and the origin. We reflect the portion of the graph in quadrant I across the y axis, then reflect everything across the x axis.

77. Reflecting a point (x, y) across the x axis yields the point $(x, -y)$. Reflecting this point through the origin yields the point $(-x, y)$. This point is the same point that would result from reflecting the original point across the y axis. Therefore, if the graph is unchanged by reflecting across the x axis and across the origin, it will be unchanged by reflecting across the y axis and will necessarily have symmetry with respect to the y axis.

79. No. For example, the graph of $xy = 1$ is symmetric with respect to the origin, and the equation is unchanged when x is replaced by $-x$ and y is replaced by $-y$ to obtain $(-x)(-y) = 1$ or $xy = 1$. However, it is not symmetric with respect to the x axis, as is seen when only y is replaced by $-y$ to obtain $x(-y)=1$ or $-xy = 1$.

81.

P	$R = (10 - p)p$
5	25
6	24
7	21
8	16
9	9
10	0

83. (A) \$6.00 on the price scale corresponds to 3,000 cases on the demand scale.

(B) The demand decreases from 3,000 to 2,600 cases, that is, by 400 cases.

(C) The demand increases from 3,000 to 3,600 cases, that is, by 600 cases.

(D) Demand decreases with increasing price and increases with decreasing price. To increase demand from 2,000 to 4,000 cases, a price decrease from \$6.90 to \$5.60 is necessary.

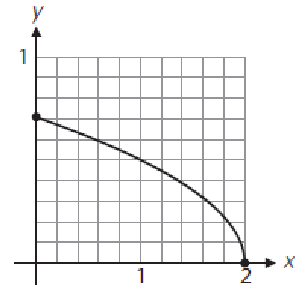
85. (A) 9:00 is halfway from 6 AM to noon, and corresponds to a temperature of 53°.

(B) The highest temperature occurs halfway from noon to 6 PM, at 3 PM. This temperature is 68°.

(C) This temperature occurs at 1 AM, 7 AM, and 11 PM.

87. (A) There is no obvious symmetry. A table of values yields the following approximate values:

x	0	0.5	1	1.5	2
v	0.7	0.6	0.5	0.35	0



(B) For a displacement of 2 cm, the ball is stationary ($v = 0$). As the vertical displacement approaches 0, the ball gathers speed until $v = 0.5\sqrt{2} \approx 0.7$ m/sec. The velocity carries the ball through the equilibrium position ($x = 0$) to rise again to displacement 2 cm and velocity 0.

Section 2-2

1. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the square of the lengths of the other two sides.

3. The x coordinate of the midpoint is the average (or arithmetic mean) of the x coordinate of the endpoints. The y coordinate of the midpoint is the average of the y coordinates of the endpoints.

5. $d = \sqrt{(4-1)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Midpoint $= \left(\frac{1+4}{2}, \frac{0+4}{2}\right) = \left(\frac{5}{2}, 2\right)$

7. $d = \sqrt{(5-0)^2 + (10-(-2))^2} = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = 13$

Midpoint $= \left(\frac{0+5}{2}, \frac{-2+10}{2}\right) = \left(\frac{5}{2}, 4\right)$

9. $d = \sqrt{(3-(-6))^2 + (4-(-4))^2} = \sqrt{9^2 + 8^2} = \sqrt{81+64} = \sqrt{145}$

Midpoint $= \left(\frac{-6+3}{2}, \frac{-4+4}{2}\right) = \left(\frac{-3}{2}, 0\right)$

11. $d = \sqrt{(-6-(-2))^2 + (-3-(-1))^2} = \sqrt{20} = 2\sqrt{5}$

Midpoint $= \left(\frac{(-6)+(-2)}{2}, \frac{(-3)+(-1)}{2}\right) = (-4, -2)$

13. $(x-0)^2 + (y-0)^2 = 7^2$
$x^2 + y^2 = 49$

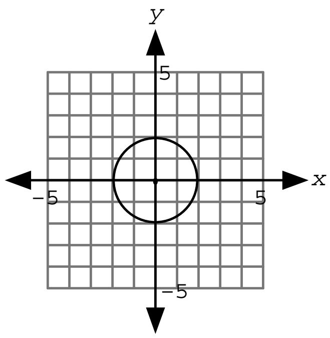
15. $(x-2)^2 + (y-3)^2 = 6^2$
$(x-2)^2 + (y-3)^2 = 36$

17. $[x-(-4)]^2 + (y-1)^2 = (\sqrt{7})^2$
$(x+4)^2 + (y-1)^2 = 7$

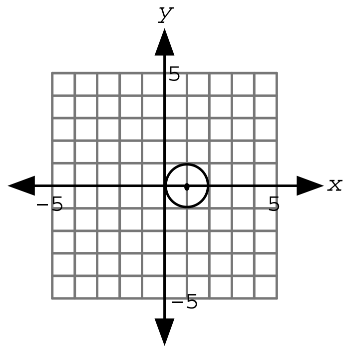
$\boxed{\text{Common Error: not } (x-4)^2}$

19. $[x-(-3)]^2 + [y-(-4)]^2 = (\sqrt{2})^2$
$(x+3)^2 + (y+4)^2 = 2$

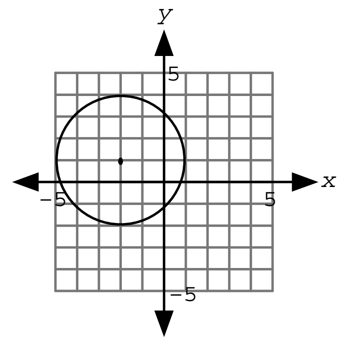
21. This is a circle with center (0, 0) and radius 2.
$x^2 + y^2 = 4$



23. This is a circle with center (1, 0) and radius 1.
$(x - 1)^2 + y^2 = 1$



25. This is a circle with center (–2, 1) and radius 3.
$(x - (-2))^2 + (y - 1)^2 = 9$
or $(x + 2)^2 + (y - 1)^2 = 9$



27. (A) $-2 = \dfrac{a_1 + 1}{2}$ (B) $6 = \dfrac{a_2 + 3}{2}$

$-4 = a_1 + 1$ $\quad 12 = a_2 + 3$

$-5 = a_1$ $\quad 9 = a_2$

(C) From parts (A) and (B), $A = (-5, 9)$

$$d(A, M) = \sqrt{(-2-(-5))^2 + (6-9)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$d(M, B) = \sqrt{(1-(-2))^2 + (3-6)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$$

As expected, the distances are the same.

29. The distance formula requires that $\sqrt{[x-(-4)]^2 + (7-1)^2} = 10$

Solving, we have

$$\begin{aligned}
\sqrt{(x+4)^2 + (6)^2} &= 10 \\
(x + 4)^2 + (6)^2 &= 10^2 \\
(x + 4)^2 + 36 &= 100 \\
(x + 4)^2 &= 64 \\
x + 4 &= \pm 8 \\
x &= -4 \pm 8 \\
x &= -12, 4
\end{aligned}$$

31. The distance formula requires that $\sqrt{[2-(-1)]^2 + (y-4)^2} = 3$

Solving, we have

$$\begin{aligned}
\sqrt{(3)^2 + (y-4)^2} &= 3 \\
(3)^2 + (y - 4)^2 &= (3)^2 \\
(y - 4)^2 &= 0 \\
y - 4 &= 0 \\
y &= 4
\end{aligned}$$

33. This is a circle with center (0, 2) and radius 2; that is, the set of all points that are 2 units away from (0, 2).
$x^2 + (y - 2)^2 = 4$

35. This is a circle with center (1, 1) and radius 4; that is, the set of all points that are 4 units away from (1, 1).
$(x - 1)^2 + (y - 1)^2 = 16$

37. $M = \left(\frac{-4.3+9.6}{2}, \frac{5.2+(-1.7)}{2}\right) = \left(\frac{5.3}{2}, \frac{3.5}{2}\right) = (2.65, 1.75)$

$$d(A, M) = \sqrt{(2.65-(-4.3))^2+(1.75-5.2)^2} = \sqrt{6.95^2+(-3.45)^2} = \sqrt{48.3025+11.9025}$$
$$= \sqrt{60.205} = 7.76$$
$$d(M, B) = \sqrt{(9.6-2.65)^2+(-1.7-1.75)^2} = \sqrt{6.95^2+(-3.45)^2} = \sqrt{48.3025+11.9025}$$
$$= \sqrt{60.205} = 7.76$$
$$d(A, B) = \sqrt{(9.6-(-4.3))^2+(-1.7-5.2)^2} = \sqrt{13.9^2+(-6.9)^2} = \sqrt{193.21+47.61}$$
$$= \sqrt{240.82} = 15.52$$
$$\frac{1}{2}d(A, B) = \frac{1}{2}(15.52) = 7.76$$

39. Write $B = (b_1, b_2)$. -5 is the average of 25 and b_1, so $-5 = \frac{25+b_1}{2}$

$$-10 = 25 + b_1$$
$$-35 = b_1$$

-2 is the average of 10 and b_2, so $-2 = \frac{10+b_2}{2}$

$$-4 = 10 + b_2$$
$$-14 = b_2$$

So $B = (-35, -14)$.

$$d(A, M) = \sqrt{(-5-25)^2+(-2-10)^2} = \sqrt{(-30)^2-(12)^2} = \sqrt{900+144} = \sqrt{1{,}044} = 32.3$$
$$d(M, B) = \sqrt{(-35-(-5))^2+(-14-(-2))^2} = \sqrt{(-30)^2+(-12)^2} = \sqrt{900+144} = \sqrt{1{,}044} = 32.3$$
$$d(A, B) = \sqrt{(-35-25)^2+(-14-10)^2} = \sqrt{(-60)^2+(-24)^2} = \sqrt{3{,}600+576} = \sqrt{4{,}176} = 64.6$$
$$\frac{1}{2}d(A, B) = \frac{1}{2}(64.6) = 32.3$$

41. Write $A = (a_1, a_2)$. -8 is the average of a_1 and 2, so $-8 = \frac{a_1+2}{2}$

$$-16 = a_1 + 2$$
$$-18 = a_1$$

-6 is the average of a_2 and 4, so $-6 = \frac{a_2+4}{2}$

$$-12 = a_2 + 4$$
$$-16 = a_2$$

So $A = (-18, -16)$.

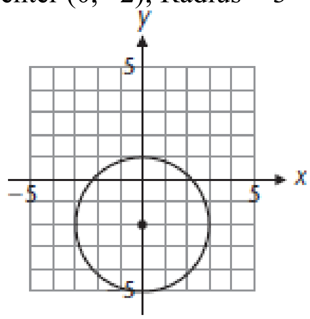
$$d(A, M) = \sqrt{(-8-(-18))^2+(-6-(-16))^2} = \sqrt{10^2+10^2} = \sqrt{100+100} = \sqrt{200} = 14.14$$
$$d(M, B) = \sqrt{(2-(-8))^2+(4-(-6))^2} = \sqrt{10^2+10^2} = \sqrt{100+100} = \sqrt{200} = 14.14$$
$$d(A, B) = \sqrt{(2-(-18))^2+(4-(-16))^2} = \sqrt{20^2+20^2} = \sqrt{400+400} = \sqrt{800} = 28.28$$

$$\frac{1}{2}d(A, B) = \frac{1}{2}(28.28) = 14.14$$

43. $x^2 + (y + 2)^2 = 9$

$(x - 0)^2 + (y - (-2))^2 = 3^2$

Center $(0, -2)$; Radius = 3

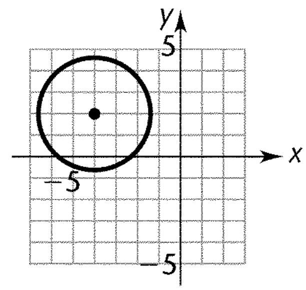


45.

$$(x + 4)^2 + (y - 2)^2 = 7$$

$$[x - (-4)]^2 + (y - 2)^2 = (\sqrt{7})^2$$

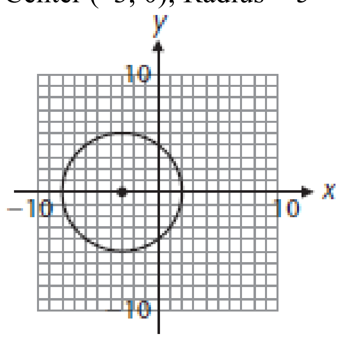
Center $(-4, 2)$; Radius = $\sqrt{7}$



47.

$$\begin{aligned} x^2 + 6x + y^2 &= 16 \\ x^2 + 6x + 9 + y^2 &= 16 + 9 \\ (x + 3)^2 + y^2 &= 25 \end{aligned}$$

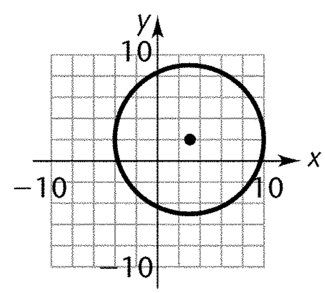
Center $(-3, 0)$; Radius = 5



49.

$$\begin{aligned} x^2 + y^2 - 6x - 4y &= 36 \\ x^2 - 6x + y^2 - 4y &= 36 \\ x^2 - 6x + 9 + y^2 - 4y + 4 &= 36 + 9 + 4 \\ (x - 3)^2 + (y - 2)^2 &= 49 \end{aligned}$$

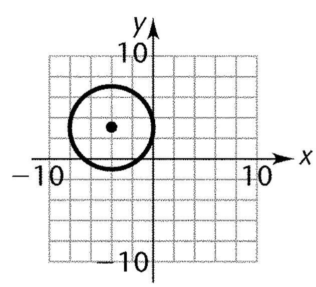
Center $(3, 2)$; Radius = 7



51.

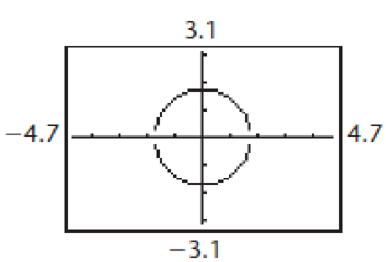
$$\begin{aligned} 3x^2 + 3y^2 + 24x - 18y + 24 &= 0 \\ x^2 + y^2 + 8x - 6y + 8 &= 0 \\ x^2 + 8x + y^2 - 6y &= -8 \\ x^2 + 8x + 16 + y^2 - 6y + 9 &= -8 + 16 + 9 \\ (x + 4)^2 + (y - 3)^2 &= 17 \end{aligned}$$

Center $(-4, 3)$; Radius = $\sqrt{17}$



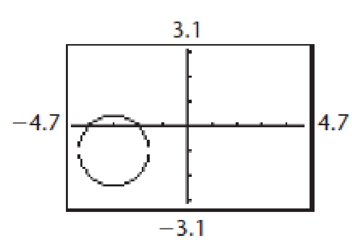
53.

$$\begin{aligned} x^2 + y^2 &= 3 \\ y^2 &= 3 - x^2 \\ y &= \pm\sqrt{3 - x^2} \end{aligned}$$



55.

$$\begin{aligned} (x + 3)^2 + (y + 1)^2 &= 2 \\ (y + 1)^2 &= 2 - (x + 3)^2 \\ y + 1 &= \pm\sqrt{2 - (x + 3)^2} \\ y &= -1 \pm \sqrt{2 - (x + 3)^2} \end{aligned}$$



57. Let $A = (-3, 2)$, $B = (1, -2)$, $C = (8, 5)$

$d(A, B) = \sqrt{(1-(-3))^2+(-2-2)^2} = \sqrt{4^2+(-4)^2} = \sqrt{16+16} = \sqrt{32}$

$d(B, C) = \sqrt{(8-1)^2+(5-(-2))^2} = \sqrt{7^2+7^2} = \sqrt{49+49} = \sqrt{98}$

$d(A, C) = \sqrt{(8-(-3))^2+(5-2)^2} = \sqrt{11^2+3^2} = \sqrt{121+9} = \sqrt{130}$

Notice that $(d(A, B))^2 + (d(B, C))^2 = 32 + 98 = 130 = (d(A, C))^2$

Since these distances satisfy the Pythagorean Theorem, the three points are vertices of a right triangle. The segment connecting A and C is the hypotenuse (it's the longest side) so we need to find its midpoint.

$$M = \left(\frac{-3+8}{2}, \frac{2+5}{2}\right) = \left(\frac{5}{2}, \frac{7}{2}\right)$$

The vertex opposite the hypotenuse is B

$d(M, B) = \sqrt{\left(1-\frac{5}{2}\right)^2+\left(-2-\frac{7}{2}\right)^2} = \sqrt{\left(\frac{-3}{2}\right)^2+\left(\frac{-11}{2}\right)^2} = \sqrt{\frac{9}{4}+\frac{121}{4}} = \sqrt{\frac{130}{4}} = \sqrt{32.5}$

59. Perimeter = sum of lengths of all three sides

$= \sqrt{[1-(-3)]^2+[(-2)-1]^2} + \sqrt{(4-1)^2+[3-(-2)]^2} + \sqrt{[4-(-3)]^2+(3-1)^2}$

$= \sqrt{16+9} + \sqrt{9+25} + \sqrt{49+4} = \sqrt{25} + \sqrt{34} + \sqrt{53}$ = 18.11 to two decimal places

61. $d(P_1, M) = \sqrt{\left(\frac{x_1+x_2}{2}-x_1\right)^2+\left(\frac{y_1+y_2}{2}-y_1\right)^2}$

$= \sqrt{\left(\frac{x_1+x_2-2x_1}{2}\right)^2+\left(\frac{y_1+y_2-2y_1}{2}\right)^2} = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}$

$d(M, P_2) = \sqrt{\left(x_2-\frac{x_1+x_2}{2}\right)^2+\left(y_2-\frac{y_1+y_2}{2}\right)^2}$

$= \sqrt{\left(\frac{2x_2-x_1-x_2}{2}\right)^2+\left(\frac{2y_2-y_1-y_2}{2}\right)^2} = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}$

$\frac{1}{2}d(P_1, P_2) = \frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{\frac{1}{4}\left[(x_2-x_1)^2+(y_2-y_1)^2\right]}$

Note: The $\frac{1}{2}$ becomes $\frac{1}{4}$ when moved inside the radical symbol.

$= \sqrt{\frac{(x_2-x_1)^2}{4}+\frac{(y_2-y_1)^2}{4}} = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}$

All three of these distances are equal.

63. The center of the circle is at the midpoint of the given diameter. From the midpoint formula, then, the center is at

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{-4+6}{2}, \frac{3+3}{2}\right) = (1, 3) = (h, k)$$

The radius of the circle is the distance from this midpoint to either endpoint. From the distance formula, then, the radius is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(6-1)^2+(3-3)^2} = 5 = r$$

Substitute into the standard form of the equation of a circle.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-1)^2 + (y-3)^2 = 5^2$$
$$(x-1)^2 + (y-3)^2 = 25$$

65. The center of the circle is at the midpoint of the given diameter. From the midpoint formula, then, the center is at

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{4+0}{2}, \frac{0+10}{2}\right) = (2, 5) = (h, k)$$

The radius of the circle is the distance from this midpoint to either endpoint. From the distance formula, then, the radius is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(4-2)^2+(0-5)^2} = \sqrt{4+25} = \sqrt{29} = r$$

Substitute into the standard form of the equation of a circle.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-2)^2 + (y-5)^2 = (\sqrt{29})^2$$
$$(x-2)^2 + (y-5)^2 = 29$$

67. The center of the circle is at the midpoint of the given diameter. From the midpoint formula, then, the center is at

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{11+3}{2}, \frac{(-2)+(-4)}{2}\right) = (7, -3) = (h, k)$$

The radius of the circle is the distance from this midpoint to either endpoint. From the distance formula, then, the radius is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(3-7)^2+[(-4)-(-3)]^2} = \sqrt{16+1} = \sqrt{17} = r$$

Substitute into the standard form of the equation of a circle.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-7)^2 + (y-(-3))^2 = (\sqrt{17})^2$$
$$(x-7)^2 + (y+3)^2 = 17$$

69. The radius of the circle is the distance from the given center to the given point. From the distance formula, then, the radius is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2-0)^2+[(-4)-5]^2} = \sqrt{4+81} = \sqrt{85} = r$$

Substitute into the standard form of the equation of a circle.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-0)^2 + (y-5)^2 = (\sqrt{85})^2$$
$$x^2 + (y-5)^2 = 85$$

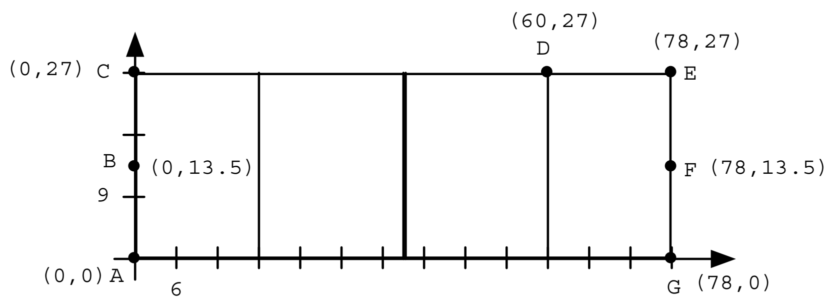
71. The radius of the circle is the distance from the given center to the given point. From the distance formula, then, the radius is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{[8-(-2)]^2+[(-7)-9]^2} = \sqrt{100+256} = \sqrt{356} = r$$

Substitute into the standard form of the equation of a circle.

$$(x-h)^2+(y-k)^2 = r^2$$
$$[x-(-2)]^2+(y-9)^2 = (\sqrt{356})^2$$
$$(x+2)^2+(y-9)^2 = 356$$

73.



$$d(B, D) = \sqrt{(60-0)^2+(27-13.5)^2} = \sqrt{60^2+13.5^2} = \sqrt{3600+182.25} = \sqrt{3,782.25} = 62 \text{ ft.}$$

$$d(F, C) = \sqrt{(78-0)^2+(13.5-27)^2} = \sqrt{78^2+(-13.5)^2} = \sqrt{6,084+182.25} = \sqrt{6,266.25} = 79 \text{ ft.}$$

75. Using the hint, we note that $(2, r-1)$ must satisfy $x^2 + y^2 = r^2$, that is

$$2^2+(r-1)^2 = r^2$$
$$4+r^2-2r+1 = r^2$$
$$-2r+5 = 0$$
$$r = \frac{5}{2} \text{ or } 2.5 \text{ ft.}$$

77. (A) From the drawing, we can write:

$$\begin{pmatrix}\text{Distance from tower}\\ \text{to town B}\end{pmatrix} = 2\times\begin{pmatrix}\text{Distance from tower}\\ \text{to town A}\end{pmatrix}$$

$$\begin{pmatrix}\text{Distance from } (x,y)\\ \text{to } (36,15)\end{pmatrix} = 2\times\begin{pmatrix}\text{Distance from } (x,y)\\ \text{to } (0,0)\end{pmatrix}$$

$$\sqrt{(36-x)^2+(15-y)^2} = 2\sqrt{(0-x)^2+(0-y)^2}$$
$$\sqrt{(36-x)^2+(15-y)^2} = 2\sqrt{x^2+y^2}$$
$$(36-x)^2+(15-y)^2 = 4(x^2+y^2)$$
$$1,296-72x+x^2+225-30y+y^2 = 4x^2+4y^2$$
$$1,521 = 3x^2+3y^2+72x+30y$$
$$507 = x^2+y^2+24x+10y$$
$$144+25+507 = x^2+24x+144+y^2+10y+25$$
$$676 = (x+12)^2+(y+5)^2$$

The circle has center $(-12, -5)$ and radius 26.

(B) All points due east of Town A have y coordinate 0 in this coordinate system. The points on the circle for which $y = 0$ are found by substituting $y = 0$ into the equation of the circle and solving for x.

$$\begin{aligned} (x+12)^2 + (y+5)^2 &= 676 \\ (x+12)^2 + 25 &= 676 \\ (x+12)^2 &= 651 \\ x + 12 &= \pm\sqrt{651} \\ x &= -12 \pm \sqrt{651} \end{aligned}$$

Choosing the positive square root so that x is greater than –12 (east rather than west) we have $x = -12 + \sqrt{651} \approx 13.5$ miles.

Section 2-3

1. Given $Ax + By = C$ as the equation, the x intercept is found by setting $y = 0$ and solving to obtain $x = C/A$. The y intercept is found by setting $x = 0$ and solving to obtain $y = C/B$. If either A or B is 0 there is no corresponding intercept.

3. m is then the slope and b is the y intercept for the line.

5. If the two equations are $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$, then the lines will be parallel if $A_2/A_1 = B_2/B_1$ but this ratio is not equal to C_2/C_1. Also note that two distinct vertical lines are parallel and so are two distinct horizontal lines, so if $A_2/A_1 \neq C_2/C_1$ and $B_2 = B_1 = 0$, the lines are parallel; likewise if $B_2/B_1 \neq C_2/C_1$ and $A_2 = A_1 = 0$ the lines are parallel.

7. The vertical segment has length 3 so rise = 3. The horizontal segment has length 5 so run = 5.

Slope = $\frac{\text{rise}}{\text{run}} = \frac{3}{5}$. (2, 2) is on the graph.

Use the point-slope form.

$$\begin{aligned} y - 2 &= \frac{3}{5}(x-2) \quad \text{(Multiply both sides by 5)} \\ 5y - 10 &= 3(x-2) \\ 5y - 10 &= 3x - 6 \\ -3x + 5y &= +4 \\ 3x - 5y &= -4 \end{aligned}$$

Common Error: Multiplying both $\frac{3}{5}$ and $(x - 2)$ by 5 on the right side.

9. The vertical segment has length 2 so rise = 2. The horizontal segment has length 8 so run = 8.

Slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{8} = \frac{1}{4}$. (–4, 1) is on the graph.

Use the point-slope form.

$$\begin{aligned} y - 1 &= \frac{1}{4}(x-(-4)) \quad \text{(Multiply both sides by 4)} \\ 4y - 4 &= (x+4) \\ -x + 4y &= 8 \\ x - 4y &= -8 \end{aligned}$$

11. The vertical segment has length 3 and goes downward so rise = –3. The horizontal segment has length 5 so run = 5.

Slope = $\frac{\text{rise}}{\text{run}} = \frac{-3}{5}$. (–4, 2) is on the graph.

Use the point-slope form.

$$\begin{aligned} y - 2 &= \frac{-3}{5}(x-(-4)) \quad \text{(Multiply both sides by 5)} \\ 5y - 10 &= -3(x+4) \\ 5y - 10 &= -3x - 12 \\ 3x + 5y &= -2 \end{aligned}$$

13. The x intercept is –2. The y intercept is 2. From the point (–2, 0) to the point (0, 2), the value of y increases by 2 units as the value of x increases by 2 units. Thus slope $= \frac{\text{rise}}{\text{run}} = \frac{2}{2} = 1$.

Equation: $y = mx + b$

$y = 1x + 2$ or $y = x + 2$

15. The x intercept is –2. The y intercept is –4. From the point (–2, 0) to the point (0, –4) the value of y decreases by 4 units as the value of x increases by 2 units. Thus, the slope $= \frac{\text{rise}}{\text{run}} = \frac{-4}{2} = -2$.

Equation: $y = mx + b$

$y = -2x + (-4)$ or $y = -2x - 4$

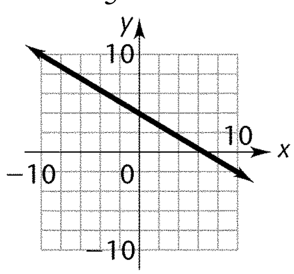
17. The x intercept is 3. The y intercept is –1. From the point (0, –1) to the point (3, 0) the value of y increases by 1 unit as the value of x increases by 3 units. Thus, the slope $= \frac{\text{rise}}{\text{run}} = \frac{1}{3}$.

Equation: $y = mx + b$

$y = \frac{1}{3}x + (-1)$ or $y = \frac{1}{3}x - 1$

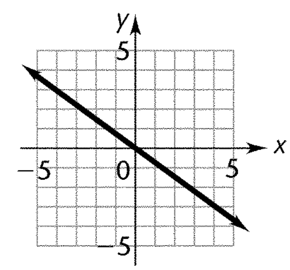
19. $y = -\frac{3}{5}x + 4$ slope $-\frac{3}{5}$

x	y
0	4
5	1
–5	7



21. $y = -\frac{3}{4}x$ slope $-\frac{3}{4}$

x	y
0	0
4	–3
–4	3

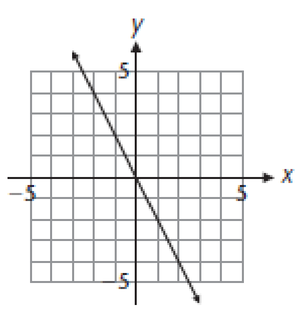


23. $4x + 2y = 0$

$2y = -4x$

$y = -2x$ slope –2

x	y
0	0
1	–2
2	–4

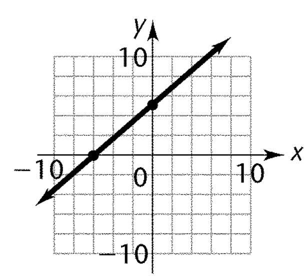


25. $4x - 5y = -24$

$-5y = -4x - 24$

$y = \frac{4}{5}x + \frac{24}{5}$ slope $= \frac{4}{5}$

x	y
–1	4
4	8
9	12

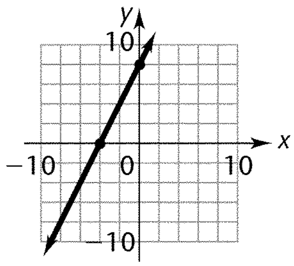


27. $\frac{y}{8} - \frac{x}{4} = 1$

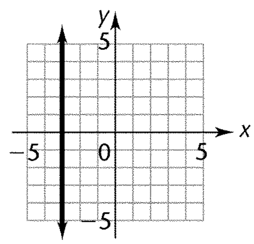
$\frac{y}{8} = \frac{x}{4} + 1$

$y = 2x + 8$ slope 2

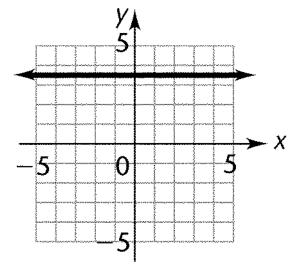
x	y
0	8
–4	0
–8	–8



29. $x = -3$
slope not defined
vertical line



31. $y = 3.5$
slope 0
horizontal line



33. Slope and y intercept are given;
We use slope-intercept form.

$y = -3x + 7$
$3x + y = 7$

35. Slope and y intercept are given;
We use slope-intercept form.

$y = \frac{7}{2}x - \frac{1}{3}$
$6y = 21x - 2$
$-21x + 6y = -2$
$21x - 6y = 2$

37. The equation of this horizontal line is $y = \frac{2}{3}$ or $3y = 2$.

39. A point and the slope are given; we use point-slope form.

$y - 3 = -2(x - 0)$
$y - 3 = -2x$
$y = -2x + 3$

41. A point and the slope are given; we use point-slope form.

$y - 4 = \frac{3}{2}[x - (-5)]$
$y - 4 = \frac{3}{2}[x + 5]$
$y - 4 = \frac{3}{2}x + \frac{15}{2}$
$y = \frac{3}{2}x + \frac{23}{2}$

43. A point and the slope are given; we use point-slope form.

$y - (-3) = -\frac{1}{2}[x - (-2)]$
$y + 3 = -\frac{1}{2}[x + 2]$
$y + 3 = -\frac{1}{2}x - 1$
$y = -\frac{1}{2}x - 4$

45. A point and the slope are given; we use point-slope form:

$y - 4 = -3(x - 0)$
$y - 4 = -3x$
$y = -3x + 4$

47. A point and the slope are given; we use point-slope form:

$y - 4 = -\frac{2}{5}[x - (-5)]$
$y - 4 = -\frac{2}{5}[x + 5]$
$y - 4 = -\frac{2}{5}x - 2$
$y = -\frac{2}{5}x + 2$

49. Two points are given; we first find the slope, then we use point-slope form.

$m = \frac{-2-6}{5-1} = \frac{-8}{4} = -2$ $\quad y - 6 = (-2)(x - 1)$
$y - 6 = -2x + 2$
$y = -2x + 8$

or $\quad y - (-2) = (-2)(x - 5)$
$y + 2 = -2x + 10$
$y = -2x + 8$

Thus, it does not matter which point is chosen in substituting into the point-slope form; both points must give rise to the same equation.

51. We proceed as in problem 49.

$$m = \frac{8-0}{-4-2} = \frac{8}{-6} = -\frac{4}{3}$$

$$y - 0 = -\frac{4}{3}(x-2)$$

$$y = -\frac{4}{3}x + \frac{8}{3}$$

53. We proceed as in problem 49.

$$m = \frac{4-4}{5-(-3)} = \frac{0}{8} = 0$$

$$y - 4 = 0(x-5)$$
$$y - 4 = 0$$
$$y = 4$$

55. We proceed as in problem 49.

$$m = \frac{-3-6}{4-4} = \frac{-9}{0}$$

slope is undefined.
A vertical line through (4, 6) has equation $x = 4$.

57. We proceed as in problem 49, using (–4, 0) and (0, 3) as the two given points.

$$m = \frac{3-0}{0-(-4)} = \frac{3}{4}$$

$$y - 0 = \frac{3}{4}[x-(-4)]$$

$$y = \frac{3}{4}x + 3$$

59. A line parallel to $y = 3x - 5$ will have the same slope, namely 3. We now use the point slope form.

$$y - 4 = 3[x-(-3)]$$
$$y - 4 = 3x + 9$$
$$y = 3x + 13$$
$$-3x + y = 13$$
$$3x - y = -13$$

61. A line perpendicular to $y = -\frac{1}{3}x$ will have slope satisfying $-\frac{1}{3}m = -1$, or $m = 3$.

We use the point-slope form.

$$y - (-3) = 3(x-2)$$
$$y + 3 = 3x - 6$$
$$9 = 3x - y$$
$$3x - y = 9$$

63. A line parallel to $3x - 2y = 4$ will have the same slope. The slope of $3x - 2y = 4$, or $2y = 3x - 4$, or $y = \frac{3}{2}x - 2$ is $\frac{3}{2}$. We use the point-slope form,

$$y - 0 = \frac{3}{2}(x-5)$$

$$y = \frac{3}{2}x - \frac{15}{2}$$

$$\frac{15}{2} = \frac{3}{2}x - y$$

$$15 = 3x - 2y$$

$$3x - 2y = 15$$

Alternatively, we could notice that a line parallel to $3x - 2y = 4$ will have an equation of the form $3x - 2y = C$.
Since the required line must contain (5, 0), (5, 0) must satisfy its equation. Therefore, $3(5) - 2(0) = C$. Since $15 = C$, the equation desired is $3x - 2y = 15$.

65. A line perpendicular to $x + 3y = 9$, which has slope $-\frac{1}{3}$, will have slope satisfying $-\frac{1}{3}m = -1$, or $m = 3$.

We use the point-slope form.

$$y - (-4) = 3(x-0)$$
$$y + 4 = 3x$$
$$4 = 3x - y$$
$$3x - y = 4$$

67. slope of $AB = \frac{-1-2}{4-0} = -\frac{3}{4}$

slope of $DC = \frac{-5-(-2)}{1-(-3)} = -\frac{3}{4}$

Therefore $AB \parallel DC$.

69. slope of $AB = -\frac{3}{4}$

slope of $BC = \frac{-5-(-1)}{1-4} = \frac{4}{3}$

(slope AB)(slope BC) $= \left(-\frac{3}{4}\right)\left(\frac{4}{3}\right) = -1$

Therefore $AB \perp BC$.

71. midpoint of $AD = \left(\frac{0+(-3)}{2}, \frac{2+(-2)}{2}\right) = \left(-\frac{3}{2}, 0\right)$

slope of $AD = \frac{-2-2}{-3-0} = \frac{4}{3}$

We require the equation of a line, through the midpoint of AD, which is perpendicular to AD. Its slope will satisfy $\frac{4}{3}m = -1$, or $m = -\frac{3}{4}$. We use the point-slope form.

$$y - 0 = -\frac{3}{4}\left[x - \left(-\frac{3}{2}\right)\right]$$
$$y = -\frac{3}{4}\left(x + \frac{3}{2}\right)$$
$$y = -\frac{3}{4}x - \frac{9}{8}$$
$$8y = -6x - 9$$
$$6x + 8y = -9$$

73. Two points are given; we first find the slope, then use the point-slope form.

$$m = \frac{0-b}{a-0} = -\frac{b}{a} \quad a \neq 0$$
$$y - b = -\frac{b}{a}(x - 0)$$
$$y - b = -\frac{bx}{a}$$

Divide both sides by b, then $(b \neq 0)$

$$\frac{y-b}{b} = -\frac{x}{a}$$
$$\frac{y}{b} - 1 = -\frac{x}{a}$$
$$\frac{y}{b} = 1 - \frac{x}{a}$$
$$\frac{x}{a} + \frac{y}{b} = 1$$

75. The circle has center (0, 0). The radius drawn from (0, 0) to the given point (3, 4) has slope given by

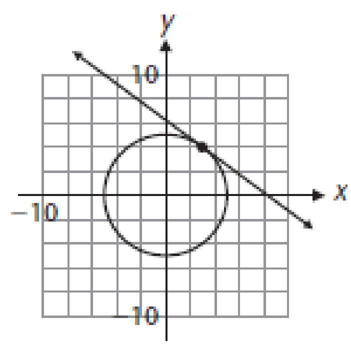
$$M_R = \frac{4-0}{3-0} = \frac{4}{3}.$$

Therefore, the slope of the tangent line is given by

$$\frac{4}{3}m = -1 \quad \text{or} \quad m = -\frac{3}{4}$$

We require the equation of a line through (3, 4) with slope $-\frac{3}{4}$. We use the point-slope form.

$$y - 4 = -\frac{3}{4}(x - 3)$$
$$y - 4 = -\frac{3}{4}x + \frac{9}{4}$$
$$4y - 16 = -3x + 9$$
$$3x + 4y = 25$$



77. The circle has center (0, 0). The radius drawn from (0, 0) to the given point (5, –5) has slope given by

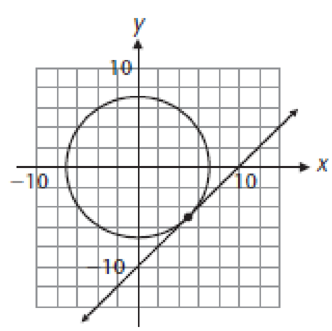
$$M_R = \frac{-5-0}{5-0} = -1$$

Therefore, the slope of the tangent line is given by

$$(-1)m = -1 \quad \text{or} \quad m = 1$$

We require the equation of a line through (5, –5) with slope 1. We use the point-slope form.

$$y - (-5) = 1(x - 5)$$
$$y + 5 = x - 5$$
$$x - y = 10$$



79. The circle has center (3, –4). The radius drawn from (3, –4) to the given point (8, –16) has slope given by

$$M_R = \frac{-16-(-4)}{8-3} = -\frac{12}{5}$$

Therefore, the slope of the tangent line is given by

$$\left(-\frac{12}{5}\right)m = -1 \quad \text{or} \quad m = \frac{5}{12}$$

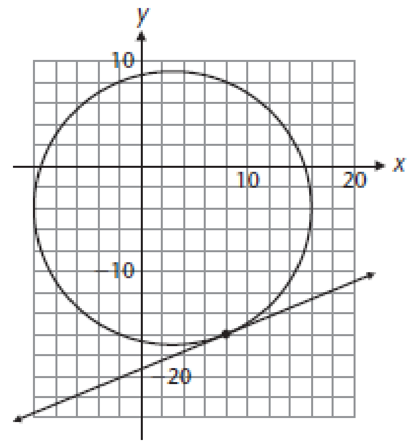
We require the equation of a line through (8, –16) with slope $\frac{5}{12}$. We use the point-slope form.

$$y-(-16) = \frac{5}{12}(x-8)$$

$$y + 16 = \frac{5}{12}x - \frac{40}{12}$$

$$12y + 192 = 5x - 40$$

$$232 = 5x - 12y$$



81. (A)

x	0	5,000	10,000	15,000	20,000	25,000	30,000
$212 - 0.0018x = B$	212	203	194	185	176	167	158

(B) The boiling point drops 9°F for each 5,000 foot increase in altitude.

83. Total cost = Fixed cost + variable cost. If x = the number of doughnuts produced, variable cost is $0.12x$ ($0.12 per doughnut times the number of doughnuts produced.) If C represents total cost,

$C = 124 + 0.12x$ Plug in 250 for C:

$250 = 124 + 0.12x$

$126 = 0.12x$

$1{,}050 = x$

The shop can produce 1,050 doughnuts for $250.

85. (A) We write $s = mw + b$. If $w = 0$, $s = 0$, hence $0 = m \cdot 0 + b$, $b = 0$.
If $w = 5$, $s = 2$, hence $2 = m \cdot 5$, $m = 0.4$.
$s = 0.4w$

(B) $s = 0.4(20) = 8$ inches

(C) Solve $3.6 = 0.4w$ to obtain $w = 9$ pounds.

87. (A) We write $F = mC + b$. If $C = 0$, $F = 32$, hence $32 = m \cdot 0 + b$, $b = 32$.
If $C = 100$, $F = 212$, hence
$212 = m \cdot 100 + 32$,
$100m = 180$, $m = \frac{9}{5}$.

$$F = \frac{9}{5}C + 32$$

(B) If $C = 20$, $F = \frac{9}{5}(20) + 32 = 68^\circ$F

If $F = 86$, solve

$$86 = \frac{9}{5}C + 32$$

$$54 = \frac{9}{5}C$$

$$C = \frac{5}{9}(54) = 30^\circ\text{C}$$

89.(A) If h is linearly related to t, then we are looking for an equation whose graph passes through $(t_1, h_1) = (9, 23)$ and $(t_2, h_2) = (24, 40)$. We find the slope, and then we use the point-slope form to find the equation.

$$m = \frac{h_2 - h_1}{t_2 - t_1} = \frac{40-23}{24-9} = \frac{17}{15} \approx 1.13$$

$$h - h_1 = m(t - t_1)$$

$$h - 23 = \frac{17}{15}(t-9)$$

$$h - 23 = \frac{17}{15}t - \frac{51}{5}$$

$$h = \frac{17}{15}t - \frac{51}{5} + 23 = 1.13t + 12.8$$

(B) We are asked for t when $h = 50$.

$50 = 1.13t + 12.8$

$37.2 = 1.13t$

$t = 32.9$ hours

91. (A) If L is linearly related to t, then we are looking for an equation whose graph passes through $(t_1, L_1) = (0, 49.2)$ and $(t_2, L_2) = (100, 77.3)$. We find the slope, then we use the point-slope form to find the equation.

$$m = \frac{L_2 - L_1}{t_2 - t_1} = \frac{77.3 - 49.2}{100 - 0} = 0.281$$

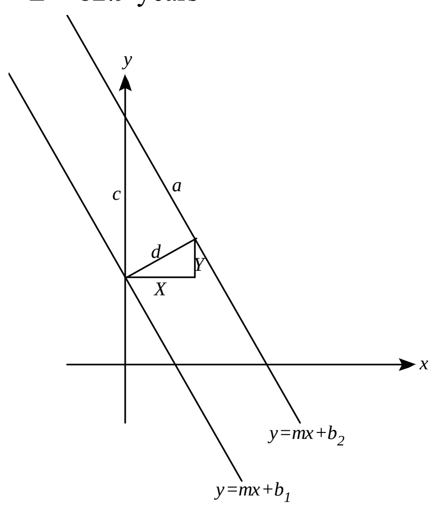
$$\begin{aligned} L - L_1 &= m(t - t_1) \\ L - 49.2 &= 0.281(t - 0) \\ L - 49.2 &= 0.281t \\ L &= 0.281t + 49.2 \end{aligned}$$

(B) We are asked for L when $t = 120$.

$L = 0.281(120) + 49.2$

$L = 82.9$ years

93.



In general, we can show that $d = \dfrac{c}{\sqrt{1+m^2}}$ as follows:

The Pythagorean Theorem gives: $d^2 + a^2 = c^2$

The two triangles shown are similar, hence corresponding sides are proportional. Thus

$$\frac{a}{d} = \frac{x}{y}$$

The slope of the line segment labeled d is the negative reciprocal of m.

Thus $\dfrac{y}{x} = -\dfrac{1}{m}$

$$\frac{x}{y} = -m$$

It follows that $a = \dfrac{x}{y} d = -md$.

$$\begin{aligned} \text{Hence } d^2 + (-md)^2 &= c^2 \\ d^2(1 + m^2) &= c^2 \\ d^2 &= \frac{c^2}{1+m^2} \\ d &= \frac{c}{\sqrt{1+m^2}} \end{aligned}$$

In particular, avenue A is shown to have a rise of –5000 and a run of 4000, hence $m = -\dfrac{5000}{4000} = -1.25$. The equation of avenue A is then (using the slope-intercept form $y = mx + b$) $y = -1.25x + 5000$. Avenue B has the same slope, and y intercept 4000. Substituting in the above formula, with $c = 5000 - 4000 = 1000$, yields

$$d_1 = \frac{1000}{\sqrt{1+(-1.25)^2}} = 625 \text{ ft.}$$

Section 2-4

1. The process of mathematical modeling consists of three steps.

Step 1: Construct the mathematical model, a mathematics problem that, when solved, will provide information about the real-world problem.
Step 2: Solve the mathematical model.
Step 3: Interpret the solution to the mathematical model in terms of the original real-world problem.

3. Calculate the values y_1 and y_2 associated with x_1 and x_2 respectively,

Then, $\text{average rate of change} = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

5. (A) If cost y is linearly related to the number of golf clubs x_1 then we are looking for an equation whose graph passes through $(x_1, y_1) = (80, 8{,}147)$ and $(x_2, y_2) = (100, 9{,}647)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9{,}647 - 8{,}147}{100 - 80} = 75$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 8{,}147 &= 75(x - 80) \\ y - 8{,}147 &= 75x - 6{,}000 \\ y &= 75x + 2{,}147 \end{aligned}$$

(B) The slope of 75 is the rate of change of cost with respect to production, \$75 per golf club.

(C) Increasing production by 1 unit increases cost by \$75.

7. (A) The rate of change of height with respect to DBH is 4.06 feet per inch.

(B) Increasing DBH by 1 inch increases height by 4.06 feet.

(C) Substitute $d = 12$ into $h = 4.06d + 24.1$ to obtain

$$\begin{aligned} h &= 4.06(12) + 24.1 \\ h &= 73 \text{ feet} \end{aligned}$$

(D) Substitute $h = 100$ into $h = 4.06d + 24.1$ and solve.

$$\begin{aligned} 100 &= 4.06d + 24.1 \\ 75.9 &= 4.06d \\ d &= 19 \text{ inches} \end{aligned}$$

9. (A) Robinson: The rate of change of weight with respect to height is 3.7 pounds per inch.
Miller: The rate of change of weight with respect to height is 3 pounds per inch.

(B) 5'6" = 6 inches over 5 feet
Substitute $h = 6$ into each model.
Robinson: $w = 108 + 3.7(6) = 130.2$ pounds
Miller: $w = 117 + 3.0(6) = 135$ pounds

(C) Substitute $w = 140$ into each model and solve.
Robinson: $140 = 108 + 3.7h$
$32 = 3.7h$
$h = 9$ inches, predicting 5'9".
Miller: $140 = 117 + 3.0h$
$23 = 3.0h$
$h = 8$ inches, predicting 5'8".

11. If speed s is linearly related to temperature t, then we are looking for an equation whose graph passes through $(t_1, s_1) = (32, 741)$ and $(t_2, s_2) = (72, 771)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{s_2 - s_1}{t_2 - t_1} = \frac{771 - 741}{72 - 32} = 0.75$$
$$s - s_1 = m(t - t_1)$$
$$s - 741 = 0.75(t - 32)$$
$$s - 741 = 0.75t - 24$$
$$s = 0.75t + 717$$

The speed of sound at sea level increases by 0.75 mph for each 1˚F change in temperature.

13. If percentage m is linearly related to time t, then we are looking for an equation whose graph passes through $(t_1, m_1) = (0, 25.7)$ and $(t_2, m_2) = (6, 23.9)$. We find the slope and then use the point-slope form to find the equation. M is used for slope to avoid confusion.

$$M = \frac{m_2 - m_1}{t_2 - t_1} = \frac{23.9 - 25.7}{6 - 0} = -0.3$$
$$m - m_1 = M(t - t_1)$$
$$m - 25.7 = -0.3\,(t - 0)$$
$$m = -0.3t + 25.7$$

To find t when $m = 18$, substitute $m = 18$ and solve.

$$18 = -0.3t + 25.7$$
$$-7.7 = -0.3t$$
$$t = 25.67$$

Rounding up to 26, 26 years after 2000 will be 2026.

15. (A) If value V is linearly related to time t, then we are looking for an equation whose graph passes through $(t_1, V_1) = (0, 142{,}000)$ and $(t_2, V_2) = (10, 67{,}000)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{V_2 - V_1}{t_2 - t_1} = \frac{67{,}000 - 142{,}000}{10 - 0} = -7{,}500$$
$$V - V_1 = m(t - t_1)$$
$$V - 142{,}000 = -7{,}500(t - 0)$$
$$V = -7{,}500t + 142{,}000$$

(B) The tractor's value decreases at the rate of \$7,500 per year.

(C) When $t = 6$, $V = -7{,}500(6) + 142{,}000 =$ \$97,000.

17. (A) If price R is linearly related to cost C, then we are looking for an equation whose graph passes through $(C_1, R_1) = (85, 112)$ and $(C_2, R_2) = (175, 238)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{R_2 - R_1}{C_2 - C_1} = \frac{238 - 112}{175 - 85} = 1.4$$
$$R - R_1 = m(C - C_1)$$
$$R - 112 = 1.4(C - 85)$$
$$R - 112 = 1.4C - 119$$
$$R = 1.4C - 7$$

(B) The slope is 1.4. This is the rate of change of retail price with respect to cost.

(C) To find C when $R = 185$, substitute $R = 185$ and solve.

$$185 = 1.4C - 7$$
$$192 = 1.4C$$
$$C = \$137$$

19. (A) If temperature T is linearly related to altitude A, then we are looking for an equation whose graph passes through $(A_1, T_1) = (0, 70)$ and $(A_2, T_2) = (18, -20)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{T_2 - T_1}{A_2 - A_1} = \frac{-20 - 70}{18 - 0} = -5$$
$$T - T_1 = m(A - A_1)$$
$$T - 70 = -5(A - 0)$$
$$T = -5A + 70$$

21. (A) If altitude a is linearly related to time t, then we are looking for an equation whose graph passes through $(t_1, a_1) = (0, 2{,}880)$ and $(t_2, a_2) = (120, 0)$. We find the slope and then use the point-slope form to find the equation.

$$m = \frac{a_2 - a_1}{t_2 - t_1} = \frac{0 - 2{,}880}{120 - 0} = -24$$
$$a - a_1 = m(t - t_1)$$
$$a - 2{,}880 = -24(t - 0)$$
$$a = -24t + 2{,}880$$

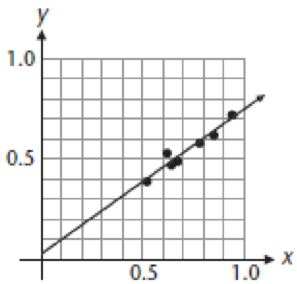
(B) To find A when $T = 0$, substitute $T = 0$ and solve.

$0 = -5A + 70$

$A = 14$, that is, 14,000 feet

(B) Since altitude is decreasing at the rate of 24 feet per second, this is the rate of descent.

23. (A)



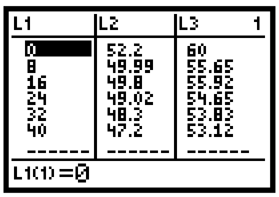
(B) Substitute $x = 1.3$ into $y = 0.72x + 0.03$ to obtain

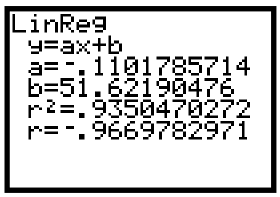
$y = 0.72(1.3) + 0.03 \approx 0.97$ million

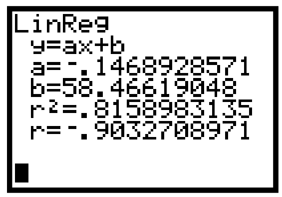
(C) Substitute $x = 1.8$ into $y = 0.72x + 0.03$ to obtain

$y = 0.72(1.8) + 0.03 \approx 1.3$ million

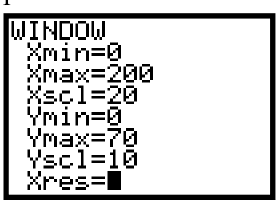
25. The entered data is shown here along with the results of the linear regression calculations.

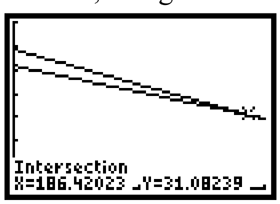






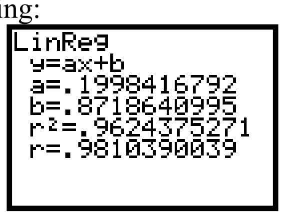
The linear regression model for men's 100-meter freestyle data is seen to be $y = -0.1102x + 51.62$. The linear regression model for women's 100- meter freestyle data is seen to be $y = -0.1469x + 58.47$. A plausible window is shown here, along with the results of the intersection calculation.

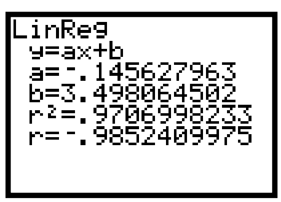




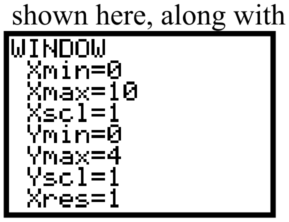
The fact that the lines intersect indicates that, according to this model, the women will eventually catch up with the men.

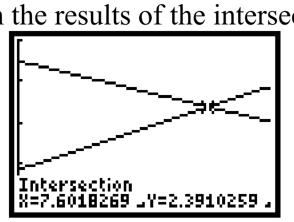
27. Entering the data (note: price is entered as y) and applying the linear regression routine yields the following:





The linear regression model for the price-supply data is seen to be $y = 0.200x + 0.872$. The linear regression model for the price-demand data is seen to be $y = -0.146x + 3.50$. A plausible window is shown here, along with the results of the intersection calculation.

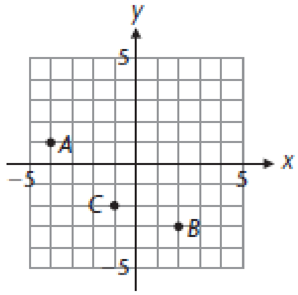




The intersection for $y = 2.39$ implies an equilibrium price of \$2.39.

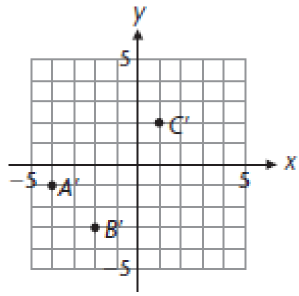
CHAPTER 2 REVIEW

1.



(2- 1)

2.



The reflection of A in the x axis is $A'(-4, -1)$.
The reflection of B in the y axis is $B'(-2, -3)$.
The reflection of C in the origin is $C'(1, 2)$. (2-1)

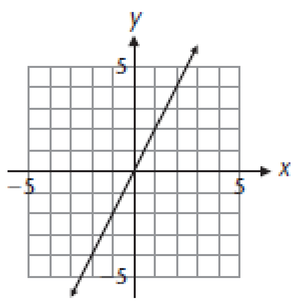
3. (A) $y = 2x$

Test y axis
Replace x with $-x$:
$y = 2(-x)$
$y = -2x$

Test x axis
Replace y with $-y$:
$-y = 2x$
$y = -2x$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = 2(-x)$
$y = 2x$

x	y
0	0
1	2
2	4



The graph has symmetry with respect to the origin. We reflect the portion of the graph in quadrant I through the origin, using the origin symmetry.

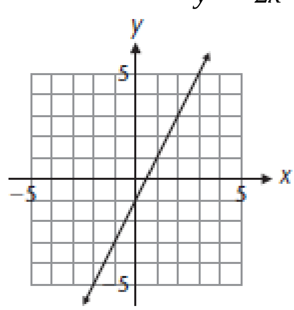
(B) $y = 2x - 1$

Test y axis
Replace x with $-x$:
$y = 2(-x) - 1$
$y = -2x - 1$

Test x axis
Replace y with $-y$:
$-y = 2x - 1$
$y = -2x + 1$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = 2(-x) - 1$
$y = 2x + 1$

x	y
0	–1
1	1
–1	–3
2	3
–2	–5



The graph has none of these three symmetries.

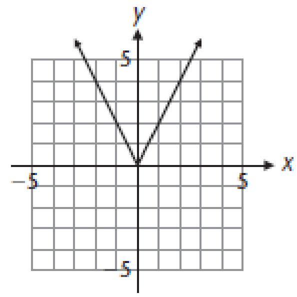
(C) $y = 2|x|$

Test y axis
Replace x with $-x$:
$y = 2|-x|$
$y = 2|x|$

Test x axis
Replace y with $-y$:
$-y = 2|x|$
$y = -2|x|$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = 2|-x|$
$y = -2|x|$

x	y
0	0
1	2
2	4



The graph has symmetry with respect to the y axis. We reflect the portion of the graph in quadrant I through the y axis, using the y axis symmetry.

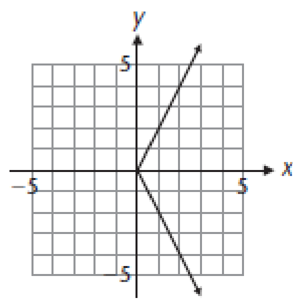
(D) $|y| = 2x$

Test y axis
Replace x with $-x$:
$|y| = 2(-x)$
$|y| = -2x$

Test x axis
Replace y with $-y$:
$|-y| = 2x$
$|y| = 2x$

Test origin
Replace x with $-x$ and y with $-y$:
$|-y| = 2(-x)$
$|y| = -2x$

x	y
0	0
1	2
2	4



The graph has symmetry with respect to the x axis. We reflect the portion of the graph in quadrant I through the x axis, using the x axis symmetry.

(2-1)

4. (A) –2 (B) –2, 1 (C) –3, 2 *(2-1)*

5. (A) $d(A, B) = \sqrt{[4-(-2)]^2 + (0-3)^2}$
$= \sqrt{36+9} = \sqrt{45}$

(B) $m = \dfrac{0-3}{4-(-2)} = \dfrac{-3}{6} = -\dfrac{1}{2}$

(C) The slope m_1 of a line perpendicular to AB

must satisfy $m_1\left(-\dfrac{1}{2}\right) = -1.$

Therefore, $m_1 = 2$. *(2-2, 2-3)*

6. (A) Center at (0, 0) and radius $\sqrt{7}$

$x^2 + y^2 = r^2$
$x^2 + y^2 = (\sqrt{7})^2$
$x^2 + y^2 = 7$

(B) Center at (3, –2) and radius $\sqrt{7}$

$(h, k) = (3, -2)$
$r = \sqrt{7}$
$(x-h)^2 + (y-k)^2 = r^2$
$(x-3)^2 + [y-(-2)]^2 = (\sqrt{7})^2$
$(x-3)^2 + (y+2)^2 = 7$ *(2-2)*

7. $(x+3)^2 + (y-2)^2 = 5$
$[x-(-3)]^2 + (y-2)^2 = (\sqrt{5})^2$

Center: $C(h, k) = (-3, 2)$ Radius: $r = \sqrt{5}$ *(2-2)*

8. (A) –4 is the average of a_1 and 2, so $-4 = \dfrac{a_1 + 2}{2}$

$-8 = a_1 + 2$
$-10 = a_1$

(B) 3 is the average of a_2 and –5, so $3 = \dfrac{a_2 + (-5)}{2}$

$6 = a_2 - 5$
$11 = a_2$

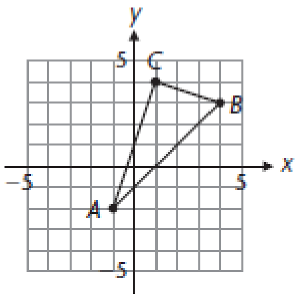
(C) $A = (-10, 11)$

$$d(A, M) = \sqrt{(-10-(-4))^2 + (11-3)^2} = \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$$

$$d(M, B) = \sqrt{(-4-2)^2 + (3-(-5))^2} = \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10 \qquad (2\text{-}2)$$

9. $A = (-1, -2)$, $B = (4, 3)$, $C = (1, 4)$

(A)



(B) $d(A, B) = \sqrt{(4-(-1))^2 + (3-(-2))^2} = \sqrt{5^2 + 5^2} = \sqrt{25+25} = \sqrt{50}$

$d(B, C) = \sqrt{(1-4)^2 + (4-3)^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

$d(A, C) = \sqrt{(1-(-1))^2 + (4-(-2))^2} = \sqrt{2^2 + 6^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

Perimeter = sum of lengths of all three sides

$= \sqrt{50} + \sqrt{10} + 2\sqrt{10} = 16.56$ to two decimal places

(C) Since $(\sqrt{50})^2 = (\sqrt{10})^2 + (2\sqrt{10})^2$, that is, $50 = 10 + 40$, the triangle is a right triangle.

(D) Midpoint of $AB = \left(\dfrac{-1+4}{2}, \dfrac{-2+3}{2}\right) = (1.5, 0.5)$

Midpoint of $BC = \left(\dfrac{4+1}{2}, \dfrac{3+4}{2}\right) = (2.5, 3.5)$

Midpoint of $AC = \left(\dfrac{-1+1}{2}, \dfrac{-2+4}{2}\right) = (0, 1)$ (2-3)

10. The points at two of the vertices of the triangle are $(-4, 3)$ and $(1, 1)$. The line moves 2 units down between these points so the rise is -2. The line moves 5 units to the right between these points so the run is 5.

The slope is thus $m = \dfrac{-2}{5}$.

$$y - 1 = \frac{-2}{5}(x - 1)$$

$$y - 1 = -\frac{2}{5}x + \frac{2}{5}$$

$$5y - 5 = -2x + 2$$

$$2x + 5y = 7 \qquad (2\text{-}3)$$

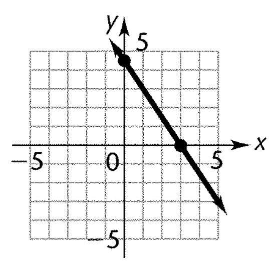
11. $3x + 2y = 9$

$2y = -3x + 9$

$y = -\dfrac{3}{2}x + \dfrac{9}{2}$

slope: $-\dfrac{3}{2}$

x	y
0	$\frac{9}{2}$
3	0
1	3



(2-3)

12. The line passes through the two given points, $(6, 0)$ and $(0, 4)$. Thus, its slope is given by

$$m = \frac{0-4}{6-0} = \frac{-4}{6} = -\frac{2}{3}$$

The equation of the line is, therefore, using the point-slope form,

$$y - 0 = -\frac{2}{3}(x - 6)$$

or $3y = -2(x - 6)$

or $3y = -2x + 12$.

$2x + 3y = 12$ (2-3)

13. $y = mx + b$ $\quad m = -\dfrac{2}{3}$ $\quad b = 2$

$$y = -\frac{2}{3}x + 2 \qquad (2\text{-}3)$$

14. *vertical:* $x = -3$, slope not defined;
horizontal: $y = 4$, slope = 0 (*2-3*)

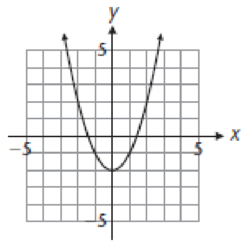
15. $y = x^2 - 2$

Test y axis
Replace x with $-x$:
$y = (-x)^2 - 2$
$y = x^2 - 2$

Test x axis
Replace y with $-y$:
$-y = x^2 - 2$
$y = -x^2 + 2$

Test origin
Replace x with $-x$ and y with $-y$:
$-y = (-x)^2 - 2$
$y = -x^2 + 2$

x	y
0	-2
1	-1
2	2



The graph has symmetry with respect to the y axis.
We reflect the portion of the graph in quadrant I through the y axis, using the y axis symmetry.

(*2-1*)

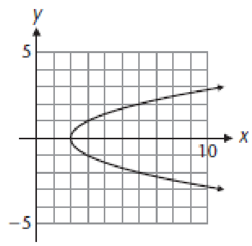
16. $y^2 = x - 2$

Test y axis
Replace x with $-x$:
$y^2 = -x - 2$

Test x axis
Replace y with $-y$:
$(-y)^2 = x - 2$
$y^2 = x - 2$

Test origin
Replace x with $-x$ and y with $-y$:
$(-y)^2 = -x - 2$
$y^2 = -x - 2$

x	y
2	0
3	± 1
6	± 2



The graph has symmetry with respect to the x axis. We reflect the portion of the graph in quadrant I through the x axis, using the x axis symmetry.

(*2-1*)

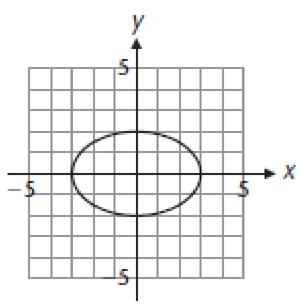
17. $9y^2 + 4x^2 = 36$

Test y axis
Replace x with $-x$:
$9y^2 + 4(-x)^2 = 36$
$9y^2 + 4x^2 = 36$

Test x axis
Replace y with $-y$:
$9(-y)^2 + 4x^2 = 36$
$9y^2 + 4x^2 = 36$

Origin symmetry follows automatically.

x	y
0	± 2
1	$\pm\frac{1}{3}\sqrt{32} \approx \pm 1.9$
2	$\pm\frac{1}{3}\sqrt{20} \approx \pm 1.5$
3	0



The graph has symmetry with respect to the x axis, the y axis, and the origin. $y = \pm\frac{1}{3}\sqrt{36 - 4x^2}$

We reflect the portion of the graph of $y = \frac{1}{3}\sqrt{36 - 4x^2}$ in quadrant I through the y axis, then reflect everything through the x axis.

(*2-1*)

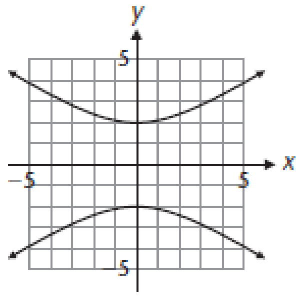
18. $9y^2 - 4x^2 = 36$

Test y axis
Replace x with $-x$:
$9y^2 - 4(-x)^2 = 36$
$9y^2 - 4x^2 = 36$

Test x axis
Replace y with $-y$:
$9(-y)^2 - 4x^2 = 36$
$9y^2 - 4x^2 = 36$

Origin symmetry follows automatically.

x	y
0	± 2
1	$\pm\frac{1}{3}\sqrt{40} \approx \pm 2.1$
2	$\pm\frac{1}{3}\sqrt{52} \approx \pm 2.4$
3	$\pm\frac{1}{3}\sqrt{72} \approx \pm 2.8$
4	$\pm\frac{10}{3} \approx \pm 3.3$



The graph has symmetry with respect to the x axis, the y axis, and the origin. $y = \pm\frac{1}{3}\sqrt{36+4x^2}$

We reflect the portion of the graph of $y = \frac{1}{3}\sqrt{36+4x^2}$ in quadrant I through the y axis, then reflect everything through the x axis.

(2-1)

19. The graph is a circle with center (2, 0) and radius 2.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-2)^2 + (y-0)^2 = 2^2$$
$$(x-2)^2 + y^2 = 4$$

(2-2)

20. (A) Since two points are given, we find the slope, then apply the point-slope form.

$$m = \frac{-3-3}{0-(-4)} = \frac{-6}{4} = -\frac{3}{2}$$

$$y - 3 = -\frac{3}{2}[x-(-4)]$$
$$2(y-3) = -3(x+4)$$
$$2y - 6 = -3x - 12$$
$$3x + 2y = -6$$

(B) $d(P, Q) = \sqrt{(-3-3)^2 + [0-(-4)]^2}$

$$= \sqrt{36+16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

(2-2, 2-3)

21. The line $6x + 3y = 5$, or $3y = -6x + 5$, or $y = -2x + \frac{5}{3}$, has slope -2.

(A) We require a line through $(-2, 1)$, with slope -2.

Applying the point-slope form, we have

$$y - 1 = -2[x-(-2)]$$
$$y - 1 = -2x - 4$$
$$y = -2x - 3$$

(B) We require a line with slope m satisfying $-2m = -1$, or $m = \frac{1}{2}$. Again applying the point-slope form, we have

$$y - 1 = \frac{1}{2}[x-(-2)]$$
$$y - 1 = \frac{1}{2}x + 1$$
$$y = \frac{1}{2}x + 2$$

(2-3)

22. We are given $C(h, k) = (3, 0)$.
To find r we use the distance formula.
r = distance from the center to $(-1, 4)$

$$= \sqrt{[(-1)-3]^2 + (4-0)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$

Then the equation of the circle is

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-3)^2 + (y-0)^2 = (\sqrt{32})^2$$
$$(x-3)^2 + y^2 = 32$$

(2-2)

23.

$$x^2 + y^2 + 4x - 6y = 3$$
$$(x^2 + 4x + ?) + (y^2 - 6y + ?) = 3$$
$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = 3 + 4 + 9$$
$$(x+2)^2 + (y-3)^2 = 16$$
$$[x-(-2)]^2 + (y-3)^2 = 4^2$$

Center: $C(h, k) = C(-2, 3)$ Radius $r = \sqrt{16} = 4$

(2-2)

24. Let (x, y) be a point equidistant from (3, 3) and (6, 0). Then

$$\sqrt{(x-3)^2+(y-3)^2} = \sqrt{(x-6)^2+(y-0)^2}$$
$$(x-3)^2+(y-3)^2 = (x-6)^2+y^2$$
$$x^2-6x+9+y^2-6y+9 = x^2-12x+36+y^2$$
$$-6x-6y+18 = -12x+36$$
$$6x-6y = 18$$
$$x-y = 3$$

This is the equation of a line. (*2-2, 2-3*)

25. If $m = 0$, the equations are reduced to $-y = b$ (a horizontal line) and $x = b$ (a vertical line). In this case, the graphs are perpendicular. Otherwise, $m \neq 0$. Solving for y yields

$$mx - y = b \qquad x + my = b$$
$$mx = y + b \qquad my = -x + b$$
$$y = mx - b \qquad y = -\frac{1}{m}x + \frac{b}{m}$$

The first line has slope m and the second has slope $-\frac{1}{m}$.

The graphs are perpendicular in this case also. (*2-3*)

26.
$$x^2-4x+y^2-2y-3 = 0$$
$$(x^2-4x+?)+(y^2-2y+?) = 3$$
$$(x^2-4x+4)+(y^2-2y+1) = 3+4+1$$
$$(x-2)^2+(y-1)^2 = 8$$
$$(x-2)^2+(y-1)^2 = (\sqrt{8})^2$$

Center: (2, 1); radius: $\sqrt{8} = 2\sqrt{2}$ (*2-2*)

27. The line tangent to the circle is perpendicular to the radius drawn to the point of tangency. The radius drawn to (4, 3) is a line through (4, 3) and (2, 1) and therefore has slope

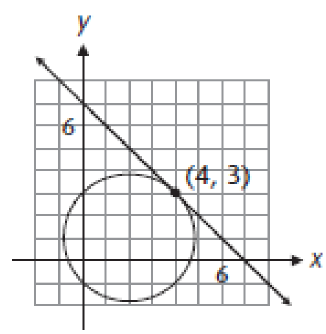
$$m_1 = \frac{1-3}{2-4} = 1$$

The tangent line therefore has slope

$m_2 = -\frac{1}{m_1} = -\frac{1}{1} = -1$ and passes through (4, 3).

Applying the point-slope form yields

$$y-3 = (-1)(x-4)$$
$$y-3 = -x+4$$
$$y = -x+7$$



(*2-2, 2-3*)

28. The radius of a circle is the distance from the center to any point on the circle. Since the center of this circle is (4, –3) and (1, 2) is a point on the circle, the radius is given by

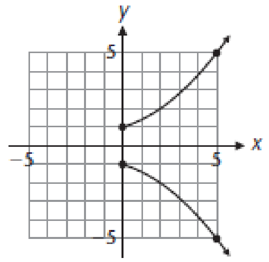
$$r = \sqrt{(1-4)^2+(2-(-3))^2} = \sqrt{9+25} = \sqrt{34}$$

Hence the equation of the circle is given by

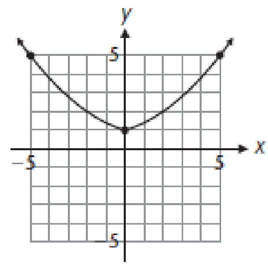
$$(x-4)^2+(y-(-3))^2 = (\sqrt{34})^2$$
$$(x-4)^2+(y+3)^2 = 34$$

(*2-2*)

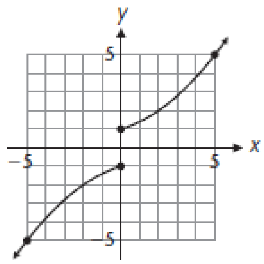
29. (A)



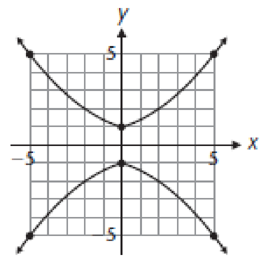
(B)



(C)



(D)



(2-1)

30. $b = 5h$ *(2-3)*

31. $h = 0.25b$ *(2-3)*

32. (A) If value V is linearly related to time t, then we are looking for an equation whose graph passes through $(t_1, V_1) = (0, 12{,}000)$ and $(t_2, V_2) = (8, 2{,}000)$. We find the slope, and then we use the point-slope form to find the equation.

$$m = \frac{V_2 - V_1}{t_2 - t_1} = \frac{2{,}000 - 12{,}000}{8 - 0} = -1{,}250$$

$$V - V_1 = m(t - t_1)$$
$$V - 12{,}000 = -1{,}250(t - 0)$$
$$V = -1{,}250t + 12{,}000$$

(B) Substitute $t = 5$ to obtain $V = -1{,}250(5) + 12{,}000 = \$5{,}750$. *(2-4)*

33. If x is the number of CD's produced, then the variable cost is $5x$. Since the fixed cost is \$24,900, the cost C of producing x CD's is given by $C = 5x + 24{,}900$. To find how many CD's can be produced for \$62,000, substitute $C = 62{,}000$ and solve.

$$62{,}000 = 5x + 24{,}900$$
$$37{,}100 = 5x$$
$$x = 7{,}420 \text{ CD's}$$

(2-4)

34. (A) The rate of change of height with respect to DBH is 2.9.
(B) Increasing DBH by 1 inch increases height by 2.9 feet.
(C) Substitute $d = 3$ into $h = 2.9d + 30.2$ to obtain

$$h = 2.9(3) + 30.2$$
$$h = 38.9$$
$$h = 39 \text{ feet to the nearest foot.}$$

(D) Substitute $h = 45$ into $h = 2.9d + 30.2$ and solve

$$45 = 2.9d + 30.2$$
$$14.8 = 2.9d$$
$$d = 5 \text{ inches}$$

(2-4)

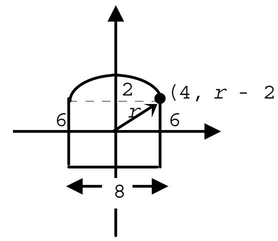
35. (A) The rate of change of body surface area with respect to weight is 0.3433.

(B) Increasing the weight by 100 grams increases the BSA by 0.3433(100) = 34.33 square centimeters.

(C) Substitute wt = 15,000 grams into BSA = 1,321 + 0.3433 wt to obtain
BSA = 1,321 + 0.3433(15,000) = 6470.5 cm^2. *(2-4)*

36. In the sketch, we note that the point $(4, r - 2)$ is on the circle with equation $x^2 + y^2 = r^2$, hence $(4, r - 2)$ must satisfy this equation.

$$4^2 + (r - 2)^2 = r^2$$
$$16 + r^2 - 4r + 4 = r^2$$
$$-4r + 20 = 0$$
$$r = 5 \text{ feet}$$



(2-2)

37. (A) $H = 0.7(220 - A)$

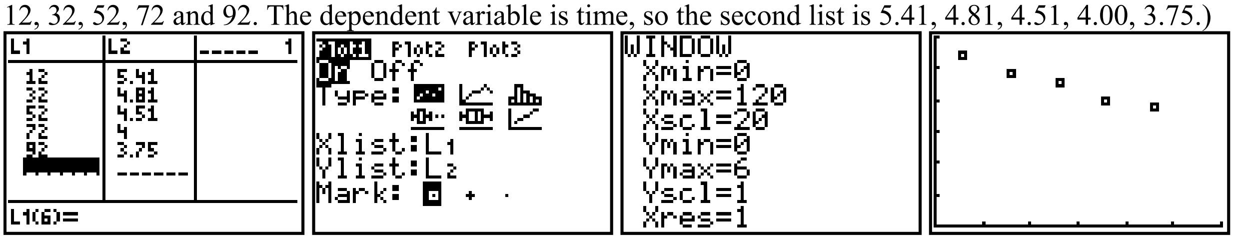
(B) To find H when $A = 20$, substitute $A = 20$ into $H = 0.7(220 - A)$ to obtain $H = 0.7(220 - 20) = 140$ beats per minute.

(C) To find A when $H = 126$, substitute $H = 126$ into $H = 0.7(220 - A)$ and solve.

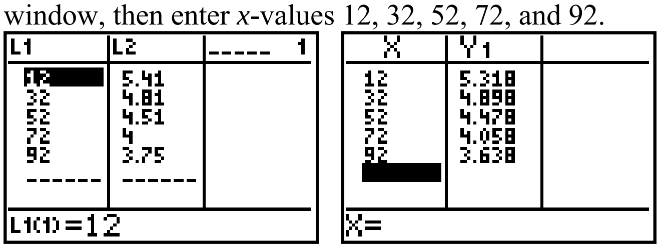
$$126 = 0.7(220 - A)$$
$$180 = 220 - A$$
$$-40 = -A$$
$$A = 40 \text{ years old.}$$

(2-4)

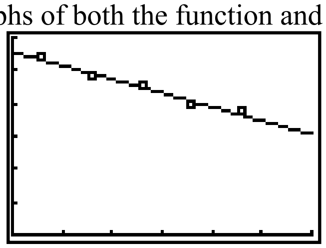
38. (A) In this example, the independent variable is years since 1900 so the first list under the STAT EDIT screen comes from the first column of Table 1. (Note that x is years after 1900, so the values we input are 12, 32, 52, 72 and 92. The dependent variable is time, so the second list is 5.41, 4.81, 4.51, 4.00, 3.75.)



The second screen shows the appropriate plot settings; the third shows a good choice of viewing window; the last is the scatter plot of the data from the table. To compare the algebraic function, we enter $y_1 = -0.021x + 5.57$ in the equation editor, choose the "ask" option for independent variable in the table setup window, then enter x-values 12, 32, 52, 72, and 92.



Comparing the two tables, we see that the corresponding times match very closely. Finally, we look at the graphs of both the function and the scatter plot:



As expected, the graph of the function matches the points from the scatter plot very well.

(B) The year 2024 corresponds to $x = 124$. Substitute $x = 124$ to obtain $-0.021(124) + 5.57 = 2.966$ minutes *(2-4)*