#### Topic 7: Introductory Calculus

#### Chapter 6: Introducing Differential Calculus

Time Frame: Weeks 13 to 18

### Day 1: A 17 6.1 Introduction to Differentiation

##### Day 2: A 19 6.2 The Gradient Function

##### Day 3: A 25 6.2 The Gradient Function

##### Day 4: A 27 6.3 The Gradient of a Curve at a Given Point

Day 5: A 2 6.4 The Tangent Line

Day 6: A 4 6.4 The Normal Line

Day 7: A 8 6.5 Rates of Change

Day 8: A 10 6.6 Local maximum and minimum points

Day 9: A 12 6.7 Modeling with Calculus

Day 10: A 6 6.7 Modeling with Calculus

Day 11: A 8 6.7 Modeling with Calculus

Day 12: A 12 Test on Calculus

Day 13: A 15 Error Analysis of Test

Math Analysis / IB Math Standard

Review Sheet for Chapter 6: Introducing Differential Calculus

You should be able to do the following things on this test:

Draw tangent lines on curves *Section 6.1*

Understand the derivative as the gradient [rate of change] of a function *Section 6.1*

Find the gradient of a line between two points on a function and compare it

to the gradient at a given point on a function. *Section 6.1*

Describe a gradient as increasing, decreasing, or zero *Section 6.1*

Use the power rule to find the first derivative of a function *Section 6.2*

Find the gradient of a curve for a given value of x *Sections 6.2 & 6.3*

Find values of x when is given *Sections 6.2 & 6.3*

Find the tangent line to a curve for a given value of x *Section 6.4*

Find the normal line to a curve for a given value of x *Section 6.4*

Apply differential calculus to real-life problems involving rates of change *Section 6.5*

Use the derivative to identify the intervals where a function is increasing ( *Section 6.6*

Use the derivative to identify the intervals where a function is decreasing *Section 6.6*

Use the derivative to identify the horizontal tangent lines to a function ( *Section 6.6*

Use the derivative to identify all maximum and minimum points of a function *Section 6.6*

Use differential calculus to model and solve optimization problems

\* perimeter, area, volume, surface area, height, cost, etc... *Section 6.7*

Math Analysis / IB Math Standard

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Use the power rule to find the first derivative of a function *Section 6.2*

Find the gradient of a curve for a given value of x *Sections 6.2 & 6.3*

Find values of x when is given *Sections 6.2 & 6.3*

Find the tangent line to a curve for a given value of x *Section 6.4*

Find the normal line to a curve for a given value of x *Section 6.4*

Apply differential calculus to real-life problems involving rates of change *Section 6.5*

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Use the derivative to identify all maximum and minimum points of a function *Section 6.6*

Use differential calculus to model and solve optimization problems

\* perimeter, area, volume, surface area, height, cost, etc... *Section 6.7*

Week 13: Sunday 17 November

Chapter 6: Introducing Differential Calculus

CCSS: There are no CCSSs for Calculus

IB Syllabus: 7.1 Concept of the derivative as a rate of change. The tangent to a curve.

Enduring Underst: Functions can be analyzed graphically by their rates of change.

The derivative is the instantaneous rate of change at a given point.

Essen Question: How are derivatives used to analyze the behavior of a function?

Topic: 6.1 Introduction to Differentiation

Lesson Obj: Students will draw tangent lines on curves.

Students will describe the gradients of tangent lines as positive or negative, increasing or decreasing.

Students will draw graphs of rates of change for real-life situations.

Students will use the gradients of secant lines to predict the gradient of a tangent line.

Review and Intro:

1. Vocab on board: derivative, gradient, tangent.
2. Hook: Roller Coasters
   * [Roller Coaster Video](http://www.youtube.com/watch?v=HODJqEdYkF4)
   * [Tangent Line App](http://www.unc.edu/~upshaw/Java/TanSec.html)
3. What do you notice? The gradients along the curve are all different.

Core Lesson:

1. Distinguish between gradient of line and gradient of curve.
   * Examples on Smartboard.
   * Students draw tangent lines on worksheet.
2. Increasing / decreasing exploration
   * Examples on Smartboard.
   * Students draw tangent lines on worksheet.
3. Time-series graphs of rate of change
   * Examples on Smartboard.
   * Students draw tangent lines on worksheet.
4. [Exploration into secant line](http://www.calvin.edu/~rpruim/courses/m161/F01/java/SecantTangent.shtml)

Check for Understanding:

1. Check student work and discussion throughout.

Assignment: Students will complete Khan Academy Videos and Practice

6.1 Introduction to Differentiation

[Interpreting Slope of a Curve Exercise](http://www.khanacademy.org/math/calculus/differential-calculus/derivative_intro/v/interpreting-slope-of-a-curve-exercise)

[Practice: Recognizing Slope of Curves](http://www.khanacademy.org/math/calculus/differential-calculus/derivative_intro/e/recognizing_slope)

(student results are registered with me as their Coach in Khan Academy)

6.2 The Gradient Function (beginning the flipped-classroom model…)

[The Power Rule](http://www.khanacademy.org/math/calculus/differential-calculus/power_rule_tutorial/v/power-rule)

[Polynomial Derivatives](http://www.khanacademy.org/math/calculus/differential-calculus/power_rule_tutorial/v/derivative-properties-and-polynomial-derivatives)

[Practice: Power Rule](http://www.sporcle.com/games/theredradish/1minder)

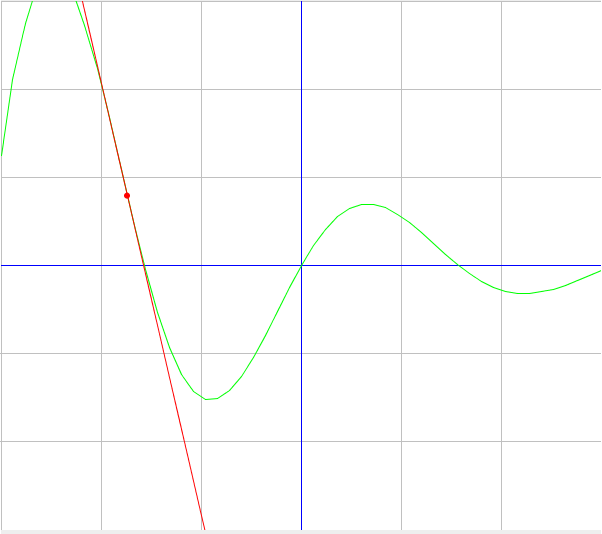
(online quiz with timer – student will print the screen and turn in)

IB Math Studies Year 2

6.1 Introduction to Differentiation

You are riding the roller coaster below, moving from left to right. Imagine sitting in the front car, looking in a straight line ahead of you. Where would you be looking at each point along the ride?

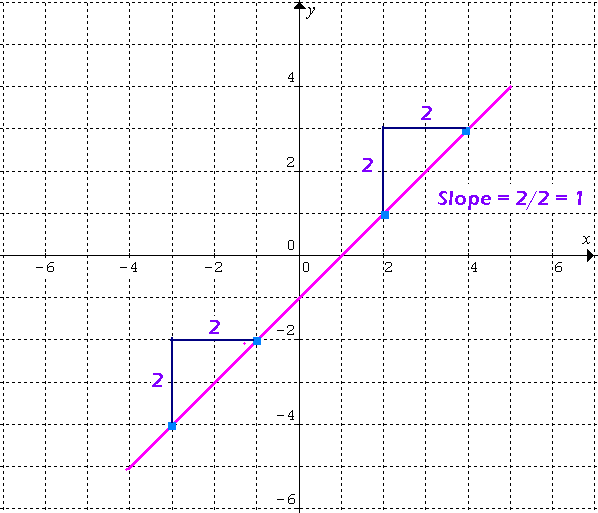
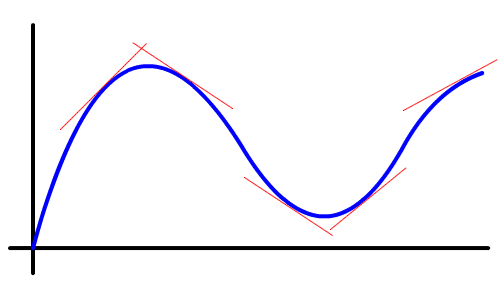
Draw a short line of sight through each point. The first one has been done for you.



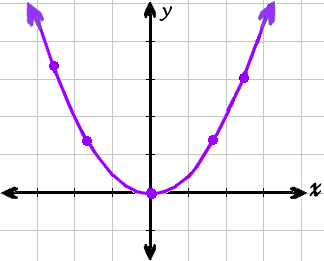
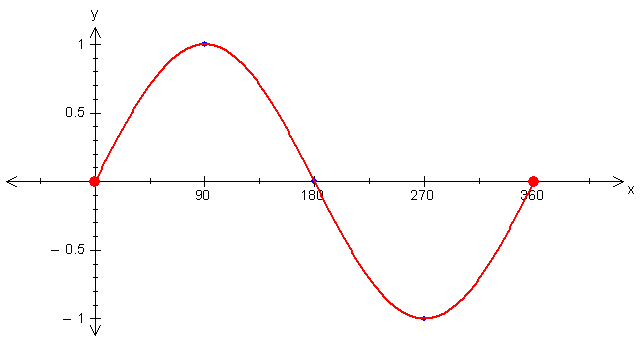
What do you notice about these lines?

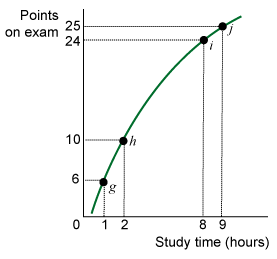
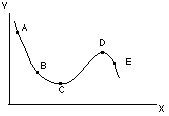
The lines you have drawn are called “tangent lines”. They touch the curve at exactly one point. The “derivative” is all about finding the gradients of these tangent lines.

The gradient of a line is constant. The gradient of a curve changes.

Draw the tangent line through each point on the curve. Identify each gradient as positive, negative, or zero.

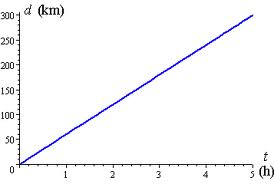
 

Gradients and derivatives are simply rates of change of a particular function .

They can be written in several ways:

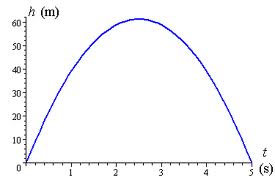
Rates of change can be positive, zero, or negative.

In addition, they can also be constant, increasing, or decreasing.



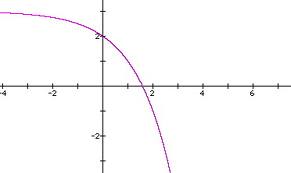
is at a rate.

The rate of change is and .



is at a rate.

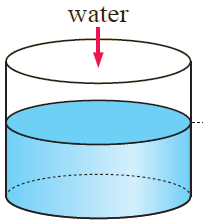
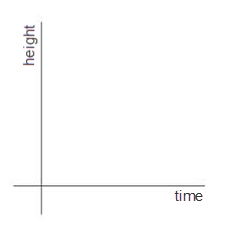
The gradient is and .



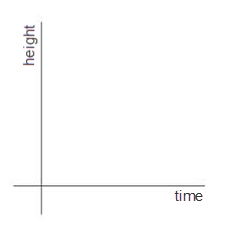
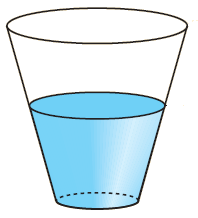
is at a rate.

The rate of change is and . .

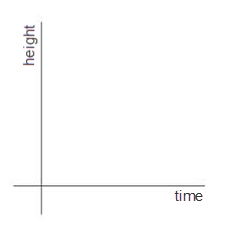
Write a time-series graph for each of these containers as they fill with water.

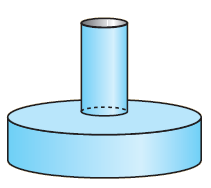


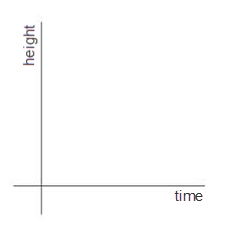
Explanation:

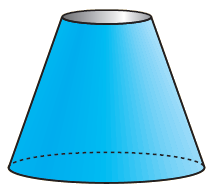


Explanation:

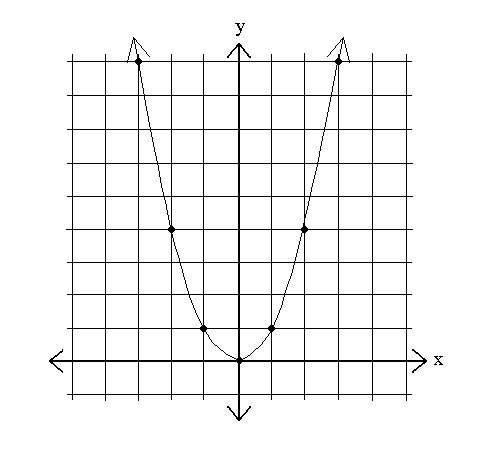


Explanation:



Explanation:

One way to find the gradient at a given point on a curve is to first find the gradient between two points on the curve. Then we move the points closer and closer together and see what happens to the gradient.



For example, let’s say we want to find the gradient of the tangent line on through

We could find the gradient between the point and other points closer and closer to

Gradient between (0 , \_\_\_\_ ) and

Gradient between (1 , \_\_\_\_ ) and

Gradient between (1.5 , \_\_\_\_ ) and

Gradient between (1.9 , \_\_\_\_ ) and

Gradient between (1.999 , \_\_\_\_ ) and

So, what do you think the gradient probably is at the point ?

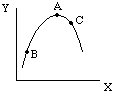
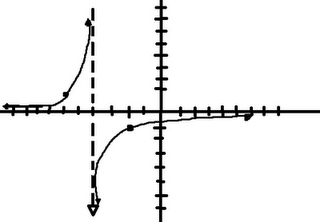
Next class we will learn how to exactly find the gradient at any given point.

IB Math Studies Year 2

6.1 Introduction to Differentiation - Practice

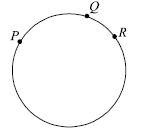
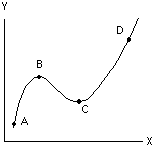
1. Draw the tangent line through each point.

Describe the gradient at each point as positive, negative, or zero.

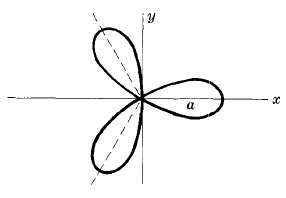
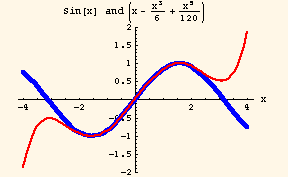
 

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2. Draw each horizontal tangent line. There are 4 in each graph.

Week 13: Tuesday 19 November

Chapter 6: Calculus

Topic: 6.2 The Gradient Function

IB Syllabus: 7.2

Lesson Obj: Students find the first derivative of polynomial functions using the power rule.

Students will find the gradient of a tangent line for given values of x and f(x).

Review and Intro:

1. Vocab on board: derivative, gradient, tangent. Homework questions.
2. Review power rule from videos with positive exponents.

Core Lesson:

1. Negative exponents.
2. Constant rule – use GDC.
3. Derivative intuition Khan Academy with students at the SmartBoard
4. <http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/e/derivative_intuition>

Check for Understanding:

1. Check student work and discussion throughout.
2. QUIZ next period

Assignment: Students will complete:

6.2 The Gradient Function

[Negative Exponent Intuition](http://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-numbers-operations/cc-8th-pos-neg-exponents/v/negative-exponent-intuition)

[Negative and Positive Exponents](http://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/negative-and-positive-exponents)

[Practice: Simplifying Expressions with Exponents](http://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/e/simplifying_expressions_with_exponents)

[Graphs of Functions and their Derivatives](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/v/graphs-of-functions-and-their-derivatives-example-1)

[Where a Function is Not Differentiable](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/v/where-a-function-is-not-differentiable)

[Identifying a Function's Derivative](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/v/identifying--a-function-s-derivative-example)

[Which Function is the Derivative?](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/v/figuring-out-which-function-is-the-the-derivative)

[Practice: Graphs of Functions and Their Derivatives](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/e/graphs-of-functions-and-their-derivatives)

IB Math Studies Year 2

6.2 The Gradient Function

Practice what you learned last night with the Power Rule:

If then

|  |  |  |
| --- | --- | --- |
|  |  | at a value |
|  |  |  |
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| 13 |  |  |
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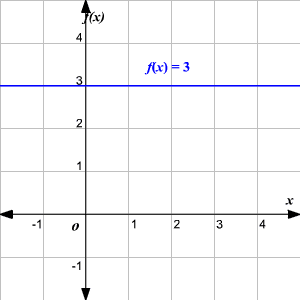
In the past, you may have studied negative exponents. They are used to represent variables that are in the denominator of a fraction.

|  |  |  |  |
| --- | --- | --- | --- |
| Fraction form | Power form | Fraction form | Power form |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

The derivatives of these types of functions are also found using the power rule. First rewrite the function using negative exponents. Then follow the same rule: Multiply by the exponent and subtract one from the exponent.

|  |  |  |  |
| --- | --- | --- | --- |
| Original  function | Rewrite the Function | Take the derivative | Rewrite the derivative |
| Fraction form | Power form | Power form | Fraction form |
|  |  |  |  |
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We also have to find the derivative of a constant function, such as .



The graph of is shown here.

What is its gradient?

What is its derivative?

Graph and .

What are their gradients?

What are their derivatives?

Based on this information, what is the derivative of any constant function?

If , then .

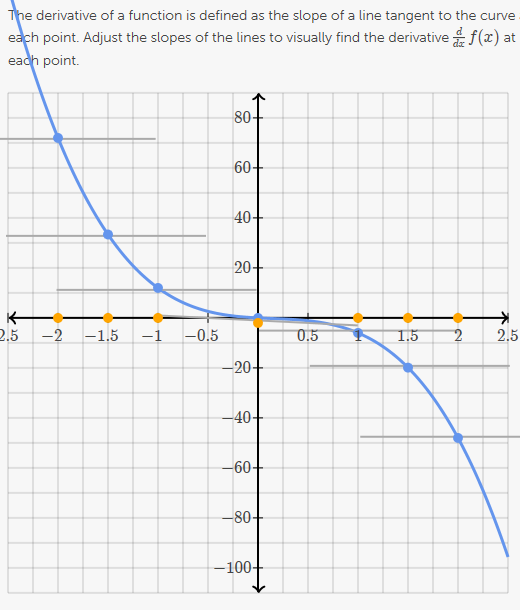
IB Practice

1. Differentiate

2. Find if .

3. Find if

Derivative Intuition: If you know a function, can you sketch its derivative?



Week 14: Wednesday 15 November

Chapter Six: Introducing Differential Calculus

Topic: 6.2 The power rule

IB Syllabus: 7.2

Lesson Obj: Students find use the power rule to find derivatives.

Students will match graphs of functions with graphs of their derivatives.

Students will find values of functions and derivatives at given points.

Review and Intro:

1. Vocab on board: derivative, gradient, tangent. Homework questions.
2. QUIZ
3. Correct quiz in class

Core Lesson:

1. Derivative card game (this took a long time for most students)

Check for Understanding:

1. Check student progress throughout

Assignment: Students will watch videos and complete online exercises:

Finding a slope at a point

[Example 1](http://www.youtube.com/watch?v=GnztUlLJtXc) [Example 2](http://www.youtube.com/watch?v=WHnRyzXXT1U)

[Practice: Derivatives 1](http://www.khanacademy.org/math/calculus/differential-calculus/derivative_intro/e/derivatives_1)

[Intuitively drawing the derivative of a function](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/v/intuitively-drawing-the-derivative-of-a-function)

[Intuitively drawing the antiderivative of a function](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/v/intuitively-drawing-the-anitderivative-of-a-function)

[Visualizing derivatives exercise](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/v/visualizing-derivatives-exercise)

[Practice: Visualizing Derivatives](http://www.khanacademy.org/math/calculus/differential-calculus/visualizing-derivatives-tutorial/e/visualizing_derivatives)

IB Practice

Let

a) i) Find the derivative of .

ii) Hence, calculate .

iii) Verify your answer to aii) using the calculator.

b) Fill in the table of values below

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | -3 | -2 | -1 | 0 | 1 | 2 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

c) On graph paper, graph for and . Use 2 cm to represent 1 unit on the -axis and 1 cm to represent 2 units on the -axis.

d) Use your GDC to write down the equation of the tangent line to at .

e) On your graph, accurately draw the tangent line to at .

f) Find the second point of intersection between the tangent line and .

Week 14: Wednesday 27 November

Chapter Six: Introducing Differential Calculus

Topic: 6.3 The Gradient of a Curve at a Given Point

IB Syllabus: 7.3

Lesson Obj: Students find the slope of the tangent line to a curve at a given point.

Review and Intro:

1. Vocab on board: derivative, gradient, tangent. Homework questions.
2. QUIZ

Core Lesson:

1. Gradient function -- exploration
2. IB Practice w/graph paper
3. Nov 2011 P1 Q14
4. Calculaughs 24-25

Check for Understanding:

1. Check student work and discussion throughout.
2. QUIZ next period

Assignment: Students will watch online videos and complete guided outline.

Finding the equation of the tangent line

[Example 1](http://www.youtube.com/watch?v=WHnRyzXXT1U) [Example 2](http://www.youtube.com/watch?v=YZniHq_kzvA)

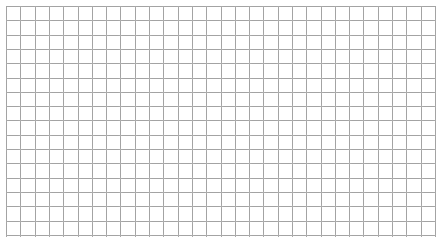
IB Math Studies Year 2

6.3 The Gradient of a Curve at a Given Point

1. a) On the grid below, accurately draw for

and .

40



-40

-3

-2

-1

3

2

1

b) Find the derivative .

c) Use the derivative to calculate the gradients of at the following points. Show your work. Plot each point on the graph above.

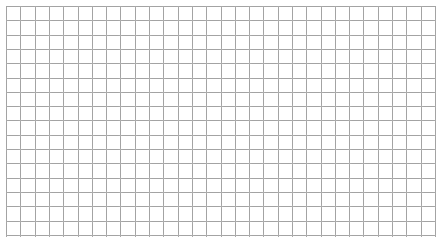
|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

d) Connect the points of .

What is the relationship between the two functions?

2. a) On the grid below, accurately draw for

and .



-3

-2

-1

-70

70

3

2

1

b) Find the derivative .

c) Use the derivative to calculate the gradients of at the following points. Show your work. Plot each point on the graph above.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

d) Connect the points of .

What is the relationship between the two functions?

IB Math Studies Year 2

Guided outline for 6.4 The Tangent Line

Notes for Example 1:

Find the equation of the line tangent to at the point .

What does give us?

Use the point-slope formula (or any other formula) to find the tangent line:

Verify your tangent line equation on the calculator!

Notes for Example 2:

Find the equation of the line tangent to when .

What does give us?

Use the point-slope formula (or any other formula) to find the tangent line:

Verify your tangent line equation on the calculator!

Practice 1:

Find the equation of the line tangent to at the point .

What does give us?

Use the point-slope formula (or any other formula) to find the tangent line:

Verify your tangent line equation on the calculator!

Practice 2:

Find the equation of the line tangent to when .

What does give us?

Use the point-slope formula (or any other formula) to find the tangent line:

Verify your tangent line equation on the calculator!

Week 15: Monday 2 December

Chapter Six: Introducing Differential Calculus

Topic: 6.4 The Tangent Line

IB Syllabus: 7.3

Lesson Obj: Students will find the gradient of a tangent line for given values of x and f(x).

Students will find values of x that have horizontal tangent lines.

Students will find the equation of a tangent line of a function at a given point

Review and Intro:

1. Vocab on board: derivative, gradient, tangent. Homework questions.
2. QUIZ

Core Lesson:

1. Tangent line -- exploration
2. Use GDC.

Check for Understanding:

1. Check student work and discussion throughout.

Assignment: Students will complete online videos and practice:

[Perpendicular line slope](http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/perpendicular-line-slope)

[Perpendicular lines 2](http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/perpendicular-lines-2)

[Practice: Equations of parallel and perpendicular lines](http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/line_relationships)

IB Math Studies Year 2

6.4 The Tangent Line

Remember that the derivative gives you the gradient of the tangent line at any point along a function. We can use this idea to solve different kinds of problems.

1. Consider the function . Find the equation of the tangent line at each value below. Show your work by hand; then verify on the graphing calculator.

a)

b)

2. Find the values of where the function has a horizontal tangent line.

2. Let

a) Find .

b) Find the gradient of the curve when .

c) Is the function increasing or decreasing when ?

d) Find the value(s) of when the gradient of the curve is 3.

e) Find the value(s) of when the curve has a horizontal tangent line.

3. A function is given as

a) Find .

b) The gradient of this function is 2 when is 6.

Write an equation in terms of and .

c) The point lies on the graph of the function.

Find a second equation in terms of and .

d) Use your GDC and your equations from parts b) and c) to find the values of and .

Week 15: Wednesday 4 December

Chapter Six: Introducing Differential Calculus

Topic: 6.4 The Normal Line

IB Syllabus: 7.3

Lesson Obj: Students find equation of a normal line to a curve through a given point.

Review and Intro:

1. Vocab on board: derivative, gradient, normal. Homework questions.
2. SmartBoard example of tangent and normal lines

Core Lesson:

1. Normal line -- exploration
2. Verify on GDC.

Check for Understanding:

1. Check student work and discussion throughout.

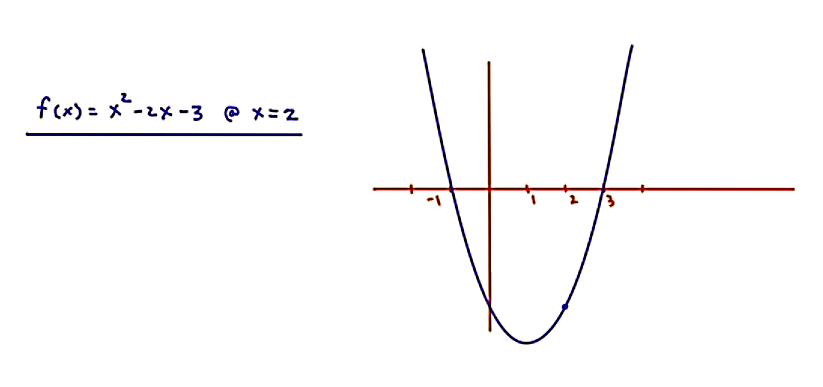
Assignment: Students will complete online videos and guided outline

[Applications of Differentiation](http://www.youtube.com/watch?v=mtwGS3cqPtE)

[Calculus: Applications of Rates of Change](http://www.youtube.com/watch?v=ofp0hwlph-w)

IB Math Studies Year 2

6.4 The Normal Line



For each function:

a) Find the equation of the tangent line through the given point

b) Find the equation of the normal line through the given point

c) Sketch the function, the tangent line, and the normal line.

Show all your work in your notebooks by hand. Then verify on the GDC.

1. at the point

2. at the point where

3. at the point where

4. where the gradient is zero

IB Math Studies Year 2

Guided outline for 6.5 Rates of Change

Applications of Differentiation Video

Rate of change of with respect to =

Rate of change of volume with respect to time =

Rate of change of area with respect to temperature =

Example 1:

Example 2:

a) How many bacteria are there after 5 hours?

b) What is the rate of change of with respect to ?

c) What is happening to the colony of bacteria after 2 hours? after 10 hours?

Explain the difference between instantaneous rate of change and average rate of change.

Calculus: Applications of Rates of Change

The cost of producing gadgets is given by .

What does the 70 represent in the cost?

What does the 0.01 represent in the cost?

Find the derivative of the cost function.

What does this “rate of change” mean in the real world?

What is happening to the cost when we produce 10 gadgets?

IB Math Studies Year 2

Guided practice for 6.5 Rates of Change

The volume of water in a container, is given by the formula

where is time measured in seconds.

a) Sketch and label a graph of this function for and

b) Find What does represent?

c) What units are used for ?

d) What is the volume of the water when ?

e) At what rate is the volume changing when ?

A company mines copper, where the mass of copper, x, is measured in thousands of tonnes. The company’s profit, P, measured in millions of dollars, depends on the amount of copper mined.

The profit is given by the function

a) What does the 2.3 represent in the function?

b) Find and . Interpret those results.

c) Find . What does represent?

d) Find the value of and when . Interpret these results.

e) Find the value of for which What is happening at that value?

Week 16: Sunday 8 December

Chapter 6: Introducing Differential Calculus

Topic: 6.5 Rates of Change

IB Syllabus: 7.1

Lesson Obj: Students will solve calculus applications related to rates of change.

Review and Intro:

1. Homework questions.
2. QUIZ / Review quiz individually while students work on Rate of Change problems

Core Lesson:

1. Students complete rate of change problems 1-9

Check for Understanding:

1. Check Student work throughout

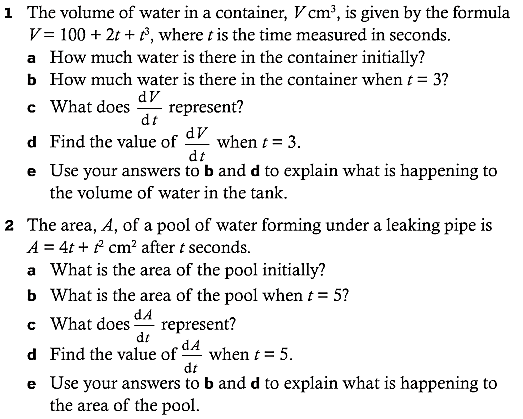
Assignment: Students will complete guided outlines and practice for:

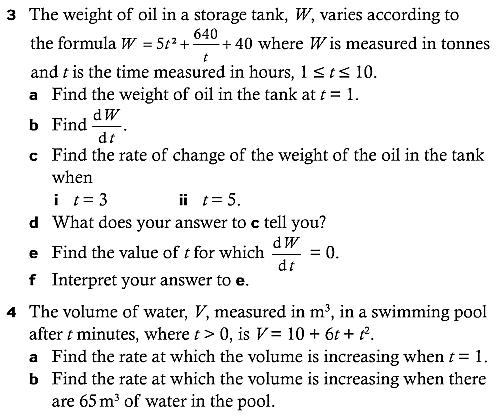
[First Derivative Test - What is it?](http://www.youtube.com/watch?v=H9t0FJBO4O4)

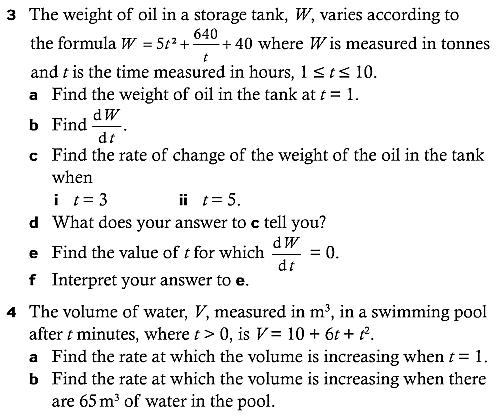
[First derivative test](http://www.youtube.com/watch?v=xNjlgXNp2jo)

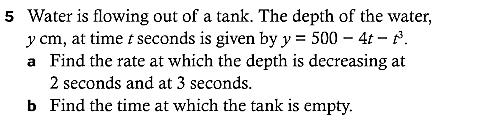
IB Math Studies Year 2

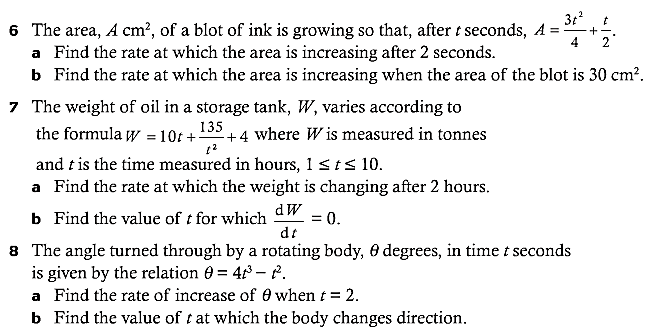
6.5 Rates of Change

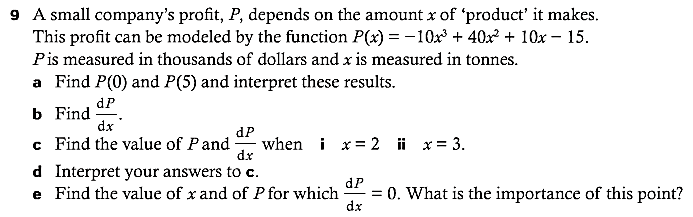


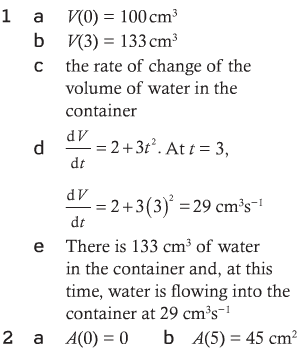


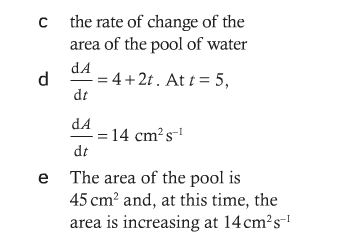


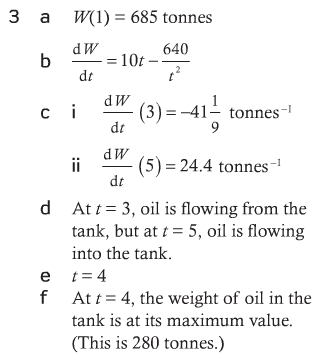


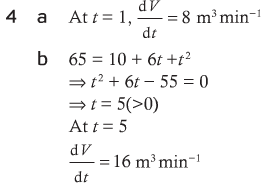


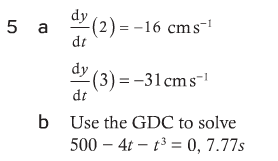


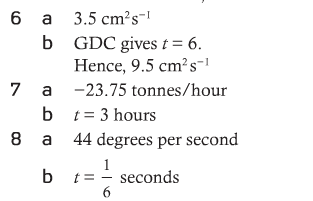


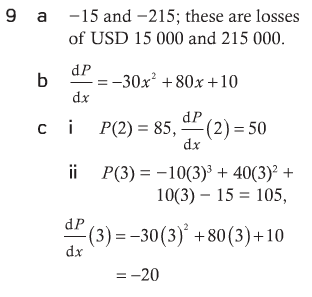












IB Math Studies Year 2

Guided outline for 6.6 Local Minimum and Maximum Points

First Derivative Test – What is it?

<http://www.youtube.com/watch?v=H9t0FJBO4O4>

|  |  |
| --- | --- |
| When do we have a local minimum? | Diagram |
| When do we have a local maximum? | Diagram |
| When do we have neither a local minimum nor a local maximum? | Diagrams |

How do we know when we have a critical point?

First derivative test

<http://www.youtube.com/watch?v=xNjlgXNp2jo>

Summarize the first derivative test in the chart below:

|  |  |  |
| --- | --- | --- |
| Condition | Critical point | Diagram |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Step 1: Use the first derivative to find the critical points.

Step 2: Use the first derivative test to identify function behavior.

Step 3: Identify each critical point.

IB Math Studies Year 2

Guided practice for 6.6 Local Minimum and Maximum Points

Use the first derivative test to identify the local extrema of .

Step 1: Use the first derivative to find the critical points.

[You should get …]

Step 2: Use the first derivative test to identify function behavior.

|  |  |
| --- | --- |
|  |  |
|  |  |

Step 3: Identify each critical point.

IB Math Studies Year 2

Guided practice for 6.6 Local Minimum and Maximum Points

Use the first derivative test to identify the local extrema of .

Step 1: Use the first derivative to find the critical points.

[You should get and …]

Step 2: Use the first derivative test to identify function behavior.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

Step 3: Identify each critical point.

Week 16: Tuesday 10 December

Chapter Six: Introducing Differential Calculus

Topic: 6.6 Local maximum and minimum points

IB Syllabus: 7.4

Lesson Obj: Students will identify the intervals where a function is increasing or decreasing.

Students will sketch a function based on derivative values.

Students will solve problems related to the maximum and minimum of functions.

Review and Intro:

1. Homework questions.
2. Derivative tells us gradient. Gradient tells us whether the function is increasing or decreasing.
3. Therefore, the derivative also tells us where the function is increasing or decreasing.
4. Two intro problems – derivative to function.

Core Lesson:

1. Student worksheets 1-5.

Check for Understanding:

1. Check student progress throughout.

Assignment: Students will watch videos and complete guided outlines.

[Optimizing box volume graphically](http://www.khanacademy.org/math/calculus/derivative_applications/calc_optimization/v/optimizing-box-volume-graphically)

[Optimizing box volume analytically](http://www.khanacademy.org/math/calculus/derivative_applications/calc_optimization/v/optimizing-box-volume-analytically)

[Minimizing the cost of a storage container](http://www.khanacademy.org/math/calculus/derivative_applications/calc_optimization/v/minimizing-the-cost-of-a-storage-container)

Additional problems if necessary:

USE

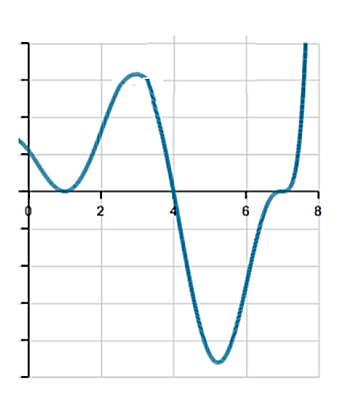
Students do a line with positive slope equaling the derivative. Analyze. Work to drawing the function.

Explore

Use graph AND derivative to create intervals chart.

IB Math Studies Year 2

6.6 Local maximum and minimum points



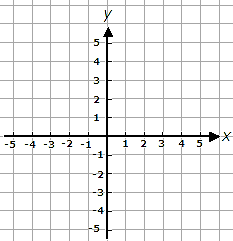
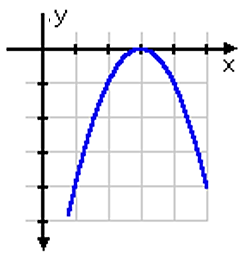
Critical values can occur wherever .

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| -value(s) |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| sketch |  |  |  |  |  |  |  |  |  |

The graph below shows the derivative of an unknown function.

Use this information to draw a possible graph of the unknown function.

Graph of Graph of

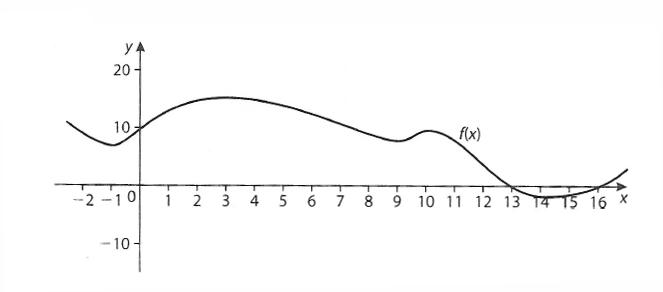
Critical values can occur wherever .

|  |  |  |  |
| --- | --- | --- | --- |
| -value(s) |  |  |  |
|  |  |  |  |
|  |  |  |  |
| sketch |  |  |  |

IB Math Studies Year 2

6.6 Local maximum and minimum points

1. Using the graph of , given below, answer the following questions.



a) Identify the values for which .

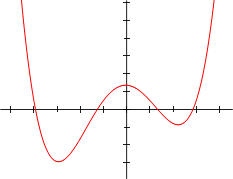
b) Identify the intervals for which .

c) Identify the intervals for which .

2. Sketch a curve which meets the following criteria:

|  |  |
| --- | --- |
| value / interval |  |
|  | >0 |
|  |  |
|  |  |
|  |  |
|  |  |

3. Create a derivative interval chart [as in problem 2] for this function.



4. Sketch a curve which meets the following criteria:

|  |  |
| --- | --- |
| value / interval |  |
|  | <0 |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

5. Consider the function .

a) Differentiate with respect to . [3]

b) Calculate when . [2]

c) Use your answer to part b) to decide whether the function,

, is increasing or decreasing at . Justify your answer. [2]

d) Solve the equation . [3]

e) The graph of has a local minimum at point .

Let be the tangent to the graph of at .

i) Write down the coordinates of . [2]

ii) Write down the gradient of . [1]

iii) Write down the equation of . [2]

f) Sketch the graph of the function , for and

Indicate clearly the point and any intercepts of the curve with the axes. [4]

g) i) On your graph draw and label the tangent . [1]

ii) intersects the graph of at a second point. [2]

Write down the -coordinate of this point of intersection.

IB Math Studies Year 2

6.7 Using Differentiation in Modeling: Optimization

Optimizing box volume graphically

<http://www.khanacademy.org/math/calculus/derivative_applications/calc_optimization/v/optimizing-box-volume-graphically>

Sketch of the box: Volume function:

Graph of the volume function:

Optimizing box volume analytically

<http://www.khanacademy.org/math/calculus/derivative_applications/calc_optimization/v/optimizing-box-volume-analytically>

Minimizing the cost of a storage container

<http://www.khanacademy.org/math/calculus/derivative_applications/calc_optimization/v/minimizing-the-cost-of-a-storage-container>

A rectangular storage container with an open top needs to have a volume of 10 m3.

The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of the material for the cheapest container.

Sketch of the box: Cost function using and :

Express as a function of : Substitute into the cost function:

Optimize: Find the critical points of the cost function (Ignore )

Weeks 16-17: Thursday 12 December / Monday 6 January / Wednesday 8 January

Chapter Fourteen: Calculus

Topic: 6.7 Modeling with Calculus

IB Syllabus: 7.5

Lesson Obj: Students will apply calculus to max/min problem.

Students will sketch a function based on derivative values.

Review and Intro:

1. Homework questions.

DAY 1: Volume and Surface Area

1. Worksheets 1-4

DAY 2: Mixed

1. Worksheets 5-9

DAY 3: Mixed

1. QUIZ
2. Worksheets 10-13

Assignment: Students will study for test on Calculus

IB Math Studies Year 2

6.7 Modeling with Calculus – Perimeter, Area, Volume, and Surface Area

1. A closed box has a square base of side and height .

a) Write down an expression for the volume, , of the box.

b) Write down an expression for the total surface area, , of the box.

The volume of the box is 1000 cm3.

c) Express in terms of .

d) Use b) and c) to show that .

e) Find .

f) Calculate the value of that gives a minimum surface area.

g) Find the surface area for this value of .

2. Ben has 200 metres of electric fence. With this fence he creates a rectangular enclosure.

a) Write an expression for the perimeter of the enclosure in terms of its length, , and width, .

b) Write an expression for the length, , in terms of the width, .

c) Use your expression from part b) to write an expression for the area, , of the enclosure in terms of its width only.

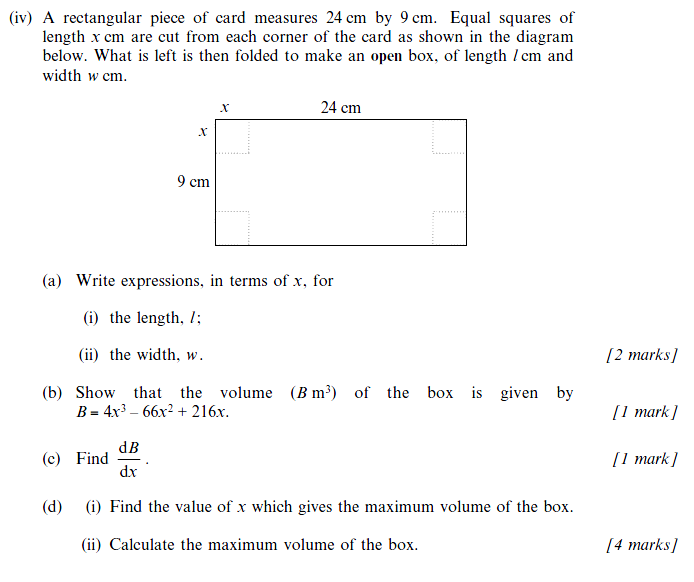
d) Find .

e) Find the dimensions of the enclosure that create the maximum

possible area.

f) What is the maximum possible area Ben can close with his electric fence?

3. A rectangular piece of card measures 24 cm by 9 cm. Equal squares of length cm are cut from each corner of the card as shown in the diagram below. What is left is then folded to make an open box, of length cm and width cm.



a) Write expressions, in terms of , for

i) the length,

ii) the width,

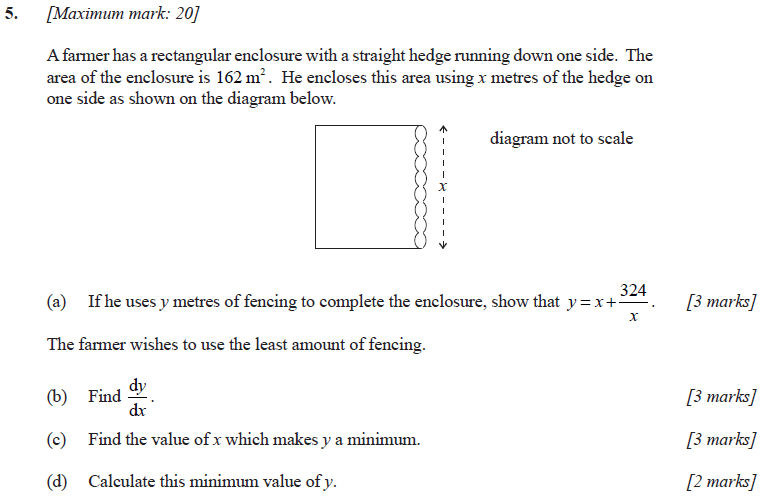
b) Show that the volume () of the box is given by

c) Find .

d) i) Find the value which gives the maximum volume of the box.

ii) Calculate the maximum volume of the box.

4. A farmer has a rectangular enclosure with a straight hedge running down one side. The area of the enclosure is 162 m2. He encloses this area using meters of the hedge on one side as shown on the diagram below.



a) If he uses meters of fencing to complete the enclosure,

show that .

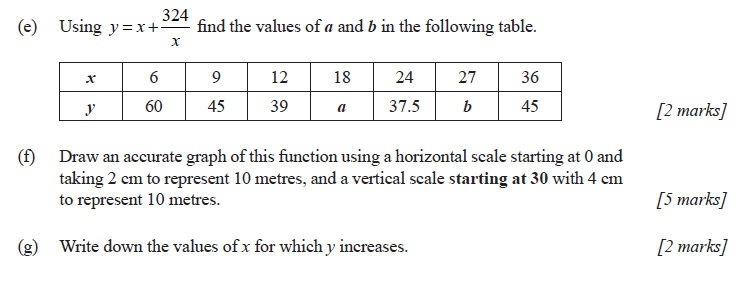
The farmer wishes to use the least amount of fencing.

b) Find

c) Find the value of which makes a minimum

d) Calculate this minimum value of .

e) Using find the values of and in the following table.

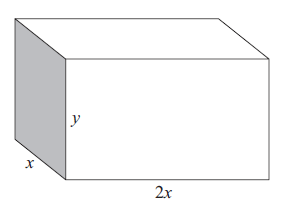


f) Draw an accurate graph of this function using a horizontal scale starting at 0 and taking 2 cm to represent 10 meters, and a vertical scale starting at 30 with 4 cm to represent 10 meters.

g) Write down the values of for which increases.

IB Math Studies Year 2

6.7 Modeling with Calculus – Mixed IB Paper 2 Problems

5. A shipping container is to be made with six rectangular faces, as shown in the diagram.

The dimensions of the container are

length

width

height .

All of the measurements are in metres.

The total length of all twelve edges is 48 metres.

a) Show that . [3]

b) Show that the volume m3 of the container is given by

[2]

c) Find . [2]

d) Find the value of for which is a maximum. [3]

e) Find the maximum volume of the container. [2]

f) Find the length and height of the container for which the volume is a maximum. [3]

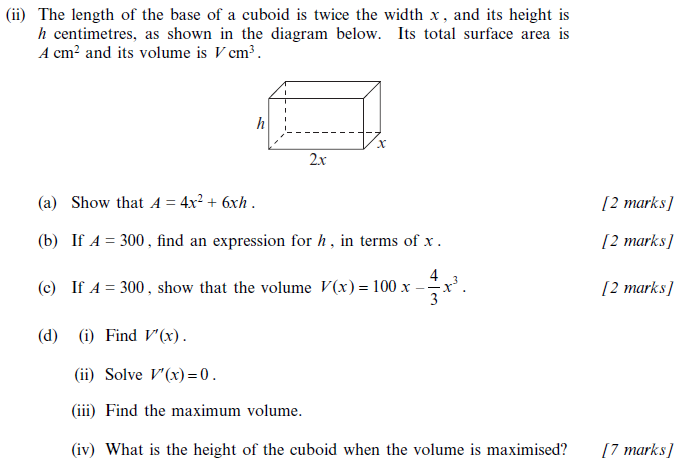
The shipping container is to be painted. One litre of paint covers an area of 15 m2. Paint comes in tins containing four litres.

g) Calculate the number of tins required to paint the shipping container. [4]

6. The length of the base of a cuboid is twice the width , and its height is

centimetres, as shown in the diagram below. Its total surface area is

cm2 and its volume is cm3.



a) Show that . [2]

b) If , find an expression for , in terms of . [2]

c) If , show that the volume is . [2]

d) i) Find .

ii) Solve .

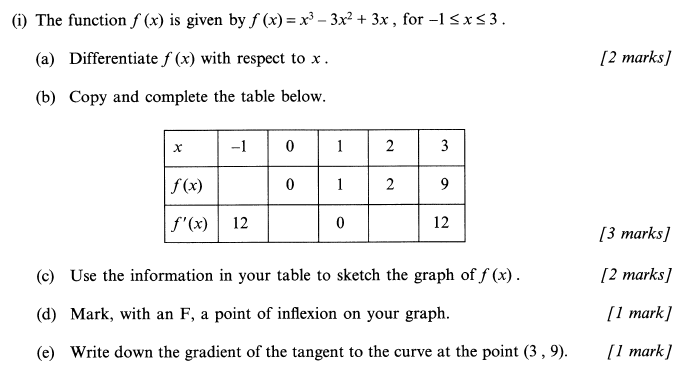
iii) Find the maximum volume.

iv) What is the height of the cuboid when the volume is maximized? [7]

7. The function is given by , for

a) Differentiate with respect to x. [2]

b) Copy and complete the table below. [3]



c) Draw an accurate sketch of , labeling all critical points. [3]

d) Write down the equation of the tangent line to the curve at

the point . [1]

8. Consider the function

a) i) Find [2]

ii) Find the gradient of the curve when [2]

b) Find the -coordinates of the points on the curve where the

gradient is equal to . [3]

c) i) Calculate the x-coordinates of the local maximum and

minimum points. [4]

ii) Hence, find the coordinates of the local minimum. [2]

d) For what values of is increasing? [2]

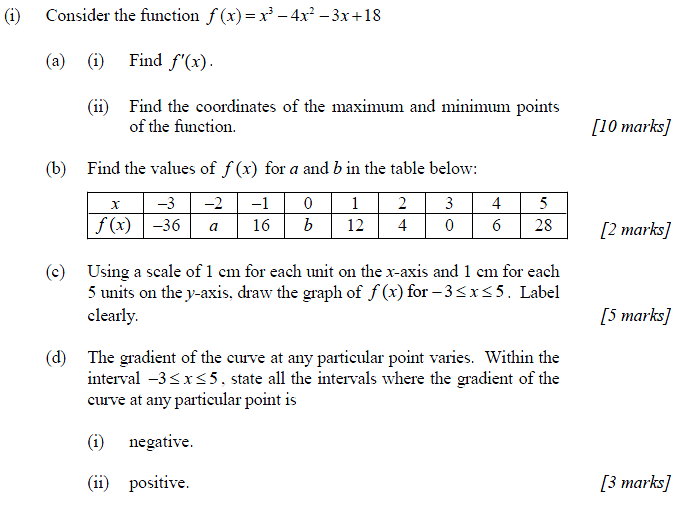
9. Consider the function .

a) i) Find [2]

ii) Find the coordinates of the maximum and minimum

points of the function. Justify your answers using [8]

b) Find the values of for and in the table below. [2]



c) Using a scale of 1 cm for each unit on the -axis and 1 cm for each 5 units on the -axis, draw the graph of for

. Label clearly. [5]

d) Within the interval , state all intervals where the gradient of the curve at any particular point is negative and positive. [3]

IB Math Studies Year 2

6.7 Modeling with Calculus – Mixed IB Paper 2 Problems

10. The number of balloons sold each year by the 123 Balloon Company can be approximated by the formula

where is the number of balloons in the millions and is the number of years since 1996.

a) How many balloons did the company sell in 2007?

b) Find .

c) What was the rate of change in the number of balloons being sold in the year 2007 to the nearest hundred thousand?

11. The number of users of a particular social networking website can be modelled by using the formula

where is the number of users in the thousands per day and is the number of months the website has been operational.

a) Fill in the table of values below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|  |  |  |  |  |  |  |  |

b) Draw an accurate sketch of from

c) Find .

d) Find the number of users being added during the second month.

12. The height (cm) of a daffodil above the ground is given by the function , where is the time in weeks after the plant has broken through the surface ().

a) Calculate the height of the daffodil after two weeks.

b) i) Find the rate of growth, .

ii) The rate of growth when is 7.2 cm per week.

Find .

iii) When will the daffodil reach its maximum height?

What height will it reach?

c) Once the daffodil has reached its maximum height, it begins to fall back towards the ground. Show that it will touch the ground after 70 days.

13. The cost of producing a mathematics textbook is $15.

It is then sold for $.

a) Find an expression for the profit made on each book sold.

A total of books is sold.

b) Show that the profit made on all the books sold is

c) i) Find .

ii) Calculate the value of to make a maximum profit.

d) Calculate the number of books sold to make this maximum profit.

IB Math Studies Year 2

6.7 Modeling with Calculus – Rates of Change [ANSWERS]

10. a)

balloons

b)

c) The derivative in the year 2007 is = 4,600,000 balloons

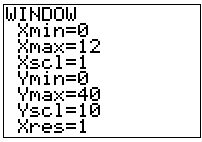
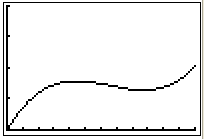
11. a) Fill in the table of values below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|  | 0 | 11.9 | 15.7 | 14.9 | 13.1 | 13.8 | 20.6 |

**y = 1000s of users per day**

**10 20 30 40**

b)

**x = months**

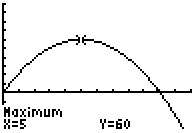
**1 2 3 4 5 6 7 8 9 10 1112**

c)

d) users

12. a) cm

b) i) .

 ii) Find when the derivative equals 7.2

**y = height (cm)**

**50 100**

iii) Max height when :

**x = weeks**

**1 2 3 4 5 6 7 8 9 10**

c) From the graph, you can see that the height is zero when = 10.

But x is measured in weeks, so 10 weeks x 7 days = 70 days.

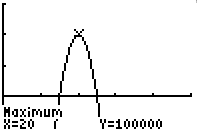
13. a) Profit on one item = revenue – cost =

b) Profit on all items is (profit on one item) x (the number of items)

**y = profit**

**50 100 150**

c) i)



ii) Maximum where .

**x = books**

**10 20 30 40 50**

d)