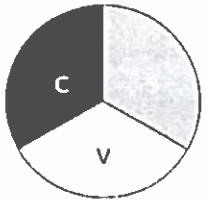
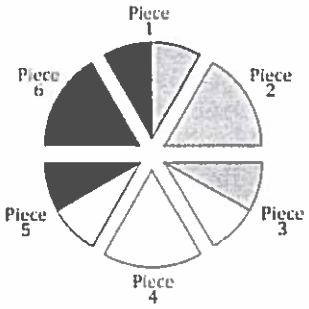
Discrete Math

Ch. 3: Practice Test: Fair Division

1. Alex buys a chocolate-strawberry-vanilla cake [shown in (i)] for $18.00. Alex values strawberry twice as much as vanilla, and values chocolate three times as much as vanilla.



(i)



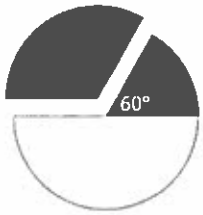
(ii)

$3v + 2v + v = 18$
$6v = 18$
$v = 3$

(a) What is the value of the chocolate part of the cake to Alex? $9.00
(b) What is the value of the strawberry part of the cake to Alex? $6.00
(c) What is the value of the vanilla part of the cake to Alex? $3.00
(d) If the cake is cut into the six 60° wedges shown in (ii), find the value to Alex of piece 5. $\frac{9}{4} + \frac{5}{4} = \frac{12}{4} = \$3.00$

2. Adam and Ben both contribute $8.00 to buy a $16.00 orange-pineapple cake [shown in (i)]. Suppose that Adam values pineapple three times as much as he values orange, and that Ben values orange twice as much as he values pineapple. They want to divide the cake using the divider-chooser method, with Adam as the divider.

(i)



(ii)

A: $3O + O = 16$ $O = \$4.00$
B: $2P + P = 16$ $P = \frac{16}{3} = \$5.33$

(a) Given that Adam is the divider, is the cut [shown in (ii)] consistent with his value system? Yes — $8.00 and $8.00
(b) Which piece would Ben choose? 180° Orange + 60° Pineapple
(c) Describe the final division of the cake, specifying which pieces each player gets and how much they value them.

Adam: 120° Pineapple → $8.00

Ben: 180° O and 60° P → 5.33 + 3.56 = $8.89

3. Four partners (Charlie, China, Chip and Dianne) want to divide a plot of land using the lone-divider method. Using a map, Dianne divides the property into four parcels (s_1, s_2, s_3, and s_4).

(a) If the chooser declarations are
Charlie: { s_1, s_2 }
China: { s_2, s_3 }
Chip: { s_2 }
Describe a possible fair division of the land.

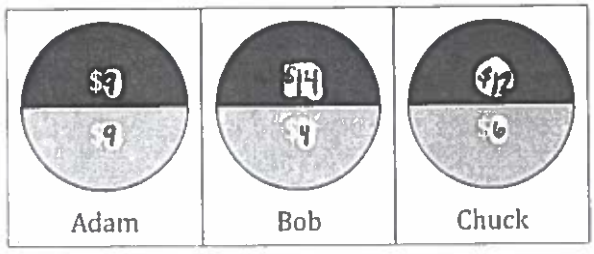
Charlie: s_1
China: s_3
Chip: s_2
Dianne: s_4

(b) If the chooser declarations are
Charlie: { s_1, s_2 }
China: { s_2 }
Chip: { s_1, s_2 }
Describe how to proceed to find a fair division of the land.

Charlie:
China: s_2
Chip:

s_1/s_3/s_4 will have to be divided again.

4. Suppose that three players (Adam, Bob, and Chuck) each contribute $6.00 to buy a $18.00 cake. They decide to divide the cake using the lone-chooser method. The dollar amounts of the cake in each player's eyes are given in the following figure.



Chuck: $\frac{9}{3} = \$3$

$s_1 \to 45°$ Choc
$s_2 \to 45°$ Choc
$s_3 \to 90°$ Str.

Bob: $\frac{9}{5} = \$2$

$\frac{7}{90} = \frac{5}{x}$ $x = 38.6$
$s_1 \to 38.6°$ Choc
$s_2 \to 38.6°$ Choc
$s_3 \to 12.8°$ C/90° S

Suppose that Bob and Chuck are the dividers and Adam is the chooser. In the first division, Bob cuts the cake vertically through the center and Chuck picks the right half.

(a) Draw a possible second division that Bob might make of the left half of the cake.
(b) Draw a possible second division that Chuck might make of the right half of the cake.
(c) Based on the second divisions you gave in (a) and (b), describe a possible final division of the cake, specifying which pieces each player gets and how much they value them.

Adam: 90° S, 12.8° C/90° Str. → 4.50 + 4.50 + .85 = $9.85
Bob: 77.2° C → $6.00
Chuck: 90° C → $6.00

5. A cake is to be divided among seven players (P_1, P_2, P_3, P_4, P_5, P_6, and P_7) using the last-diminisher method. The players play in a fixed order with P_1 first, P_2 second, etc. In round 1, only P_1 and P_3 are diminishers. In round 2, there are no diminishers. In round 3, everyone with an opportunity to diminish does so. In round 4, there are no diminishers.

(a) Which player cuts at the beginning of round 5? P_4
(b) Which player has the last chance to diminish in round 5? P_7

Players.
$P_1 \to$ Rd. 2
$P_2 \to$ Rd. 4
$P_3 \to$ Rd. 1
P_4
$P_5 \to$ Rd. 3

6. Four players (A, B, C, and D) wish to divide up five items using the method of sealed bids. Let's say that the following table represents how each player honestly values each of the five items:

	A	B	C	D
Item 1	$22,000	$20,000	$18,000	$17,000
Item 2	8,000	10,000	9,000	7,000
Item 3	12,000	14,000	13,000	9,000
Item 4	24,000	24,000	28,000	36,000
Item 5	36,000	50,000	53,000	55,000
FS:	25,500	29,500	36,250	31,000

Describe the final outcome of this fair division problem.

Prelim.

A - #1 (22,000) + get $3,500

B - #2 (10,000) + #3 (14000) + get $5,500

C - no items + get $30,250

D - #4 (36,000) + #5 (55000) + pay $60,000

Surplus : 60000 − 3500 − 5500 − 30250 = $20750 ÷ 4 = $5187.5

Final:

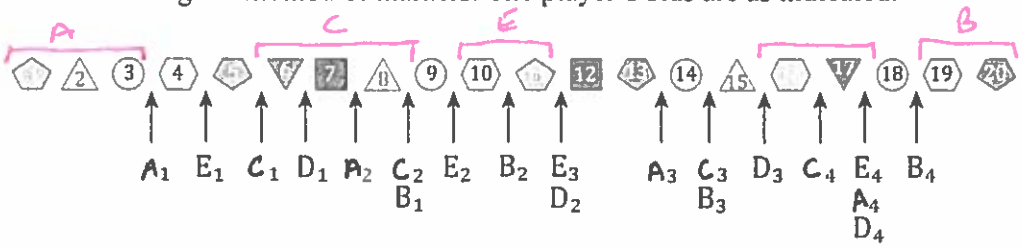
A - #1 + get $8687.50

B - #2, #3, + get $10,687.50

C - no items, get $35 437.50

D - #4, #5, + pay $54,812.50

7. Five players (A, B, C, D, and E) agree to divide the 20 items shown by lining them up in order and using the method of markers. The player's bids are as indicated.



(a) Describe the allocation of items to each player.
(b) Which items are left over?

a) A - 1, 2, 3
B - 19, 20
C - 4, 7, 8
D - 16, 17
E - 10, 11

b) 4, 5, 9, 12, 13, 14, 15, 18