9.7 Definite Integrals with Linear Motion and Other Problems

Do you remember this relationship between position, velocity and acceleration?

→ → → differentiation → → →

position ----- velocity ----- acceleration

← ← ← integration ← ← ←

We've looked at the indefinite integral before in this context, using conditions of time with position and velocity to find specific equations.

Definite Integration and Kinematics

Example 1

The acceleration a ms^{-2} of a particle moving in a straight line at time t seconds is given by $a = t + 1$.

a) Find formulae for the velocity $v(t)$ and the displacement $s(t)$, given that when $t = 0$, $s = 0$ and $v = 8$.

b) Find the distance travelled between $t = 2$ and $t = 5$.

c) Find $\int_2^5 v(t)\,dt$.

a. $v(t) = \int t+1\,dt = \frac{1}{2}t^2 + t + 8$

$s(t) = \int \frac{1}{2}t^2 + t + 8 = \frac{1}{6}t^3 + \frac{1}{2}t^2 + 8t$

b. $s(5) = 73.3$

$s(2) = 19.34$

$73.3 - 19.34 = 54.0$

c. $\int_2^5 v(t)\,dt = 54.0$

$t = a$	$t = b$	$s(a)$	$s(b)$	Displacement	$\int_a^b v(t)\,dt$

Definite Integration and Kinematics

One really cool thing about calculus is that the definite integral gives us the area under the velocity-time graph. This area represents the displacement – the distance traveled – during that time.

For an object moving in a straight line, with the velocity v given as a function of the time t, the displacement between t_1 and t_2 is given by $\int_{t_1}^{t_2} v\,dt$. This displacement is represented by the area under the $v(t)$ graph for the interval.

Definite Integration and Kinematics

Example 2

A car starts to accelerate as soon as it leaves a town. After t seconds its velocity is given by the formula $v(t) = 14 + 0.45t^2 - 0.03t^3$. Find the distance travelled while the car accelerates to its maximum velocity.

$$s(t) = \int_0^{10} 14 + 0.45t^2 - 0.03t^3 \, dt$$

$$= 14t + 0.15t^3 - \frac{0.03}{4}t^4 \Big|_0^{10}$$

$$= 215$$

$a(t) = 0.9t - 0.09t^2 = 0$

$0.09t(10 - t) = 0$

$t = 0, 10$

Definite Integration and Kinematics

Practice

Given that $v = \frac{3}{t^2}$, find the distance travelled between $t = 2$ and $t = 10$.

$$s(t) = \int_2^{10} \frac{3}{t^2}\,dt \qquad \frac{3}{t^2} = 3t^{-2}$$

$$-3t^{-1}$$

$$= -3t^{-1}\Big|_2^{10}$$

$$= -\frac{3}{10} - \left(-\frac{3}{2}\right) = \frac{6}{5}$$

Homework Assignment:

WS attached to class notes, check with answers on back