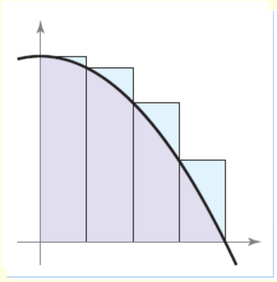
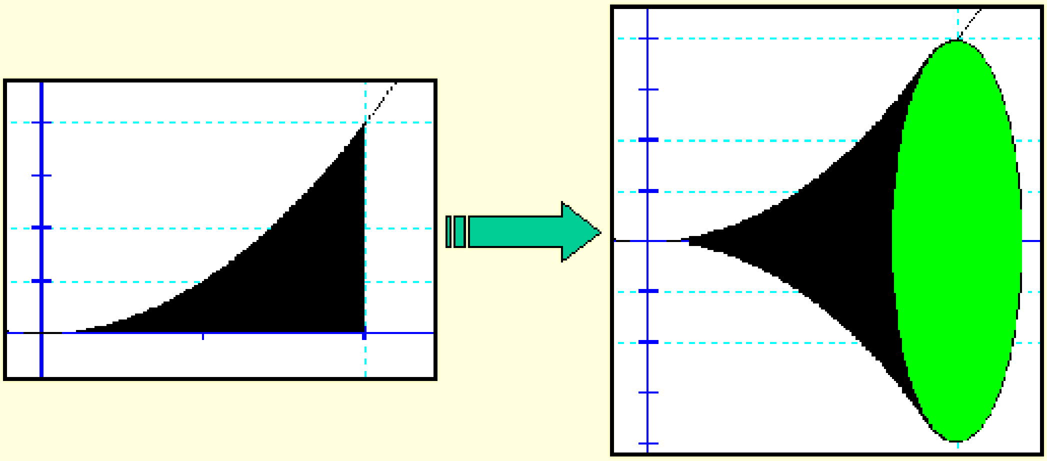
9.7 Definite Integrals with Linear Motion and Other Problems

Approximating the Area of Any Region

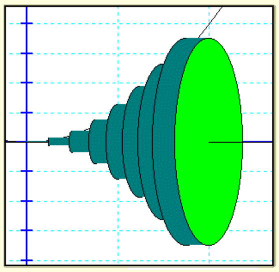


Here's how it works!

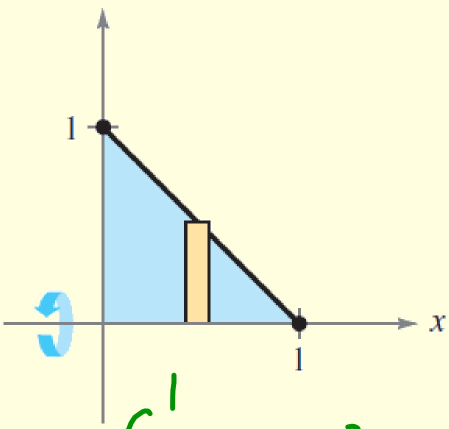
Solids of Revolution



Solids of Revolution
Using Discs



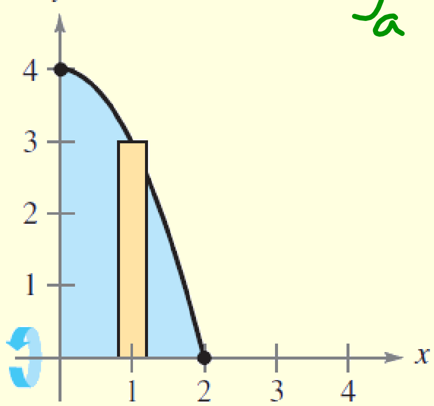
1. $y = -x + 1$



$$V = \pi \int_0^1 (-x+1)^2 \, dx$$

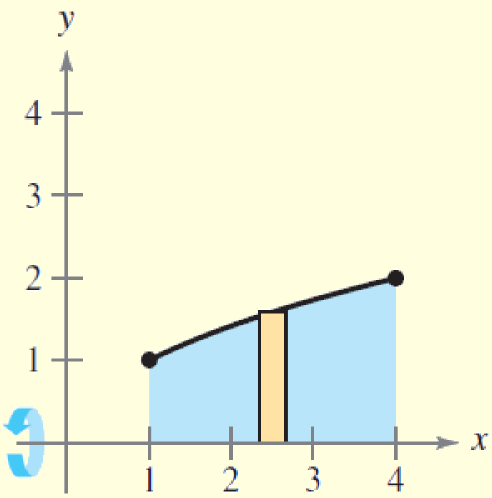
2. $y = 4 - x^2$

$$V = \int_a^b \pi y^2 \, dx$$



$$V = \pi \int_0^2 (4 - x^2)^2 \, dx$$

3. $y = \sqrt{x}$

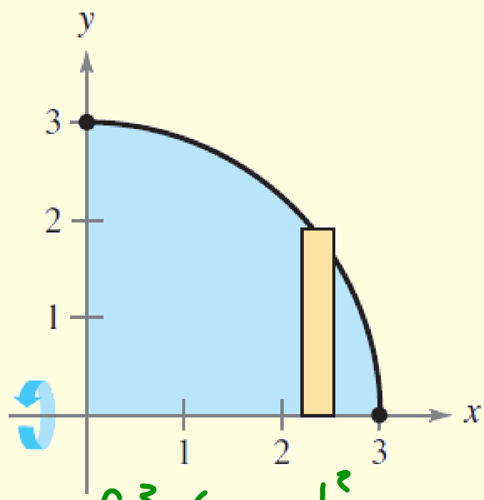


$$V = \pi \int_1^4 (\sqrt{x})^2 \, dx$$

$$= \pi \int_1^4 x \, dx$$

$$= \pi \frac{1}{2} x^2 \Big|_1^4 = \pi\left(\frac{1}{2}(4)^2 - \frac{1}{2}(1)^2\right) = 7.5\pi$$

4. $y = \sqrt{9 - x^2}$



$$V = \pi \int_0^3 \left(\sqrt{9-x^2}\right)^2 dx$$

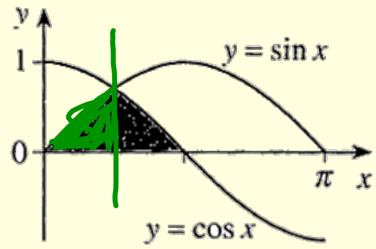
$$= \pi \int_0^3 (9-x^2)\, dx$$

Practice

$\sin x = \cos x$

$\tan x = 1$

The curves $y = \sin x$ and $y = \cos x$ enclose a region as shaded in the sketch.



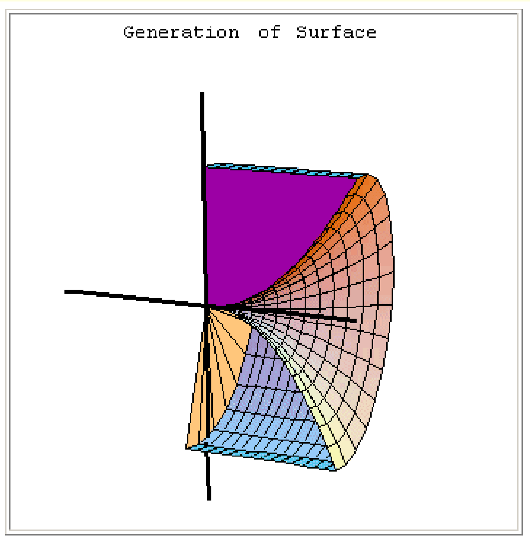
a) Find the exact area of the shaded region.

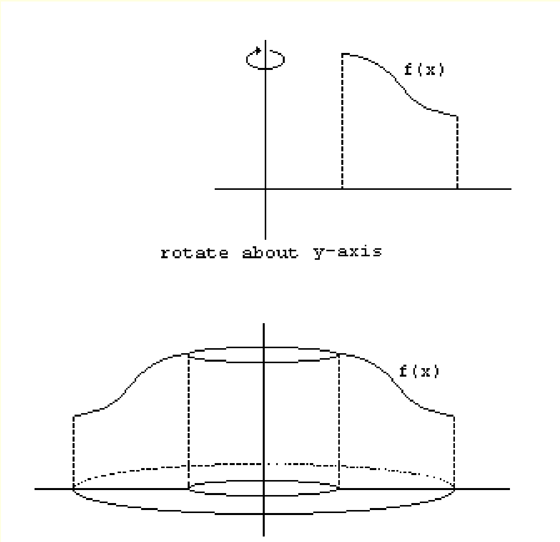
b) Find the volume generated when this region is rotated about the x-axis, correct to three significant figures.

a) $A = \int_0^{\pi/4} \sin x \, dx + \int_{\pi/4}^{\pi/2} \cos x \, dx$

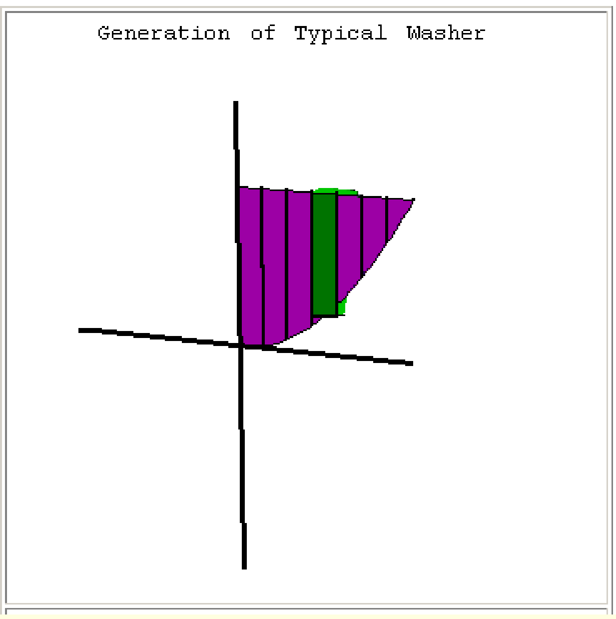
$A =$

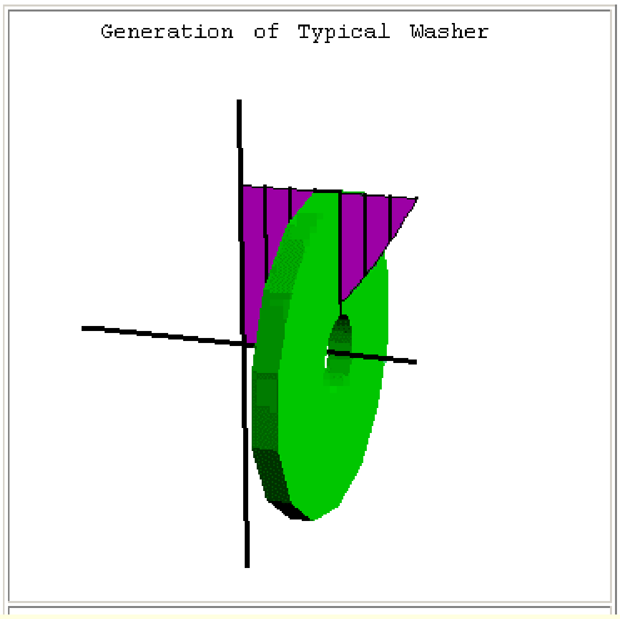
Solids of Revolution



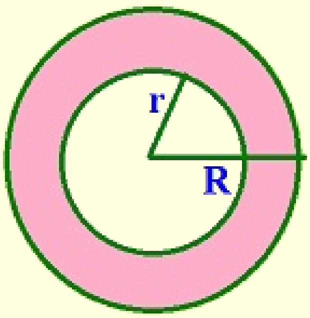


Solids of Revolution
Using Washers

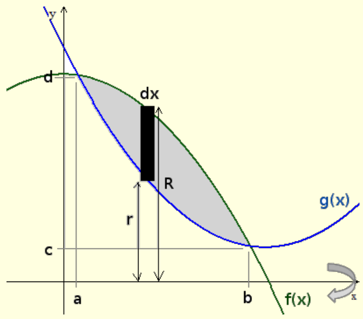




Area of a Washer

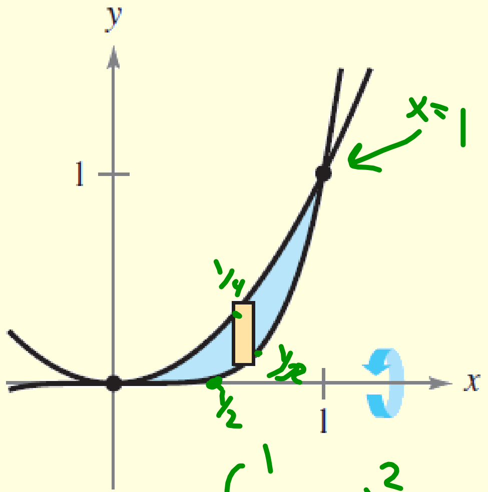


Volume of a Solid

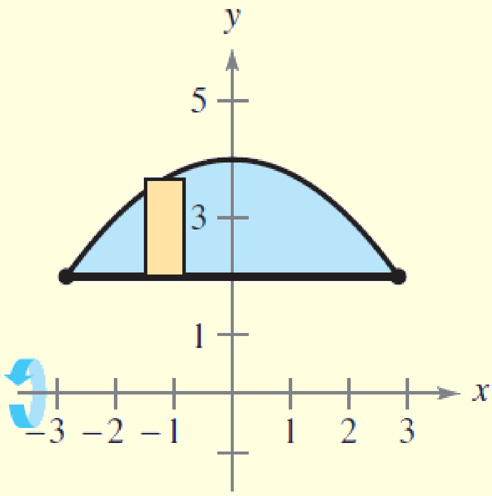


$$V = \pi \int_a^b R(x)^2 - r(x)^2 \, dx$$

5. $y = x^2, \; y = x^5$



6. $y = 2, \; y = 4 - \dfrac{x^2}{4}$



$$V = \pi \int_0^1 (x^2)^2 - (x^5)^2 \, dx$$

$$= \pi \int_0^1 x^4 - x^{10} \, dx$$

$$= \pi \left[\frac{1}{5}x^5 - \frac{1}{11}x^{11} \right] \Big|_0^1$$

$$= \pi \left(1/5 - 1/11 \right)$$

Practice

Consider the region bounded by the graphs $f(x) = \sqrt{x}$ and $g(x) = x^2$.

a) Without using a calculator, find the intersection points of the two curves.
b) Without using a calculator, sketch these graphs on the same axis.
c) Find the exact area bounded by these curves.
d) Find the exact volume of the solid of revolution formed by rotating this area around the x-axis.

Homework Assignment:

Exercise 9N