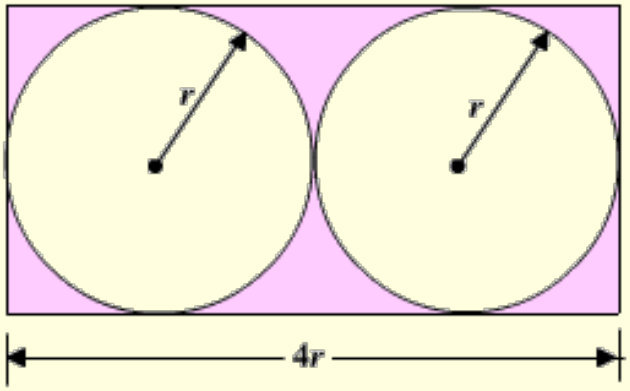
9.5 Area of a Region Between Two Curves

How do we find the area of the shaded region?



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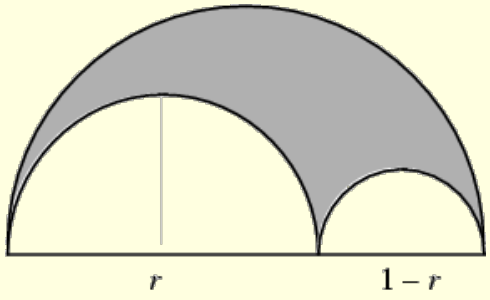

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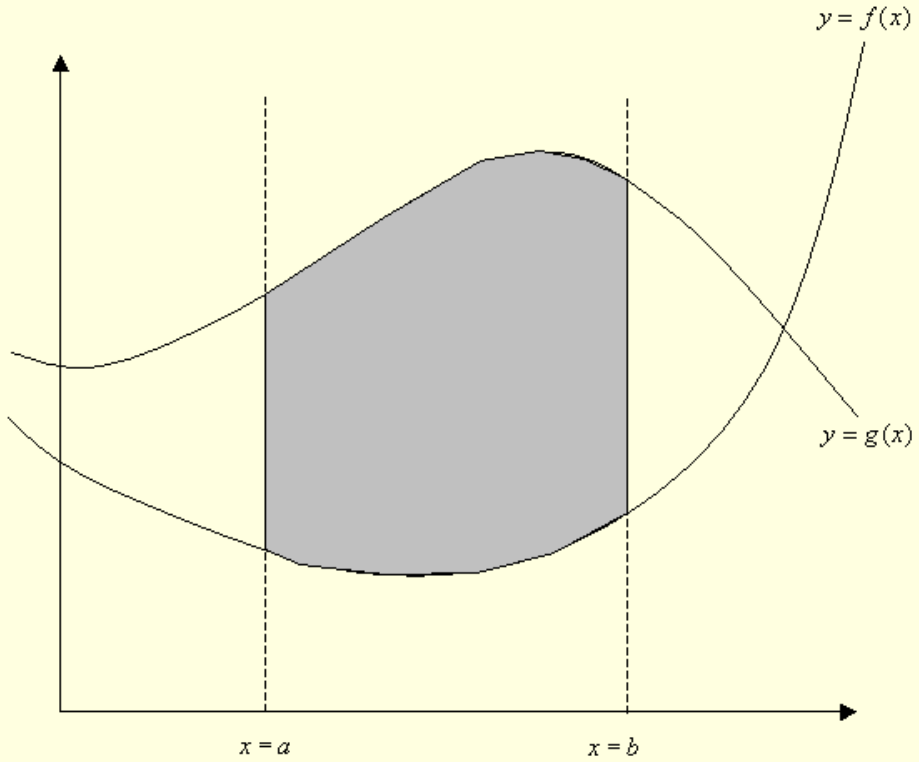

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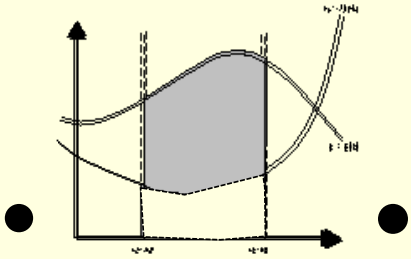


1. Find the area of the region bounded by the graphs of $y = x^2 + 2$, $x = 0$, $y = -x$, $x = 1$

2. Find the area of the region bounded by the graphs of $f(x) = x$ and $g(x) = 2 - x^2$.

3. The sine and cosine curves intersect infinitely many times, bounding regions of equal area. Find the area of one of these regions.

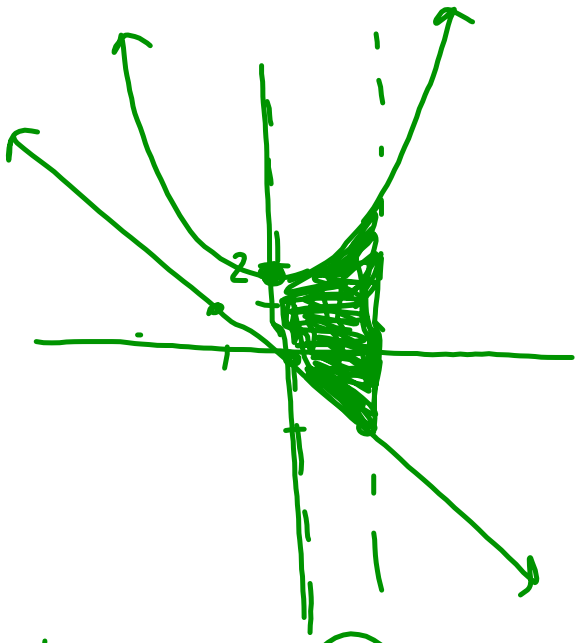
4. Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

① $y = x^2 + 2$

$y = -x$

$x = 0$

$x = 1$



$$A = \int_0^1 (x^2+2) - (-x)\,dx$$

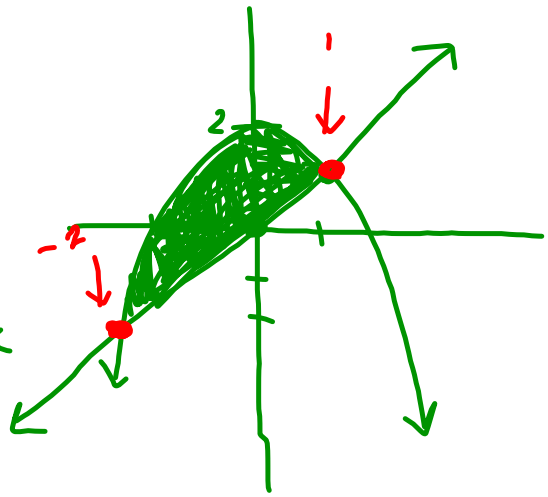
$$A = \int_0^1 x^2 + x + 2\,dx$$

$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \Big|_0^1$$

$$= \frac{1}{3}(1)^3 + \frac{1}{2}(1)^2 + 2(1) = \frac{1}{3} + \frac{1}{2} + 2 = \frac{17}{6}$$

(2) $f(x) = x$

$g(x) = 2 - x^2$



$$A = \int_{-2}^{1} (2 - x^2) - x \, dx$$

$$A = \int_{-2}^{1} -x^2 - x + 2 \, dx$$

$$A = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \Big|_{-2}^{1}$$

$$= \left(-\frac{1}{3}(-2)^3 - \frac{1}{2}(-2)^2 + 2(-2)\right) - \left[-\frac{1}{3}(1)^3 - \frac{1}{2}(1)^2 + 2(1)\right]$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 - \left[\frac{8}{3} - 2 - 4\right]$$

$$= \frac{7}{6} + \left(+\frac{10}{3}\right) = \frac{27}{6} = \frac{9}{2}$$

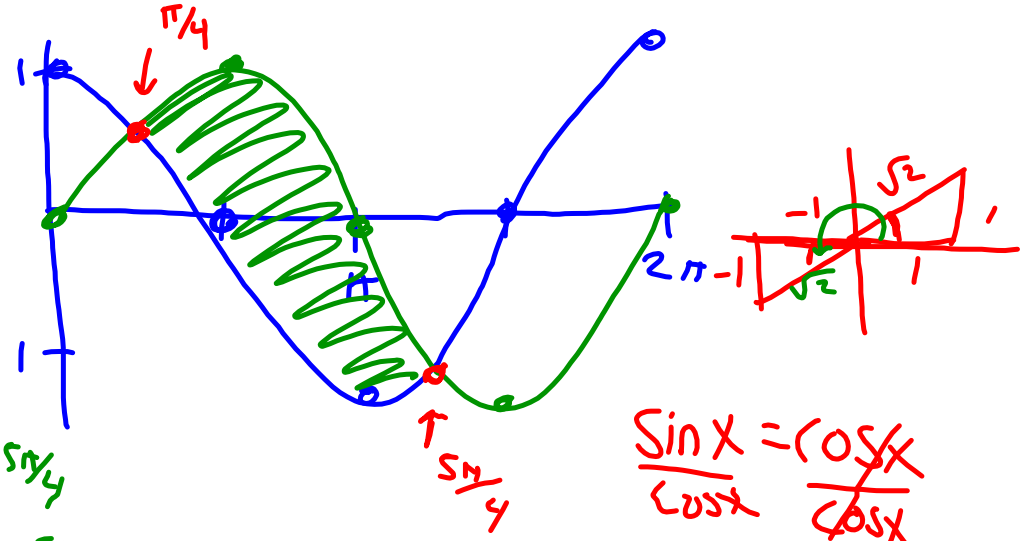
$x = 2 - x^2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

(3)



$$A = \int_{\pi/4}^{5\pi/4} \sin x - \cos x \, dx$$

$$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\tan x = \frac{1}{1}$$

$x = \pi/4$

$x = 5\pi/4$

$$A = -\cos x - \sin x \Big|_{\pi/4}^{5\pi/4}$$

$$A = -\cos 5\pi/4 - \sin 5\pi/4 - \left[-\cos \pi/4 - \sin \pi/4\right]$$

$$A = -\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right) - \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$$

$$A = \frac{4}{\sqrt{2}}$$

④ $f(x) = 3x^3 - x^2 - 10x$

$g(x) = -x^2 + 2x$

$3x^3 - x^2 - 10x = -x^2 + 2x$

$3x^3 - 12x = 0$

$3x(x^2 - 4) = 0$

$3x(x+2)(x-2) = 0$

$x = 0, -2, 2$

Homework Assignment:

Exercise 9K (5-10) and 9L (1,3,5)