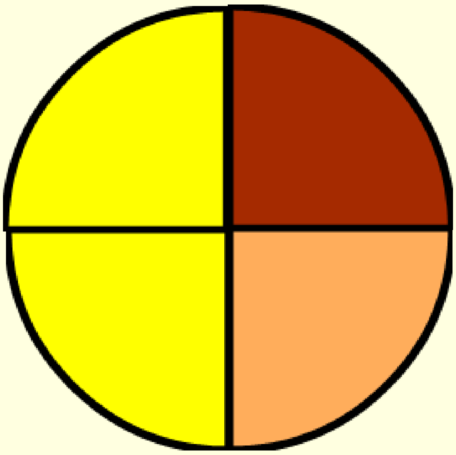
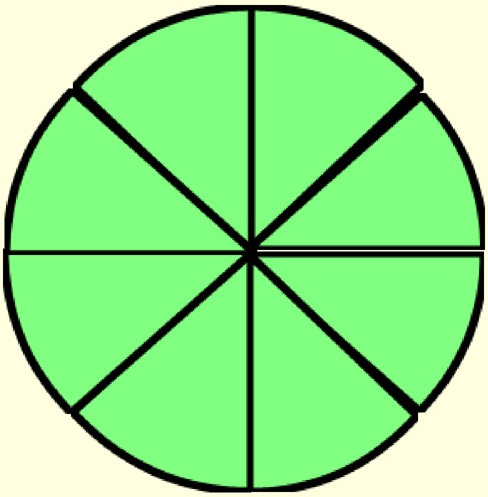
# Applications of Integrals

9.3 Area and Definite Integrals

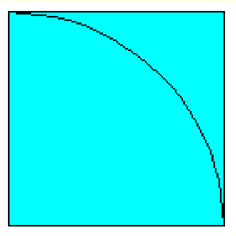
# Approximating the Area of a Circle



# Approximating the Area of a Circle



# Approximating the Area of Any Region



Here's how it works!

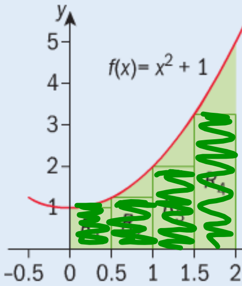
## Investigation – area and the definite integral

**1** Consider the area bounded by the function $f(x) = x^2 + 1$, $x = 0$, $x = 2$ and the $x$-axis, which is shaded in green in the graph.

**a** **i** Write down the width of each of the four rectangles shown in the graph. 0.5

**ii** Calculate the height of each of the four rectangles.

**iii** Find the sum of the areas of the four rectangles to find a lower bound of the area of the shaded region.



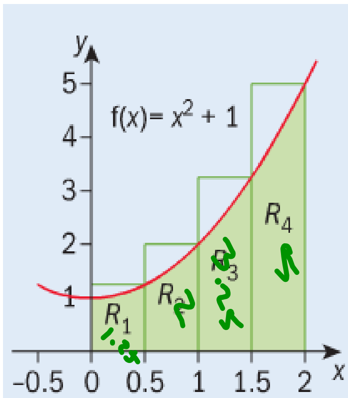

3.75

$R_1 = 1$ $R_3 = 2$

$R_2 = 1.25$ $R_4 = 3.25$

iii) 3.75 → lower.

5.75 → upper




**c** Use a GDC to evaluate the **definite integral** $\int_0^2 (x^2 + 1)dx$. Compare your result with your answers in parts **a** and **b**. = 4.67

What do you think the definite integral might represent?

Approximations for area under $f(x) = x^2 + 1$ from $x = 0$ to $x = 2$ for different numbers of rectangle.

| # Rectangles | Lower sum | Upper sum |
|---|---|---|
| 4 | 3.75 | 5.75 |
| 10 | 4.28 | 5.08 |
| 50 | 4.5872 | 4.7472 |
| 100 | 4.6268 | 4.7068 |
| 500 | 4.65867 | 4.67467 |

Exact area =

$$\int_0^2 (x^2 + 1)\,dx = \frac{14}{3} \approx 4.66667$$

Notice that both the lower and upper sums appear to approach 4.66667.

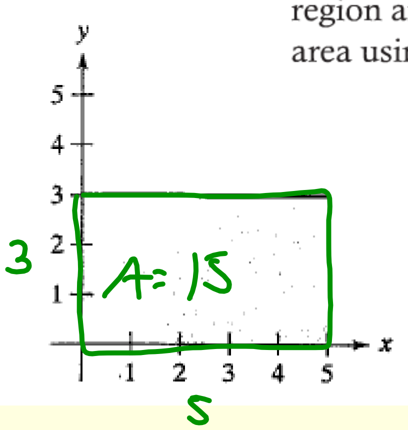
Area under a curve:

From Σ to ∫

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x)\Delta x$$

$$S = \int_{a}^{b} f(x)dx = F(b) - F(a)$$

**1.** $f(x) = 3$




Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.

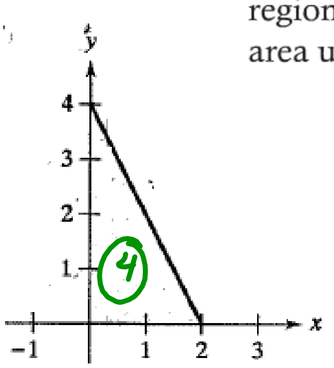
$$\int_0^5 3\,dx$$

$$= 3x \Big|_0^5$$

$$= 3(5) - 3(0)$$

$$= 15$$

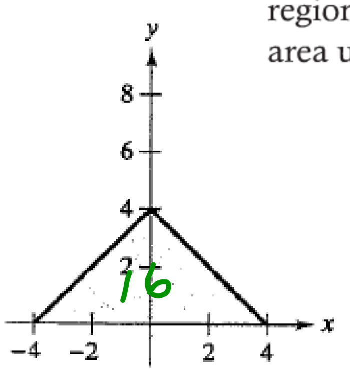
**2.** $f(x) = 4 - 2x$




Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.

$$\int_0^2 4-2x \, dx = 4$$

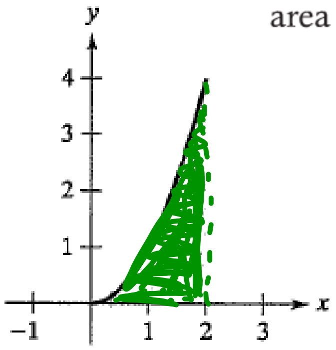
**3.** $f(x) = 4 - |x|$




Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.

$$\int_{-4}^{4} 4-|x|\,dx = 16$$

**4.** $f(x) = x^2$




Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.

$$\int_0^2 x^2\,dx$$

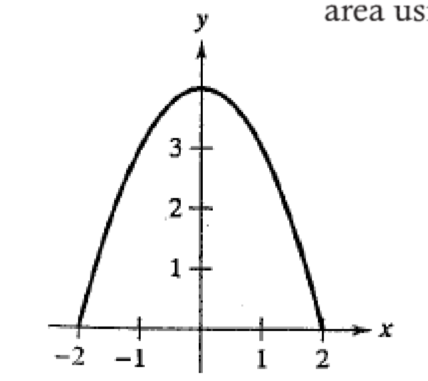
$$= \frac{1}{3}x^3 \Big|_0^2$$

$$= \frac{1}{3}(2)^3 - \frac{1}{3}(0)^3$$

$$= \frac{8}{3}$$

5. $f(x) = 4 - x^2$

Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.




$$\int_{-2}^{2} 4 - x^2\,dx = 10.7$$

$$= 4x - \tfrac{1}{3}x^3 \Big|_{-2}^{2}$$

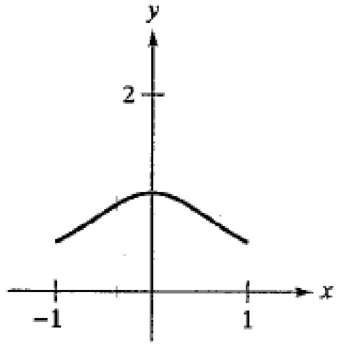
$$= 4(2) - \tfrac{1}{3}(2)^3 - \left(4(-2) - \tfrac{1}{3}(-2)^3\right)$$

$$= 8 - \tfrac{8}{3} + \left(+8 + \tfrac{-8}{3}\right)$$

$$= 16 - \tfrac{16}{3} = 32/3$$

**6.** $f(x) = \dfrac{1}{x^2 + 1}$

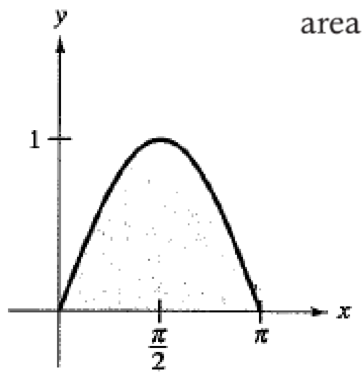
Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.




$$\int_{-1}^{1} \frac{1}{x^2+1}\,dx$$

$$= 1.57$$

**7.** $f(x) = \sin x$

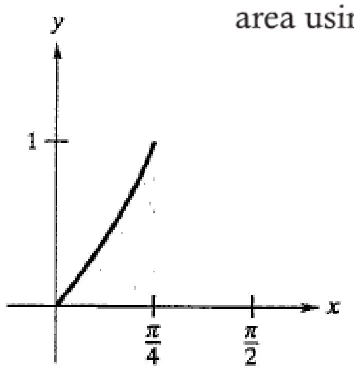



Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.

$$\int_0^{\pi} \sin x \, dx$$

$$= 2$$

**8.** $f(x) = \tan x$



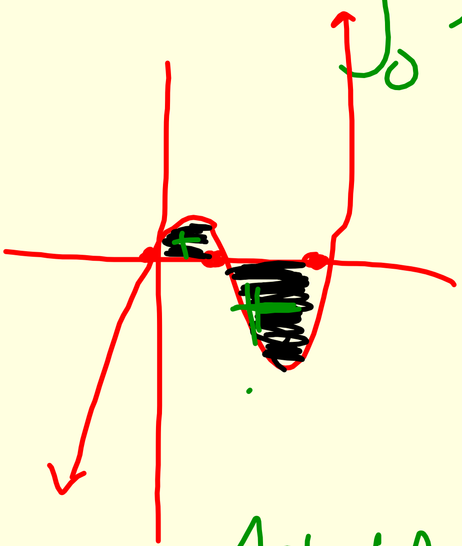

Write down a definite integral that gives the area of the shaded region and evaluate it using your GDC. Where possible find the area using a geometric formula to verify your answer.

$$\int_0^{\pi/4} \tan x \, dx$$

$$= 0.347$$

Find the area bounded by the curve y= x(x - 1) (x - 3) and the x-axis.

$$\int_0^3 x(x-1)(x-3)\,dx = -2.25$$



$$\int_0^1 x(x-1)(x-3)\,dx = 0.417$$

$$-\int_1^3 x(x-1)(x-3)\,dx = -2.67$$

Actual Area = 0.417 + 2.67
= 3.085

Exercise 9.3 G