9.2 Integration by Substitution: Rational Functions

Remember the Chain Rule: If $\int \frac{1}{x}\,dx = \ln|x| + C$ then $\int \frac{1}{u}\,du = \ln|u| + C$

Examples:

1. $\int \frac{2}{x}\,dx$

$= 2\ln x + c$

2. $\int \frac{1}{4x-1}\,dx$

$= \frac{1}{4}\ln(4x-1) + c$

3. $\int \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$\frac{du}{2} = \frac{\cancel{2}x\,dx}{\cancel{2}}$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln u + c$

$= \boxed{\frac{1}{2}\ln(x^2+1) + c}$

4. $\int \frac{9x^2+3}{x^3+x}\,dx$

$u = x^3+x$

$3du = 3(3x^2+1\,dx)$

$3du = 9x^2+3\,dx$

$= 3\int \frac{1}{u}\,du$

$= 3\ln u + c$

$= \boxed{3\ln(x^3+x) + c}$

Practice:

5. $\int \frac{x+1}{x^2+2x}\,dx$

$u = x^2+2x$

$du = 2x+2\,dx$

$\frac{1}{2}du = x+1\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln u + c$

$= \frac{1}{2}\ln(x^2+2x) + c$

6. $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$-du = {}^{+}\sin x\,dx$

$= -\int \frac{1}{u}\,du$

$= -\ln u + c$

$= -\ln(\cos x) + c$

IB Math Standard

9.2 Integration by Substitution: Exponential Functions

Remember the Chain Rule: If $\int e^x dx = e^x + C$ then $\int e^u du = e^u + C$

Examples:

1. $\int e^{3x+1} dx$

$u = 3x+1$

$\frac{1}{3} du = \frac{3}{3} dx$

$= \frac{1}{3} \int e^u du$

$= \frac{1}{3} e^u + C$

$= \frac{1}{3} e^{(3x+1)} + C$

2. $\int 5xe^{-x^2} dx$

Practice:

3. $\int \frac{e^{1/x}}{x^2} dx$

4. $\int \sin x e^{\cos x} dx$

$u = \cos x$

$-du = +\sin x dx$

$= -\int e^u du$

$= -e^u + C$

$= -e^{\cos x} + C$

5. $\int xe^{ax^2} dx$

6. $\int (e^x - e^{-x})^2 dx$

9.2 Integration by Substitution: Initial Conditions

Find the function which meets these conditions:

$f''(x) = \frac{1}{2}(e^x + e^{-x})$

$f(0) = 1, f'(0) = 0$

$f'(x) = \frac{1}{2}\int (e^x + e^{-x})\,dx$

$= \frac{1}{2}e^x - e^{-x} + c$

$0 = \frac{1}{2}e^0 - e^{-0} + c$

$0 = \frac{1}{2} - 1 + c$

$c = 1/2$

$f'(x) = \frac{1}{2}e^x - e^{-x} + \frac{1}{2}$

$F(x) = \int 1/2\, e^x - e^{-x} + 1/2\, dx$

$= \frac{1}{2}e^x + e^{-x} + 1/2\, x + c$

$1 = \frac{1}{2}e^0 + e^{-0} + \frac{1}{2}(0) + c$

$1 = \frac{1}{2} + 1 + 0 + c$

$c = -1/2$

$\boxed{F(x) = \frac{1}{2}e^x + e^{-x} + \frac{1}{2}x - 1/2}$

Find the function which meets these conditions:

$f''(x) = \sin x + e^{2x}$

$f(0) = \frac{1}{4}, f'(0) = \frac{1}{2}$