IB Math Standard

9.2 Integration by Substitution

Review of the Chain Rule: $\frac{d}{dx}(4x+5)^3 = 3(4x+5)^2 \cdot 4$ $= 12(4x+5)^2$ $\frac{d}{dx}\sin 5x = \cos 5x \cdot 5$ $= 5\cos 5x$

With integration, we also have to consider the Chain Rule: $\int f(u)du = F(u)+C$

Examples: Derivative already present

1. $\int 2x(x^2+1)^4 dx$ $u = x^2+1$, $du = 2x\,dx$

$= \int u^4 du$

$= \frac{1}{5}u^5 + C = \frac{1}{5}(x^2+1)^5 + C$

2. $\int 3x^2\sqrt{x^3+1}dx$ $u = x^3+1$, $du = 3x^2$

$= \int u^{1/2} du = \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(x^2+1)^{3/2} + C$

3. $\int 5\sin(5x-3)dx$

4. $\int \frac{\tan x + 3}{\cos^2 x}dx$ $u = \tan x + 3$, $du = \frac{1}{\cos^2 x}dx$

5. $\int 12x^3\cos(3x^4)dx$ $u = 3x^4$, $du = 12x^3 dx$

$= \int \cos u\, du = \sin u + C$

$= \sin(3x^4) + C$

6. $\int \frac{\cos x}{\sin^2 x}dx$ $u = \sin x$, $du = \cos x\, dx$

$= \int \frac{1}{u^2} du$

$= \int u^{-2} du = -1u^{-1} + C$

$= -\sin x^{-1} + C$

$= \frac{-1}{\sin x} + C$

Examples: Multiply and divide by a constant

1. $\int x(x^2+1)^4 dx$ $u = x^2+1$, $\frac{1}{2}du = \cancel{2}x\,dx$

$= \frac{1}{2}\int u^4 du$

$= \frac{1}{2}\cdot\frac{1}{5}u^5 + C = \frac{1}{10}(x^2+1)^5 + C$

2. $\int \frac{x^2\sqrt{x^3+1}}{4}dx$ $u = x^3+1$, $\frac{1}{3}du = \cancel{3}x^2 dx$

$= \frac{1}{4}\int x^2\sqrt{x^3+1}\,dx$

$= \frac{1}{4}\cdot\frac{1}{3}\int \sqrt{u}\,du = \frac{1}{12}\int u^{1/2}du$

$= \frac{1}{12}\cdot\frac{2}{3}(x^3+1)^{3/2} + C$

$= \frac{1}{18}(x^3+1)^{3/2} + C$

3. $\int x^3\sin(3x^4)dx$ $u = 3x^4$, $\frac{1}{12}du = \cancel{12}x^3 dx$

$= \frac{1}{12}\int \sin u\, du$

$= -\frac{1}{12}\cos(3x^4) + C$

4. $\int \frac{3\tan 2x}{\cos^2 2x}dx$

Check for Understanding: $\int [3x + \frac{1}{x} - 3\sin(2x-1)]\,dx$ $u = 2x-1$, $\frac{du}{2} = \frac{2dx}{2}$

$= \int 3x\,dx + \int \frac{1}{x}dx - 3\int \underbrace{\sin(2x-1)}_{\sin u}dx$

$= \frac{3x^2}{2} + \ln x - 3(\frac{1}{2})(-\cos(2x-1)) + C$

$= \frac{3}{2}x^2 + \ln x + \frac{3}{2}\cos(2x-1) + C$

9.2 Integration by Substitution: Mixed Practice

Integrate each expression using an appropriate u-substitution. Show your work.

1. $\int [16 - (2x+1)^4]\, dx$

2. $\int [\sin 5t + 12t]\, dt$

3. $\int [\cos 3x + (4x-2)^2 + 10]\, dx$

4. $\int 6t \sin(t^2 - 4)\, dt$

5. $\int [3\cos 2x + 2\sin 3x]\, dx$

6. $\int [(5t+3)^3 + \cos t \sin^3 t]\, dt$

7. $\int \sin x \cos^2 x\, dx$

8. $\int \cos 2t \sin^3 2t\, dt$

9. $\int \frac{1}{(x+5)^2}\, dx$

10. $\int \frac{1}{t+5}\, dt$