

## 9.2 Applications of Integration

$$\frac{dy}{dx} = 3x^2 - 1, F(x) \text{ passes through } (2, 4)$$

$$F(x) = \int 3x^2 - 1 \, dx$$

$$F(x) = x^3 - x - 2$$

$$4 = (2)^3 - 2 + C$$

-6      :6

$$\underline{-2 = C}$$

## Kinematics

position:  $s(t)$

$s'(t) =$  velocity:  $v(t)$

$v'(t) = s''(t) =$  acceleration:  $a(t)$



A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet. Acceleration due to gravity on the earth is  $-32$  feet  $s^{-2}$ .

- a) Find the position function giving the height  $s$  as a function of time  $t$ .  
 b) When does the ball hit the ground?

a)  $a(t) = -32$

$$v(t) = \int -32 dt = -32t + 64$$

$$s(t) = \int -32t + 64 dt = -16t^2 + 64t + 80$$

b)  $s(t) = -16t^2 + 64t + 80 = 0$

$$= -16(t^2 - 4t - 5) = 0$$

$$= -16(t-5)(t+1) = 0$$

$$(t=5) \text{ or } \cancel{t}$$

1. A tree is growing so that after  $t$  years its height is increasing at a rate of  $\frac{30}{\sqrt[3]{t}}$  cm per year. It was planted as a seedling with a height of 5 cm.
  - a) Find the height of the tree after 4 years.
  - b) After how many years will the height be 4.1 meters?
  
2. A pond with surface area  $48 \text{ m}^2$  is being invaded by a weed. At a time  $t$  months after the weed first appeared, the area of the weed on the surface is increasing at a rate of  $\frac{1}{3}t \text{ m}^2$  per month. How long will it be until the weed covers the whole surface of the pond?

3. A ball is thrown vertically upward from the ground with an initial velocity of  $60 \text{ feet s}^{-1}$ . If acceleration due to gravity is  $-32 \text{ feet s}^{-2}$ , how high will the ball go?
4. Suppose that the marginal cost of manufacturing an item when  $x$  thousand items are produced is  $\frac{dc}{dx} = 10 + 3\sqrt{x}$  thousand dollars and that the cost of producing 4000 items is 60,000 dollars. What will it cost to produce 9000 items?

5. The maker of a certain automobile advertises that it takes 13 seconds to accelerate from  $7 \text{ m s}^{-1}$  to  $22 \text{ meters s}^{-1}$ . Assuming constant acceleration, compute the following:
- the acceleration in meters per second
  - the distance the car travels during the 13 seconds

6. The acceleration,  $a \text{ ms}^{-2}$ , of a particle at time  $t$  seconds is given by  $a = \frac{1}{t} + 3 \sin 2t$ , for  $t \geq 1$ . The particle is at rest when  $t = 1$ . Find the velocity of the particle when  $t = 5$ . [7]
7. The velocity  $v \text{ ms}^{-1}$  of a moving body at time  $t$  seconds is given by  $v = 50 - 10t$ .
- a) Find its acceleration in  $\text{m s}^{-2}$ . [2]
- b) The initial displacement  $s$  is 40 metres. Find an expression for  $s$  in terms of  $t$ . [4]
8. The velocity  $v$  of a particle at time  $t$  is given by  $v = e^{-2t} + 12t$ . The displacement of the particle at time  $t$  is  $s$ . Given that  $s = 2$  when  $t = 0$ , express  $s$  in terms of  $t$ . [6]



Homework Assignment:

Complete WS 1-9 all