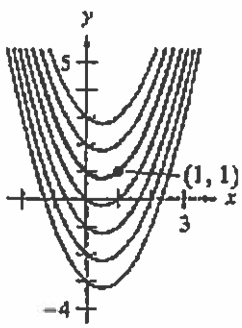
IB Math Standard

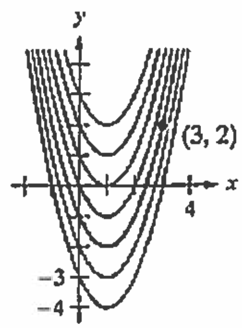
9.1 Boundary Conditions and Particular Solutions

In Exercises 45–48, find the equation for y, given the derivative and the indicated point on the curve.

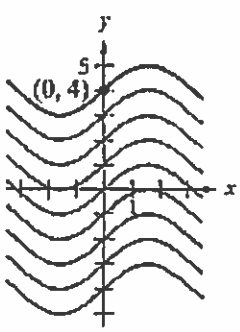
45. $\frac{dy}{dx} = 2x - 1$



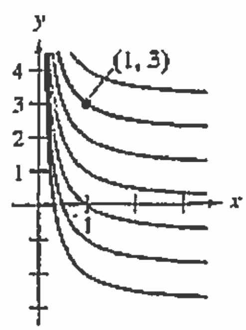
46. $\frac{dy}{dx} = 2(x - 1)$



47. $\frac{dy}{dx} = \cos x$



48. $\frac{dy}{dx} = -\frac{1}{x^2}, \quad x > 0$



49. Given that $f''(x) = 2 - \frac{2}{\sqrt{x^3}}$, $f'(1) = 0$, and $f(1) = 8$, find $f(x)$.

50. An evergreen nursery usually sells a certain shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by $\frac{dh}{dt} = 1.5t + 5$, where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when initially planted.

a) Find an equation for the height of the seedlings after t years.
b) How tall are the shrubs when they are sold?

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9.1 Boundary Conditions and Particular Solutions

1. A curve has an equation which satisfies $\frac{d^2y}{dx^2} = 6x - 4$. The point $P(2, 11)$ lies on the curve, and the gradient of the curve at the point P is 9. Determine the equation of the curve.

$\int 6x-4\,dx = 3x^2-4x+c = f'(x)$

$(2,9)$

$9 = 3(2)^2 - 4(2) + c$

$c=5$

$f'(x) = \int 3x^2-4x+5\,dx$

$f(x) = x^3-2x^2+5x+c$

$11 = (2)^3-2(2)^2+5(2)+c \quad c=1$

$\boxed{f(x) = x^3-2x^2+5x+1}$

2. The rate of growth of a population of bacteria is given by $\frac{dP}{dt} = k\sqrt{t}$, where t is the time in days, and k is a constant to be determined.

The initial size of the population is 500. After 1 day, the population has grown to 600. Use this information to determine k and C in the population function $P(t)$. Estimate the population after 7 days.

3. Find $f(x)$ if $f''(x) = \sin x$, $f'(0) = 1$, and $f(0) = 6$.

IB Math Standard

9.2 Integrals of $\frac{1}{x}$ and e^x

We have two more integral rules to examine to complete our study.

How can we find $\int \frac{1}{x}\, dx$? Let's try the integral power rule with $\int x^{-1}\, dx$...

~~$\int \frac{1}{x} dx = \ln x + C$~~ $\frac{x^{-1+1}}{0}$ not possible!

Hmmmm... That didn't work very well...

What derivative rule did we learn that is related to this? ________________

Therefore, $\int \frac{1}{x}\, dx =$ $\ln x + c$

Examples

1. $\int \frac{3}{x} dx$

$= 3\int \frac{1}{x}\, dx$

$= 3\ln x + c$

2. $\int \frac{1}{3x} dx$

$= \frac{1}{3}\int \frac{1}{x}\, dx$

$= \frac{1}{3}\ln x + c$

3. $\int \frac{3x^2+2x+1}{x} dx$

$= \int 3x + 2 + \frac{1}{x}\, dx$

$= \frac{3}{2}x^2 + 2x + \ln x + C$

If you had to guess the value of $\int e^x\, dx$, what would you guess? $\int e^x\, dx =$ $e^x + C$

Examples:

1. $\int 3e^x dx$

$= 3e^x + c$

2. $\int \frac{e^t}{3} dt$

$= \frac{1}{3}e^t + c$

3. $\int \ln(e^{2x+1}) dx$

$(2x+1)\ln e$

$= \int (2x+1)\, dx$

$= x^2 + x + C$