9.1 Antiderivatives and the Indefinite Integral

Every operation has its inverse. The inverse of differentiation is integration, or "antidifferention." When you "integrate" a function, you are doing differentiation in reverse.

If $f'(x) = 2x$, then what could the original function $f(x)$ equal?

$f(x) = x^2 + c$

Show that these statements are true:

$\int 3x\,dx = \frac{3}{2}x^2 + C$ $\qquad \int\left(-\frac{9}{x^4}\right)dx = \frac{3}{x^3} + C$ $\qquad \int -\sin x\,dx = \cos x + C$ $\qquad \int 2e^{2x}\,dx = e^{2x} + C$

$\frac{3x^{1+1}}{2}$ $\qquad 3x^{-3}$

a) $\sqrt[7]{x^3} = \int x^{3/7}\,dx = \frac{7}{10}x^{10/7} + C$

Differentiation Rules	Integration Rules
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int -\sin x\,dx = \cos x + C$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int nx^{n-1}\,dx = x^n + C$
	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ or $\frac{1}{n+1}x^{n+1} + C$

Examples

1. $\int (3x^4 - 5x^2 + x)\,dx$

$= \frac{3}{5}x^5 - \frac{5}{3}x^3 + \frac{1}{2}x^2 + C$

2. $\int (x + 2)\,dx$

$= \frac{1}{2}x^2 + 2x + C$

3. $\int 1\,dx$

$= x + C$

4. $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx$

$= -\frac{1}{2}x^{-2} + C$

5. $\int 2\sin x\,dx = 2\int \sin x\,dx$

$= -2\cos x + C$

6. $\int \sqrt{x}\,dx = \int x^{1/2}\,dx$

$= \frac{2}{3}x^{3/2} + C$

7. $\int x(x^3 + 1)\,dx = \int (x^4 + x)\,dx$

$= \frac{1}{5}x^5 + \frac{1}{2}x^2 + C$

8. $\int \frac{x+1}{\sqrt{x}}\,dx = \int \frac{x+1}{x^{1/2}}\,dx$

$= \int x^{1/2} + x^{-1/2}\,dx$

$= \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

Practice

1. $\int (13 - x)\,dx$

$= 13x - \frac{1}{2}x^2 + C$

2. $\int (8x^3 - 9x^2 + 4)\,dx$

$= 2x^4 - 3x^3 + 4x + C$

3. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$

$= \frac{2}{3}x^{3/2} + x^{1/2} + C$

4. $\int \frac{1}{x^6}\,dx$

$= \frac{1}{-5}x^{-5} + C$

$= \frac{-1}{5x^5} + C$

5. $\int \frac{x^2+2x-3}{x^4}\,dx = \int x^{-2} + 2x^{-3} - 3x^{-4}\,d$

$= -1x^{-1} + \frac{2}{-2}x^{-2} - \frac{3}{-3}x^{-3} + C$

$= -\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$

6. $\int (2t^2 - 1)\,dt$

$= \frac{2}{3}t^3 - t + C$

7. $\int (5\cos x + 4\sin x)\,dx$

$= 5\sin x - 4\cos x + C$

8. $\int (t^2 - \cos t)\,dt$

$= \frac{1}{3}t^3 - \sin t + C$