Algebra II: 8.3 Notes
Date:

Name:
Period:

Objective: Will graph rational functions with higher-degree polynomials.

KEY CONCEPT

Graphs of Rational Functions

Let $p(x)$ and $q(x)$ be polynomials with no common factors other than ± 1. The graph of the following rational function has the characteristics listed below.

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$$

1. The x-intercepts of the graph of f are the real zeros of $p(x)$.
2. The graph of f has a vertical asymptote at each real zero of $q(x)$.
3. The graph of f has at most one horizontal asymptote, which is determined by the degrees m and n of $p(x)$ and $q(x)$.

$m < n$	The line $y = 0$ is a horizontal asymptote.
$m = n$	The line $y = \frac{a_m}{b_n}$ is a horizontal asymptote.
$m > n$	The graph has no horizontal asymptote. The graph's end behavior is the same as the graph of $y = \frac{a_m}{b_n}x^{m-n}$.

① set num = 0
$p(x) = 0 \Rightarrow$ zeros / x-intercepts

② set denom = 0
$q(x) = 0 \Rightarrow$ VAsymp

exponents / highest degree

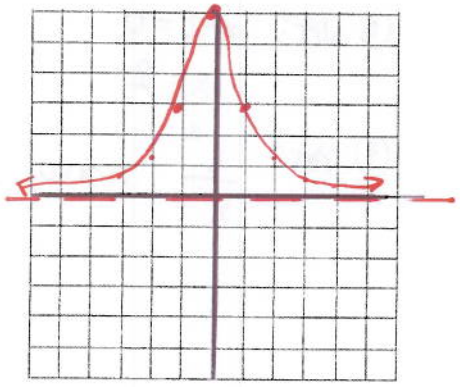
num < denom → $y = \frac{x^2+1}{x^3}$ $y = 0$

degree: num = denom → $y = \frac{4x}{x+2}$ $y = \frac{4}{1} = 4$

num > denom. → $y = \frac{3x^2-1}{x+7}$ no HA

Graph the function, identify any asymptotes, identify any zeroes, and state the domain and range.

1. $y = \frac{6}{x^2+1}$

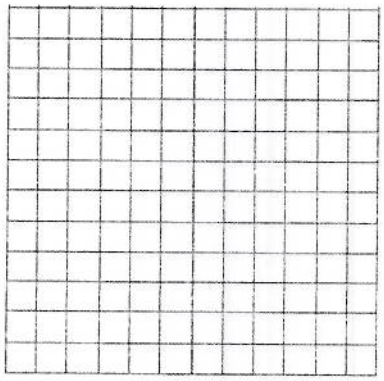


① x-int: $6 \neq 0$ none D: $\mathbb{R}$

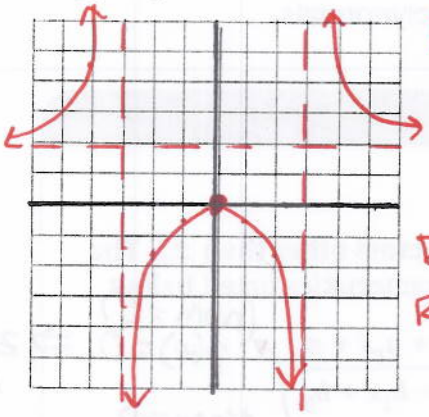
② VA: $x^2+1 \neq 0$ none R: $0 < y \leq 6$

③ HA: $\frac{x^0}{x^2}$ $y = 0$

2. $y = \frac{4}{x^2+2}$



3. $y = \frac{2x^2}{x^2 - 9}$



① $2x^2 = 0$
$x = 0$

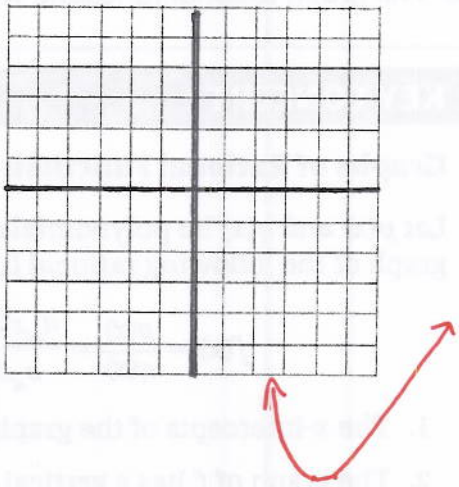
② VA: $x^2 - 9 = 0$
$x = \pm 3$

③ HA: $\frac{x^2}{x^2}$
$y = 2$

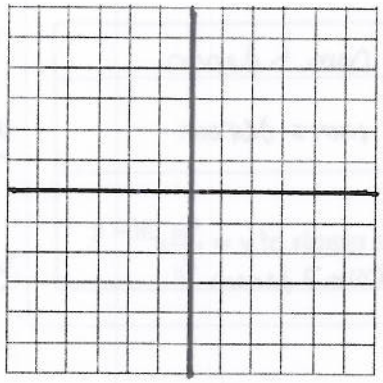
D: $\mathbb{R}, x \neq -3, 3$

R: $y \leq 0$
$y > 2$

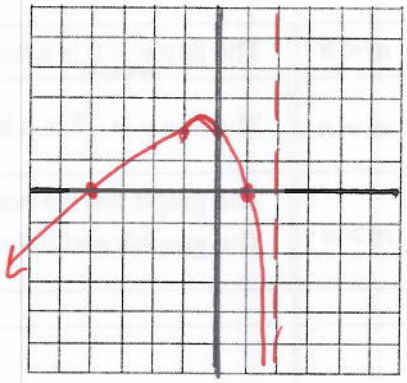
4. $y = \frac{3x^2}{x^2 - 1}$



5. $y = \frac{x^2 - 5}{x^2 + 1}$



6. $y = \frac{x^2 + 3x - 4}{x - 2}$



D: $\mathbb{R}, x \neq 2$

① x-int:
$x^2 + 3x - 4 = 0$
$(x+4)(x-1) = 0$
$x = -4, 1$

② VA:
$x - 2 = 0$
$x = 2$

③ HA: $\frac{x^2}{x}$ none

7. $y = \frac{x^2 - 2x - 3}{x - 4}$