8.2 Law of Cosines

*When do we use Sine rules???

*So... when do you think we use Cosine rules?

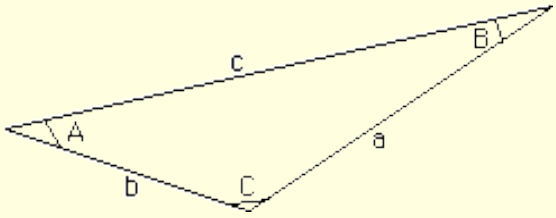
The Law of Cosines can be used to solve a triangle when all other simpler methods fail:

Right triangle: Use Pythagorean Theorem and sin, cos, or tan

A non-right triangle using an angle and a side opposite: Use the Law of Sines

Any other triangle: Use the Law of Cosines

The Law of Cosines



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

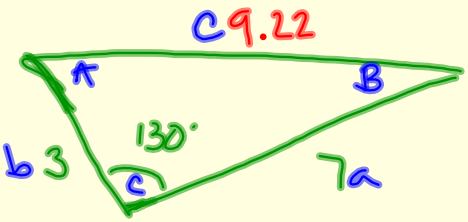
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

1. Suppose two sides of a triangle have lengths 3 cm and 7 cm, and the angle between them measures 130°. Solve the triangle.



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 7^2 + 3^2 - 2(7)(3)\cos 130°$

$c^2 = 84.997$

$c = 9.22$ cm

$\frac{\sin 130}{9.22} = \frac{\sin A}{7}$

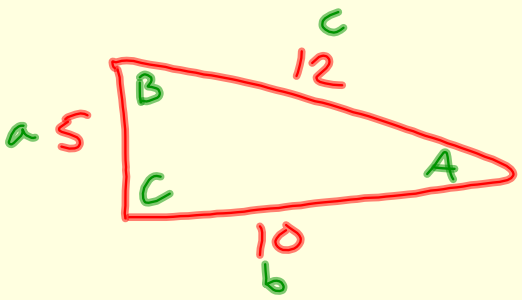
$\sin A =$

$\angle A = 35.6°$

$180 - 35.6 - 130 =$

$\angle B = 14.4°$

2. The lengths of the sides of a triangle are 5, 10, and 12. Solve the triangle.



$$\frac{12}{\sin 101} = \frac{5}{\sin A}$$

$$\sin A = 0.409$$

$\angle A = 24.1^\circ$

$\angle B = 54.9^\circ$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

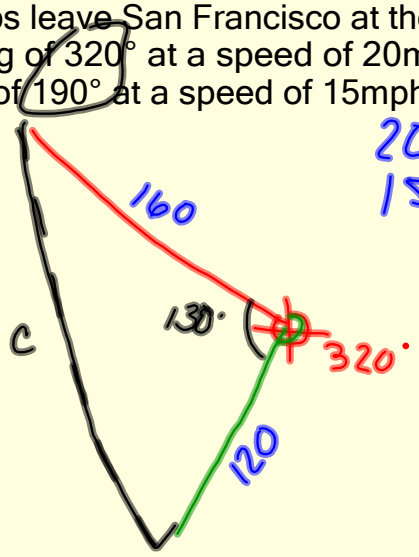
$$\cos C = \frac{(5^2+10^2-12^2)}{(2(5)(10))}$$

$$\cos C = -0.19$$

$\angle C = 101^\circ$

3. Two ships leave San Francisco at the same time. One ship travels on a bearing of 320° at a speed of 20mph. The other ship travels on a bearing of 190° at a speed of 15mph. How far apart are they after 8 hours?

$20 \cdot 8 = 160$
$15 \cdot 8 = 120$

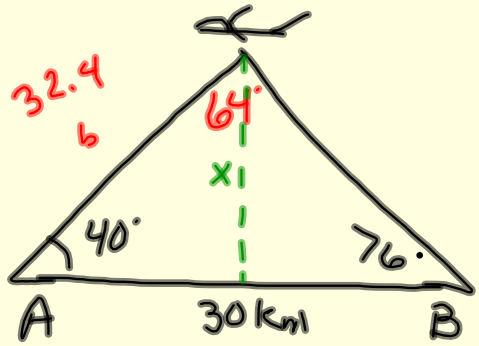


$c^2 = 120^2 + 160^2 - 2(120)(160)\cos 130$

$c^2 = 64683.044$

$c \approx 254$ miles

4. Observers on the ground at points A and B, 30 km apart, sight an airplane at angles of elevation of 40° and 76°, respectively. Find the altitude of the plane.



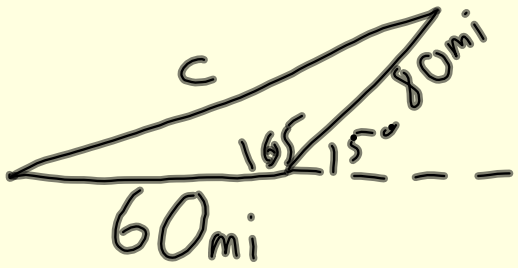
$$\frac{30}{\sin 64} = \frac{b}{\sin 76}$$

$$b = 32.4 \text{ km}$$

$$\sin 40 = \frac{x}{32.4}$$

$$x = 20.8 \text{ km}$$

5. A boat travels 60 miles due east. Then it adjusts its course 15° north and travels 80 miles. How far is the boat from its point of departure?

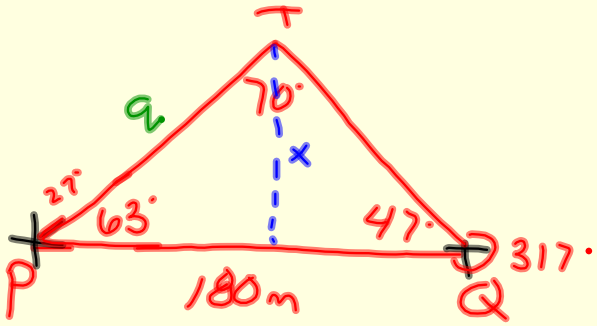


$c^2 = 60^2 + 80^2 - 2(80)(60)\cos(165)$

$c \approx 139$ miles

6. From points P and Q, 180 m apart, a tree at T is sighted on the opposite side of a deep ravine. The bearing from P to T is 27°, while the bearing from Q to T is 317°.

a) How far is P from the tree?
b) What is the shortest distance from the tree to the line PQ?



a) $\frac{\sin 70}{180} = \frac{\sin 47}{q}$

$q = 140\ m$

b) $\sin 63 = \frac{x}{140}$

$x = 125 m$

Homework Assignment
pages 525-527: 13-31 odd, 51, 65