Algebra II: 8.1 Notes
Date:

Name:
Period:

Objective: Use inverse variation and joint variation models.

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**KEY CONCEPT**

**Inverse Variation**

Two variables $x$ and $y$ show **inverse variation** if they are related as follows:

$$y = \frac{a}{x}, a \neq 0$$

The constant $a$ is the **constant of variation**, and $y$ is said to *vary inversely* with $x$.

Direct: $a = \frac{x}{y}$, $y = ax$

$a = xy$

$y = \frac{x}{2}$, $y = \frac{1}{2}x$, direct

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Tell whether x and y show a direct variation, inverse variation, or neither.

1. $xy = 7$
   $y = \frac{7}{x}$
   inverse

2. $y = x + 3$
   neither

3. $\frac{y}{4} = x$
   $y = 4x$
   direct

4. $3x = y$
   direct

5. $xy = 0.75$
   $y = \frac{0.75}{x}$
   inverse

6. $y = x - 5$
   neither

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when x = 2.

$y = \frac{a}{x}$ $a = xy$

7. $x = 4, y = 7$
   $a = (4)(7) = 28$
   $\therefore y = \frac{28}{x}$
   if $x=2$: $y = \frac{28}{2}$, $y = 14$

8. $x = 4, y = 3$
   $a = 12$
   $\therefore y = \frac{12}{x}$
   if $x=2$, $y = 6$

9. $x = 8, y = -1$
   $a = -8$
   $\therefore y = \frac{-8}{x}$
   if $x=2$: $y = -4$

10. $x = \frac{1}{2}, y = 12$
    $a = (\frac{1}{2})(12) = 6$
    $\therefore y = \frac{6}{x}$
    if $x=2$, $y = \frac{6}{2} = 3$

**KEY CONCEPT** *For Your Notebook*

**Joint Variation**

**Joint variation** occurs when a quantity varies directly with *the product of two or more* other quantities. In the equations below, $a$ is a nonzero constant.

$z = axy$ — $z$ varies jointly with $x$ and $y$. → $a = \frac{z}{xy}$

$p = aqrs$ — $p$ varies jointly with $q$, $r$, and $s$.

The variable z varies jointly with x and y. Use the given values to write an equation relating x, y, and z. Then find z when x = -2 and y = 5.

11. x = 3, y = -5, z = -75

$a = \frac{-75}{3(-5)} = 5$

$\therefore z = 5xy$

if $x = -2$ + $y = 5$:

$z = 5(-2)(5)$

$z = -50$

12. x = 1, y = 2, z = 7

$a = \frac{7}{1(2)} = \frac{7}{2}$

$\therefore z = \frac{7}{2}xy$

if $x = -2$, $y = 5$:

$z = \frac{7}{2}(-2)(5)$

$z = -35$

13. x = 4, y = -3, z = 24

$a = \frac{24}{4(-3)} = -2$

$\therefore z = -2xy$

if $x = -2$, $y = 5$:

$z = -2(-2)(5)$

$z = 20$

14. x = -2, y = 6, z = 18

$a = \frac{18}{-2(6)} = \frac{18}{-12} = -\frac{3}{2}$

$\therefore z = -\frac{3}{2}xy$

if $x = -2$, $y = 5$:

$z = -\frac{3}{2}(-2)(5)$

$z = 15$

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Write an equation for the given relationship.

15. y varies inversely with x

$y = \frac{a}{x}$

16. z varies jointly with x, y, and r

$z = axyr$

17. y varies inversely with the square of x

$y = \frac{a}{x^2}$

18. z varies directly with y and inversely with x

$z = \frac{ay}{x}$